PHYS 306: HOMEWORK ASSIGNMENT No. 6: RIGID BODY ROTATION

(March 22nd, 2016)

HOMEWORK DUE: Friday, APRIL 1ST, 2016
To be handed in during class- Late Homework will not be accepted

QUESTION (1) MOMENTS OF INERTIA: This is just to make sure you all know how to calculate moments of inertia.

1(a) Consider a hollow spherical shell with internal radius a and external radius b, with the shall material having mass density ρ. Calculate the 3 moments of inertia about the centre of mass of this system, writing them in terms of the total mass.

1(b) Now consider a solid ellipsoidal system of mass density ρ, with perpendicular axes a, b, and c, such that the outer surface of the ellipsoid obeys the equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \tag{0.1}$$

Find the moment of inertia tensor, defined in the Cartesian coordinate system, again in terms of the total mass.

1(c) Suppose we took the last ellipsoidal problem, and carved out a hollow sphere of radius R_o (with $R_o < a, b, c$) in the solid ellipsoid. What then is the moment of inertia tensor, in terms of the total mass?

QUESTION (2) USE OF EULER EQUATIONS: Let's consider a solid body with moments of inertia I_1, I_2, I_3 along its principal axis directions $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$. We assume that at $t = 0$ it is rotating with angular velocity $\mathbf{\Omega}$ having components $(\Omega_1, \Omega_2, \Omega_3)$ along these axes. Let's assume that the direction of $\mathbf{\Omega}$ is very close to that of $\mathbf{e}_3$, so that $\Omega_1, \Omega_2 \ll \Omega_3(t)$.

2(a) Let's write $\Omega_3(t) = \Omega_o(t) + \omega_3(t)$, and $\Omega_1(t) = \omega_1(t), \Omega_2(t) = \omega_2(t)$, such that $\omega_\alpha(t) \ll \Omega_o(t)$, for $\alpha = 1, 2, 3$, so that you have a small dimensionless parameter in the problem. Find, to lowest non-trivial approximation in this small parameter, the oscillatory solution for the dynamics of $\mathbf{\Omega}(t)$, assuming that at $t = 0$, $\omega_2(t = 0) = 0$, and $\omega_1(t = 0) = \bar{\omega_1}$, some constant.

2(b) Now let us go to the next lowest order in your small parameter. The result for $\mathbf{\Omega}(t)$ is rather messy. However you can certainly find the next order correction to $\Omega_3(t)$, since it is given by a simple integral over the solution you have found for the lowest-order harmonic motion. Give the explicit solution for $\Omega_3(t)$ to this next order.

2(c) What is the magnitude and direction of the angular momentum $\mathbf{L}$ for this system?

END of 7th HOMEWORK ASSIGNMENT

NB: This will be the last homework assignment for this course