PHYS 306: HOMEWORK ASSIGNMENT No. 2: TWO COUPLED OSCILLATORS

(Jan. 18th, 2016)

HOMEWORK DUE: MONDAY, JAN 25th, 2016

To be handed in during class- Late Homework will not be accepted

QUESTION (1): COUPLED SPRINGS We consider a system of "coupled springs", in which 2 masses M_1 and M_2 are free to move in a vertical plane, and hang from springs with spring constants k_1 and k_2 respectively (see Fig. 1); they are also connected by a 3rd spring of spring constant k_3.

1(a) Find the Lagrangian for this system, using coordinates of your own choosing, and show how it can be reduced to the form of a pair of simple harmonic oscillators with a simple bilinear coupling between these oscillators.

1(b) Solve for the 2 eigenfrequencies of the system, and plot the way these 2 eigenfrequencies vary as a function of the coupling strength between the oscillators (noting as above that k_1, k_2, M_1, and M_2 are arbitrary.

QUESTION (2): 2-D OSCILLATOR We consider a 2-dimensional oscillator, with coordinates q_1 and q_2, and with a Lagrangian given by

$$L(q_1, q_2; \dot{q}_1, \dot{q}_2) \;=\; \frac{1}{2}\left[(\dot{q}_1^2 + \dot{q}_2^2) - (\omega_1^2 q_1^2 + \omega_2^2 q_2^2) - g q_1 q_2\right] \tag{0.1}$$

2(a) Find the equations of motion of the 2 coordinates, and solve for the 2 eigenfrequencies of the system.

2(b) Now plot a contour map of the potential in which this 2-oscillator moves; and also plot a graph of the way in which the 2 eigenfrequencies vary as a function of the coupling strength g between the 2 coordinates.

NB: The Figure is on the following page

END of 2ND HOMEWORK ASSIGNMENT

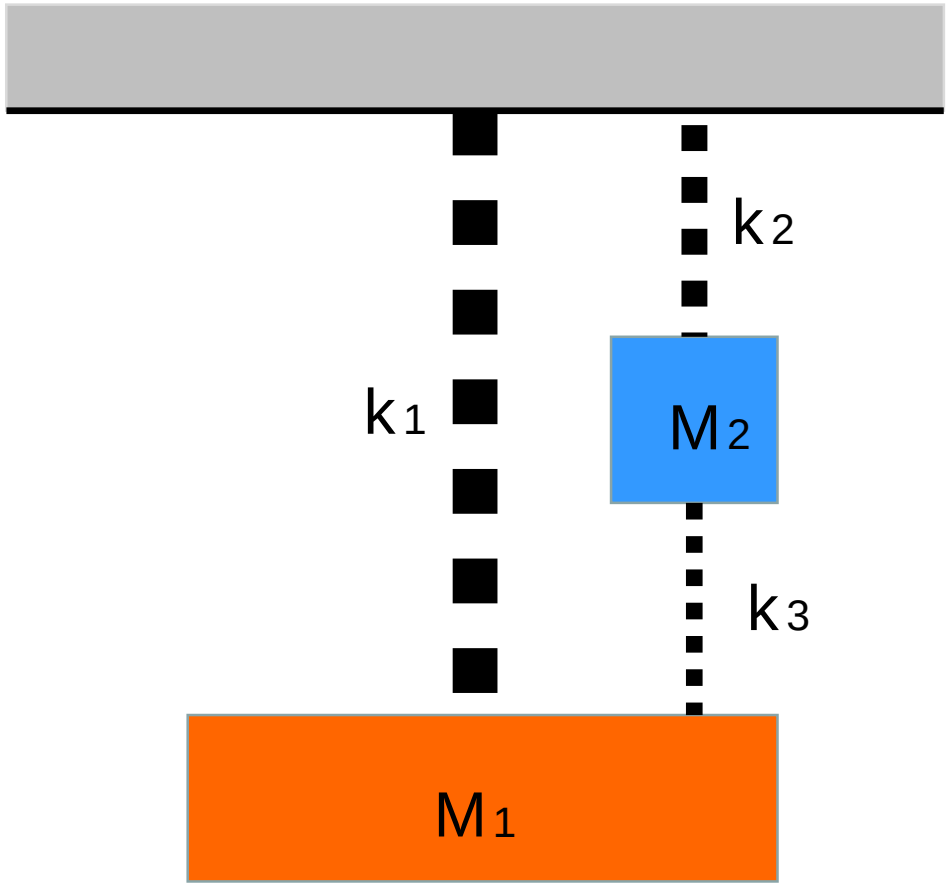

Fig. 1