An increase δA that corresponds to the flux increase $\delta\varphi$ is equivalent to a displacement of an extended state by $\delta A/B$ in the y direction. By the Stokes theorem and the definition of the vector potential we have $\delta\varphi = L_x \delta A$. Thus $\delta\varphi$ causes a motion of the entire electron gas in the y direction.

By $\delta U = NeV_H$ and $\delta\varphi = hc/e$, we have

$$I = c(\Delta U/\Delta\varphi) = cNe^2V_H/hc = (Ne^2/h)V_H \ , \tag{25}$$

so that the Hall resistance is

$$\rho_H = V_H/I = h/Ne^2 \ , \tag{26}$$

as in (22).

Fractional Quantized Hall Effect (FQHE). A quantized Hall effect has been reported for similar systems at fractional values of the index s, by working at lower temperatures and higher magnetic fields. In the extreme quantum limit the lowest Landau level is only partially occupied, and the integral QHE treated above should not occur. It has been observed,[4] however, that the Hall resistance ρ_H is quantized in units of $3h/e^2$ when the occupation of the lowest Landau level is 1/3 and 2/3, and ρ_{xx} vanishes for these occupations. Similar breaks have been reported for occupations of 2/5, 3/5, 4/5, and 2/7.

p-n JUNCTIONS

A *p-n* junction is made from a single crystal modified in two separate regions. Acceptor impurity atoms are incorporated into one part to produce the *p* region in which the majority carriers are holes. Donor impurity atoms in the other part produce the *n* region in which the majority carriers are electrons. The interface region may be less than 10^{-4} cm thick. Away from the junction region on the *p* side there are (−) ionized acceptor impurity atoms and an equal concentration of free holes. On the *n* side there are (+) ionized donor atoms and an equal concentration of free electrons. Thus the majority carriers are holes on the *p* side and electrons on the *n* side, Fig. 13.

Holes concentrated on the *p* side would like to diffuse to fill the crystal uniformly. Electrons would like to diffuse from the *n* side. But diffusion will upset the local electrical neutrality of the system.

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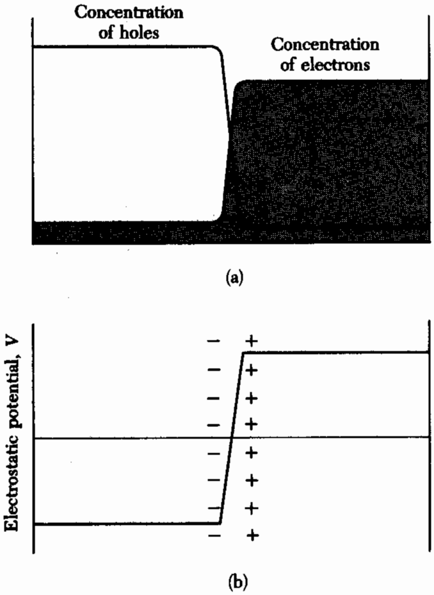
[4] D. C. Tsui, H. L. Stormer, and A. C. Gossard, Phys. Rev. Lett. 48, 1562 (1982); A. M. Chang et al., Phys. Rev. Lett. 53, 997 (1984). For a discussion of the theory see R. Laughlin in G. Bauer et al., eds., *Two-dimensional systems, heterostructures, and superlattices,* Springer, 1984.



(a)

(b)

Figure 13 (a) Variation of the hole and electron concentrations across an unbiased (zero applied voltage) junction. The carriers are in thermal equilibrium with the acceptor and donor impurity atoms, so that the product pn of the hole and electron concentrations is constant throughout the crystal in conformity with the law of mass action. (b) Electrostatic potential from acceptor (−) and donor (+) ions near the junction. The potential gradient inhibits diffusion of holes from the p side to the n side, and it inhibits diffusion of electrons from the n side to the p side. The electric field in the junction region is called the built-in electric field.

A small charge transfer by diffusion leaves behind on the p side an excess of (−) ionized acceptors and on the n side an excess of (+) ionized donors. This charge double layer creates an electric field directed from n to p that inhibits diffusion and thereby maintains the separation of the two carrier types. Because of this double layer the electrostatic potential in the crystal takes a jump in passing through the region of the junction.

In thermal equilibrium the chemical potential of each carrier type is everywhere constant in the crystal, even across the junction. For holes

$$k_BT \ln p(\mathbf{r}) + e\varphi(\mathbf{r}) = \text{constant} \ , \tag{27a}$$

where p is the hole concentration and φ the electrostatic potential. Thus p is low where φ is high. For electrons

$$k_BT \ln n(\mathbf{r}) - e\varphi(\mathbf{r}) = \text{constant} \ , \tag{27b}$$

and n will be low where φ is low.

