PHYS 306: HOMEWORK ASSIGNMENT No. 4: CENTRAL FIELD MOTION - RADIAL DYNAMICS

(Feb. 12th, 2016)

HOMEWORK DUE: MONDAY, FEB 22nd, 2016
To be handed in during class- Late Homework will not be accepted

QUESTION (1) 2-d PENDULUM: We consider a problem of a 2-d pendulum, in which a pendulum of length l, with a mass M on the end, moves in both the z- and ϕ coordinates (vertical displacement and azimuthal angle).

1(a) Find the Lagrangian $\mathcal{L}$ of this pendulum, and its equation of motion, as a function of the polar angle θ and the azimuthal angle ϕ (where $z = -l \cos\theta$).

1(b) Write the energy of the system as a function of θ and of the conserved angular momentum L of the system around the $\hat{z}$ axis. Hence find the implicit equations for the solutions to the equations of motion, written as integrals over θ.

1(c) Finally, find an algebraic equation, written in terms of $\cos\theta$, for the minimum and maximum values that θ can take, for a given value of L.

QUESTION (2) RADIAL OSCILLATIONS : using the radial eqtn. of motion, we can derive an equation for small oscillations of the system when the orbit deviates a little bit from circular (actually we can do it for small oscillations around any closed orbit, but this is a little harder). We do this here for 2 different potentials

2(a) Consider a central field with a Newtonian potential, such that $V(r) = -V_o/r$. Find first of all, when the system has a given angular momentum L, what is the value r_o of the radius for which the orbit is circular; and determine also the orbital period of this orbit (ie., for the angular motion).

Then make a Taylor expansion of the radial potential up to 2nd order in the deviations of r from r_o, and use this to find the frequency of harmonic oscillations of r around r_o. How does this frequency compare with the frequency of orbital motion when $r = r_o$?

2(b) Now do exactly the same for the $2-d$ harmonic potential, where $V(r) = kr^2/2$; ie., first find r_o for circular orbits, then find the period of revolution around the z axis, and finally find the frequency of small oscillations around r_o.

END of 4th HOMEWORK ASSIGNMENT