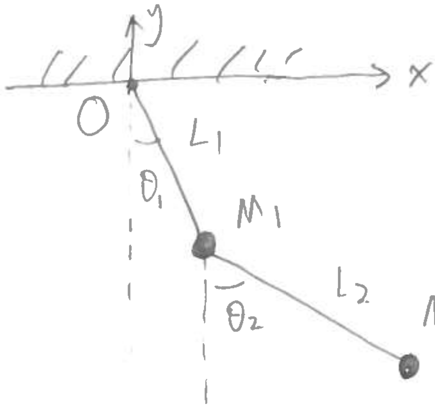
P306 : HW1, MODEL ANSWER

Question 1



1 (a) double Pendulum

Set O as the origin of the coordinate, x, y directions as left.

The position of M_1 is (x_1, y_1)

$$x_1 = L_1 \sin\theta_1$$

$$y_1 = -L_1 \cos\theta_1$$

The position of M_2 is (x_2, y_2)

$$x_2 = L_2 \sin\theta_2 + L_1 \sin\theta_1$$

$$y_1 = -L_1 \cos\theta_1 - L_2 \cos\theta_2$$

The Kinetic energy. T.

$$T = \frac{1}{2} m_1 (\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2} m_2 (\dot{x}_2^2 + \dot{y}_2^2)$$

$$= \frac{1}{2} m_1 \left(L_1^2 \cos^2\theta_1 \dot{\theta}_1^2 + L_1^2 \sin^2\theta_2 \dot{\theta}_1^2 \right)$$

$$+ \frac{1}{2} m_2 \Big((L_2 \cos\theta_2 \dot{\theta}_2 + L_1 \cos\theta_1 \dot{\theta}_1)^2$$

$$+ (L_1 \sin\theta_1 \dot{\theta}_1 + L_2 \sin\theta_2 \dot{\theta}_2)^2 \Big)$$

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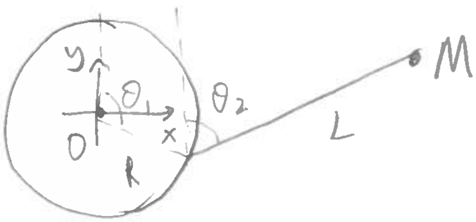
$$T = \frac{1}{2} m_1 L_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 \Big(L_1^2 \dot{\theta}_1^2 + L_2^2 \dot{\theta}_2^2$$
$$+ 2 L_1 L_2 \dot{\theta}_1 \dot{\theta}_2 \left(\cos\theta_2 \cos\theta_1 + \sin\theta_1 \sin\theta_2 \right) \Big)$$

$$= \frac{1}{2} m_1 L_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 \Big(L_1^2 \dot{\theta}_1^2 + L_2^2 \dot{\theta}_2^2$$
$$+ 2 L_1 L_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_2 - \theta_1) \Big)$$

$$V = m_1 g y_1 + m_2 g y_2$$
$$= -m_1 g L_1 \cos\theta_1 - m_2 g (L_1 \cos\theta_1 + L_2 \cos\theta_2)$$
$$L = T - V$$

1 (b) Set the origin at the center of the circle and x, y directions as left.



The position of M is

$$x = L \sin\theta_2 + R \sin\theta_1$$
$$y = R \cos\theta_1 + L \cos\theta_2$$

$$T = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2)$$
$$= \frac{1}{2} m \Big((L \cos\theta_2 \dot{\theta}_2 + R \cos\theta_1 \dot{\theta}_1)^2$$
$$+ (R \sin\theta_1 \dot{\theta}_1 + L \sin\theta_2 \dot{\theta}_2)^2 \Big)$$

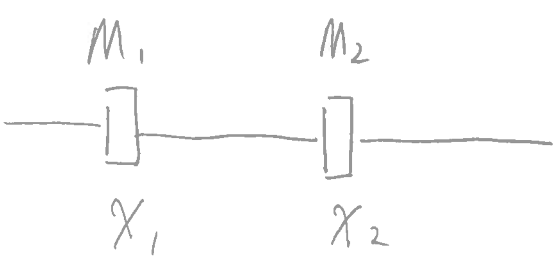
$$= \frac{1}{2} m \left(L^2 \dot{\theta}_2^2 + R^2 \dot{\theta}_1^2 + 2RL\dot{\theta}_1\dot{\theta}_2 \cos(\theta_2 - \theta_1) \right)$$

$$V = mgy$$

$$= mg(R\cos\theta_1 + L\cos\theta_2).$$

$$L = T - V$$

Question 2



The kinetic energy is

$$T = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2$$

$$V = V(x) = V(|x_2 - x_1|)$$

so $L = T - V$

$$= \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 - V(|x_2 - x_1|)$$

The EOM is

$$\frac{\partial L}{\partial x_1} = \frac{d}{dt} \frac{\partial L}{\partial \dot{x}_1}$$

$$\frac{\partial L}{\partial x_2} = \frac{d}{dt} \frac{\partial L}{\partial \dot{x}_2}$$

$$\Rightarrow \frac{\partial V(x)}{\partial x_1} = m_1 \ddot{x}_1$$

$$\frac{\partial V(x)}{\partial x_2} = m_2 \ddot{x}_2$$

here $$\frac{\partial V(x)}{\partial X_1} = \frac{\partial V\left(|x_2 - x_1|\right)}{\partial X_1}$$

$$= \frac{\partial V(x)}{\partial x} \cdot sgn(x_1 - x_2)$$

$$\frac{\partial V(x)}{\partial X_2} = \frac{\partial V\left(|x_2 - x_1|\right)}{\partial X_2}$$

$$= \frac{\partial V(x)}{\partial x} \cdot sgn(x_2 - x_1)$$

$$= - \frac{\partial V(x)}{\partial x_1}$$