The total chemical potential is constant across the crystal. The effect of the concentration gradient exactly cancels the electrostatic potential, and the net particle flow of each carrier type is zero. However, even in thermal equilibrium there is a small flow of electrons from n to p where the electrons end their lives by recombination with holes. The recombination current J_{nr} is balanced by a current J_{ng} of electrons which are generated thermally in the p region and which are pushed by the built-in field to the n region. Thus in zero external applied electric field

$$J_{nr}(0) + J_{ng}(0) = 0 \ , \tag{28}$$

for otherwise electrons would accumulate indefinitely on one side of the barrier.

Rectification

A *p-n* junction can act as a rectifier. A large current will flow if we apply a voltage across the junction in one direction, but if the voltage is in the opposite direction only a very small current will flow. If an alternating voltage is applied across the junction the current will flow chiefly in one direction—the junction has rectified the current (Fig. 14).

For back voltage bias a negative voltage is applied to the *p* region and a positive voltage to the *n* region, thereby increasing the potential difference between the two regions. Now practically no electrons can climb the potential energy hill from the low side of the barrier to the high side. The recombination current is reduced by the Boltzmann factor:

$$J_{nr}(V \text{ back}) = J_{nr}(0) \exp(-e|V|/k_BT) \ . \tag{29}$$

The Boltzmann factor controls the number of electrons with enough energy to get over the barrier.

The thermal generation current of electrons is not particularly affected by the back voltage because the generation electrons flow downhill (from *p* to *n*) anyway:

$$J_{ng}(V \text{ back}) = J_{ng}(0) \ . \tag{30}$$

We saw in (28) that $J_{nr}(0) = -J_{ng}(0)$; thus the generation current dominates the recombination current for a back bias.

When a forward voltage is applied, the recombination current increases because the potential energy barrier is lowered, thereby enabling more electrons to flow from the *n* side to the *p* side:

$$J_{nr}(V \text{ forward}) = J_{nr}(0) \exp(e|V|/k_BT) \ . \tag{31}$$

Again the generation current is unchanged:

$$J_{ng}(V \text{ forward}) = J_{ng}(0) \ . \tag{32}$$

The hole current flowing across the junction behaves similarly to the electron current. The applied voltage which lowers the height of the barrier for electrons also lowers it for holes, so that large numbers of electrons flow from the *n* region under the same voltage conditions that produce large hole currents in the opposite direction.

The electric currents of holes and electrons are additive, so that the total forward electric current is

$$I = I_s[\exp(eV/k_BT) - 1] \ , \tag{33}$$

where I_s is the sum of the two generation currents. This equation is well satis-

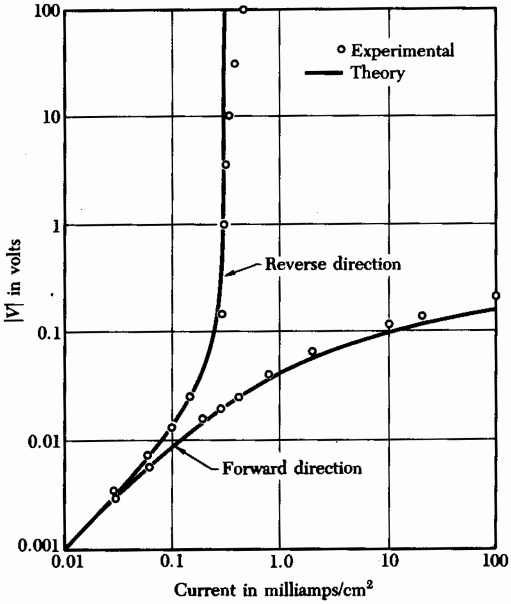


Figure 14 Rectification characteristic of a *p-n* junction in germanium, after Shockley. The voltage is plotted vertically and the current horizontally.

fied for *p-n* junctions in germanium (Fig. 14), but not quite as well in other semiconductors.

Solar Cells and Photovoltaic Detectors

Let us shine light on a *p-n* junction, one without an external bias voltage. Each absorbed photon creates an electron and a hole. When these carriers diffuse to the junction, the built-in electric field of the junction separates them at the energy barrier. The separation of the carriers produces a forward voltage across the barrier: forward, because the electric field of the photoexcited carriers is opposite to the built-in field of the junction.

The appearance of a forward voltage across an illuminated junction is called the **photovoltaic effect**. An illuminated junction can deliver power to an external circuit. Large area *p-n* junctions of silicon are used to convert solar photons to electrical energy.

Schottky Barrier

When a semiconductor is brought into contact with a metal, there is formed in the semiconductor a barrier layer from which charge carriers are severely depleted. The barrier layer is also called a depletion layer or exhaustion layer.

In Fig. 15 an *n*-type semiconductor is brought into contact with a metal. The Fermi levels are coincident after the transfer of electrons to the conduction band of the metal. Positively charged donor ions are left behind in this region