410 Tutorial 6

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1 Critical Point of Percolation in 2D

In physics and materials science, percolation refers to the movement of fluids through porous materials. Although it was originally developed to model the motion of a fluid through a porous material, it was later discovered that the model displayed many interesting features including critical behaviour and phase transitions. In particular, we will be interested in the application to modeling the transition between a solution and gelatine. Gelatine starts off as a solution containing many small monomers (small chains of molecules). As the solution is cooled, the monomers join together into polymers and eventually, one molecule becomes large enough to spain between the sides of the container and the solution transitions into a gel. At this point, the shear modulus (how the fluid resists shear forces) increases suddenly from zero to a finite value indicating a change in phase.

We can model the percolation process in a simple manner. Consider an n by n grid where each cell is occupied with probability p. The sol-gel transition corresponds to the value of p at which, for an infinite grid, there is a cluster which spans the grid from top to bottom with probability 1. In practice, since we cannot use infinite grids, this transition will be smoothed out and we will take the transition point to be the point which has a probability 0.5 of exhibiting a spanning cluster.

As programming such a simulation is more work than what I would expect you to be able to accomplish in a few hours, I have written a python program which models this process and displays the results. The purpose of the first part of this activity is as follows:

- Find the value of p which spans the cluster with probability 0.5 for a sufficiently large grid to within two significant figures.
- You may find it useful to start with a small grid to find an approximate value of p and use successfully larger grids to hone in on it.

What is particularly interesting about percolation is that at the critical point, the shortest path between the top and bottom is a fractal. That is to say that if the size of the bounding box is doubled, the length of the shortest path more than doubles. you can describe the length of this path by, l $= \alpha n^d$

where α is some constant and $1 < d < 2$ is the fractal dimension of the path. Try to find an estimate of d for this system.

2 Boundary Conditions

Once you have found the critical value and the approximate scaling exponent, modify the code such that the path can wind through the sides of the domain (e.g. loop the grid into a cylinder). Does this effect the critical value?

3 Scaling of Clusters

By setting cluster to 1, the script will use Hoshen–Kopelman algorithm to cluster all of the different regions together. make a plot of the number of clusters versus the size of the clusters. What do you notice as the system approaches the critical value? This behaviour is characteristic of systems near a critical point as is related to the correlation length of the system.