PHYS 306: HOMEWORK ASSIGNMENT No. 3: NORMAL MODES, FRICTION and RESONANCE

(Jan. 30th, 2016)

HOMEWORK DUE: WEDNESDAY, FEB 10th, 2016
To be handed in during class- Late Homework will not be accepted

QUESTION (1): NORMAL MODES FOR LINEAR MOLECULE We consider a molecule which resembles several simple molecules in Nature. A mass M_1 in the centre of a molecule is coupled identically, by 2 springs with spring constant k, to 2 identical masses of mass M_2 on either side of it; and we will for the purposes of this question, ignore all but linear motion along the axis of the molecule (see Figure).

1(a) Write down the Lagrangian for this system, and the equations of motion; and then find the eigenfrequencies of the system - there are three of them.

1(b) Find the 3 normal modes of the system (Hint: think about this physically - imagine what each mode should look like - this will help you with the math manipulations).

1(c) now take exactly the same problem as above, but now add some friction in the dynamics of the springs. We do this by adding into the equation of motion frictional force terms with a coefficient η for each of the 2 springs.

Find the new equations of motion for the system; and, without solving these equations, give an argument to show why although the motion of 2 of the normal modes is changed by the friction, nevertheless all 3 modes still remain decoupled from each other.

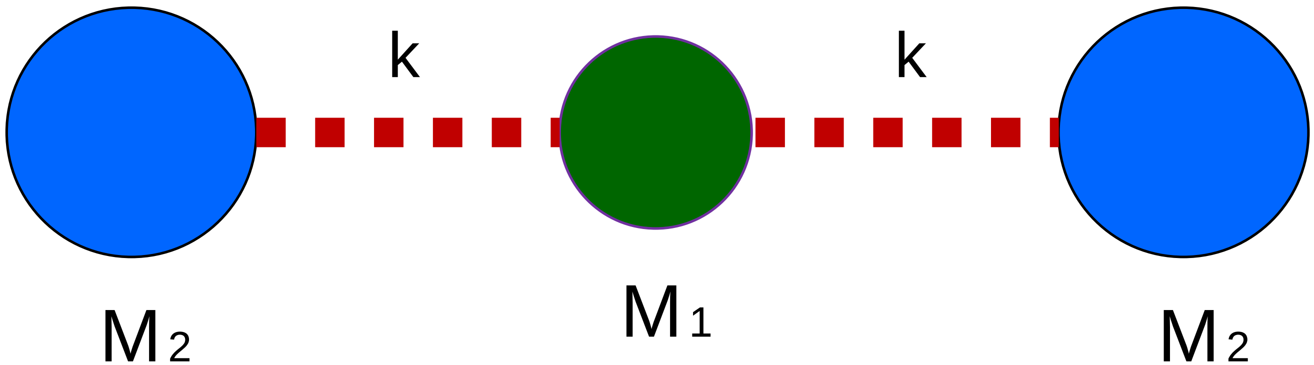
QUESTION (2): DRIVEN DAMPED OSCILLATOR Consider a damped oscillator of mass M, with natural frequency ω_o, and damping coefficient $\gamma = \eta/M$ (where η is the friction coefficient). We now apply an external force of form $F(t) = \theta(t)\, F_o e^{-At} \cos\omega t$, where $A > 0$, and $\theta(t)$ is the Heaviside step function.

Find the equation of motion and its solution as a function of time, using Fourier transforms.

NB: We could find the solution for the more general driving force $F(t) = F_o e^{At} \cos\omega t$, where A is of either sign, by summing a general solution of the homogeneous eqtn. and a particular solution of the inhomogeneous equation; but the point of this question is to use Fourier transformation.

NB: The Figure is on the following page

END of 3rd HOMEWORK ASSIGNMENT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