PHYS 306: HOMEWORK ASSIGNMENT No. 2: OSCILLATORS

(Jan. 19th, 2017)

HOMEWORK DUE: MONDAY, JAN 30th, 2017

To be handed in during class- Late Homework will not be accepted

QUESTION (1): We begin with a problem of 3 masses coupled by springs, ie., a problem of coupled oscillators, with no dissipation.

1(a) Consider a set of 3 equal masses of mass M, confined to move without friction on a circle of radius R_o; we ignore gravity. The masses are coupled by springs with identical spring constants k_o. Draw a picture of the system, and write down the Lagrangian for it. By minimizing the potential energy, show that the configuration of minimum energy has the masses equally spaced around the circle. Then find the equations of motion for each of the 3 masses, using Lagrange's equations, as functions of the displacement of the masses from their equilibrium positions.

1(b) We now wish to solve these equations of motion. First find the 3 eigenfrequencies of the system, by solving the relevant characteristic equation. Then find the solution to the eigenvalue equations, assuming the initial conditions at $t = 0$ for the masses are that (i) all three of them are stationary, and (ii) that two of the masses are together on one side of the circle, and the third is exactly on the opposite side from the other two, ie., it is displaced by 180^o around the circle from the first two.

NB: It is useful to think physically about this problem, noting the inherent symmetries in it and imagining intuitively how the motion will go.

QUESTION (2) The following problem is partly extracted from last year's exam - it concerns the extraction of energy from waves and tides.

(i) To extract power from water waves, let us take a large mass M, and attach it to the marine bottom by a spring with spring constant k. At low tide the water depth is z_1, at high tide it is z_2; we will assume that the mass is kept floating on the water surface using air floats.

What are the MKS units of k and M? Suppose the spring constant is $k = 5 \times 10^3$ in these units, and that in the absence of waves, $z_1 = 2\ m$ and $z_2 = 8\ m$. Then (a) what is the downward force on the mass from the spring at high and low tides, and (b) what is the energy stored in the spring at high and low tides? Suppose that $M = 2 \times 10^4$ (in MKS units). What is the natural frequency ω_o of oscillation of the float (give the correct units here)?

Suppose that at high tide, we have time-dependent waves of form $\Delta z(t) = A \cos \omega t$ on the water, giving a float height $z(t) = z_2 + \Delta z(t)$. Assuming that we can extract all the kinetic energy from the float motion, what would be the mean power (averaged over time) that we would get from this? If $A = 2\ m$, how much would this be? Again, give the correct units.

END of 2ND HOMEWORK ASSIGNMENT