PHYS 306: HOMEWORK ASSIGNMENT No. 3: DAMPED OSCILLATORS

(Jan. 30th, 2017)

HOMEWORK DUE: MONDAY, FEB 6th, 2017

To be handed in during class- Late Homework will not be accepted

QUESTION (1): Here we look at a single damped oscillator, made from a mass on a spring. Suppose it has a mass M_o, a friction coefficient η, and a spring constant k_o. For all times $t < 0$, we will suppose that the oscillator is at rest.

1(a) Suppose that we apply a force $F(t) = F_o\,\theta(t)$ to the oscillator, where $\theta(t)$ is the Heaviside step function. What will be the motion of the oscillator coordinate for all times $t > 0$?

1(b) Calculate from your solution just given, what is (i) the kinetic energy $T(t)$ of the mass M_o as a function of time, (ii) the potential energy $V(t)$ of the spring as a function of time, and (iii) the total energy $E(t)$ as a function of time. Draw graphs of these.

NB:

QUESTION (2) There are various ways of interpreting the Q-factor of a damped oscillator. Here we consider a damped oscillator of natural frequency ω_o, being driven by a force $F_o \cos\omega t$, once all transient dynamics has ceased, and we are left with the oscillations at a constant amplitude $A_o(\omega)$. We assume that $\gamma \ll \omega_o$, so that $Q \gg 1$.

(i) Suppose we are on resonance, so that $\omega = \omega_o$. Show that the total energy (kinetic plus potential) of the oscillator is $E = M_o(\omega A_o)^2/2$.

(ii) Show that the energy dissipated per cycle is $W = 2\pi M_o \gamma \omega A_o^2$, and that the Q-factor is then equal to $Q = 2\pi E/W$.

END of 2ND HOMEWORK ASSIGNMENT