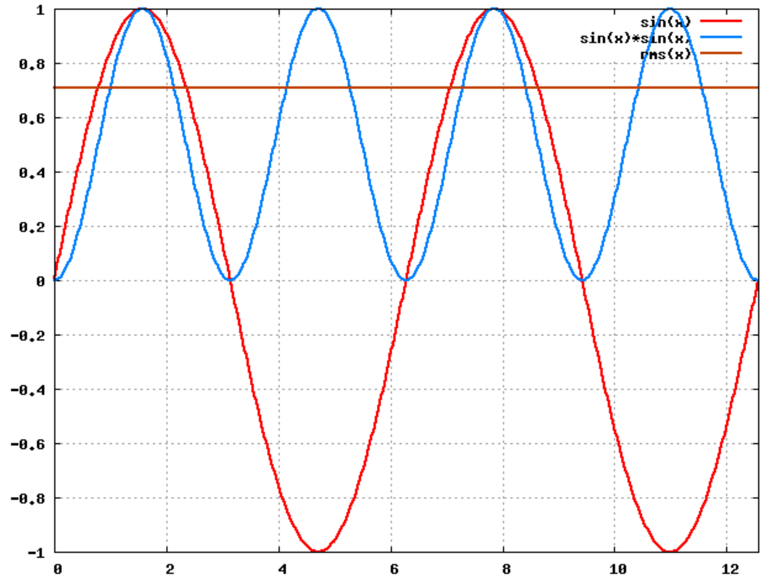
RMS – Root Mean Square

- For non-DC quantities need a measure of the voltage, current etc that captures the essence of the signal
- For repeating signals we often use a particular version of the "average" value of the signal – the RMS or Root Mean Square

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T (V(t))^2\,dt},$$

where T = period



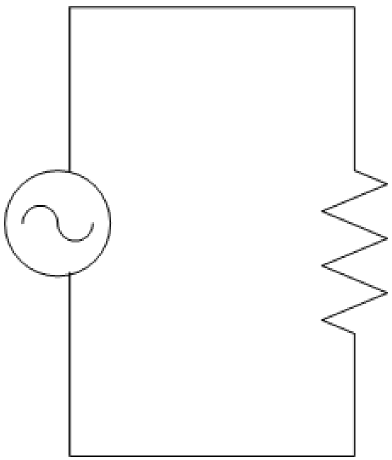
- E.g. Sine Wave Voltage
- Compare:
  - Amplitude

    $$V(t) = A\sin(\omega t) = V_{\max}\sin(\omega t)$$
  - Peak to peak

    $$V_{pp} = 2V_{\max}$$

RMS

- Why do we use this?
  - Think about the power in an AC circuit with a resistor
  - Power is dissipated regardless of which way the current is flowing



$$P(t) = V(t)I(t)$$

$$= V(t)\frac{V(t)}{R} = \frac{V^2(t)}{R}$$

- Average power
  - Same as a DC source with voltage = RMS AC voltage
  - Your meters display RMS for AC quantities

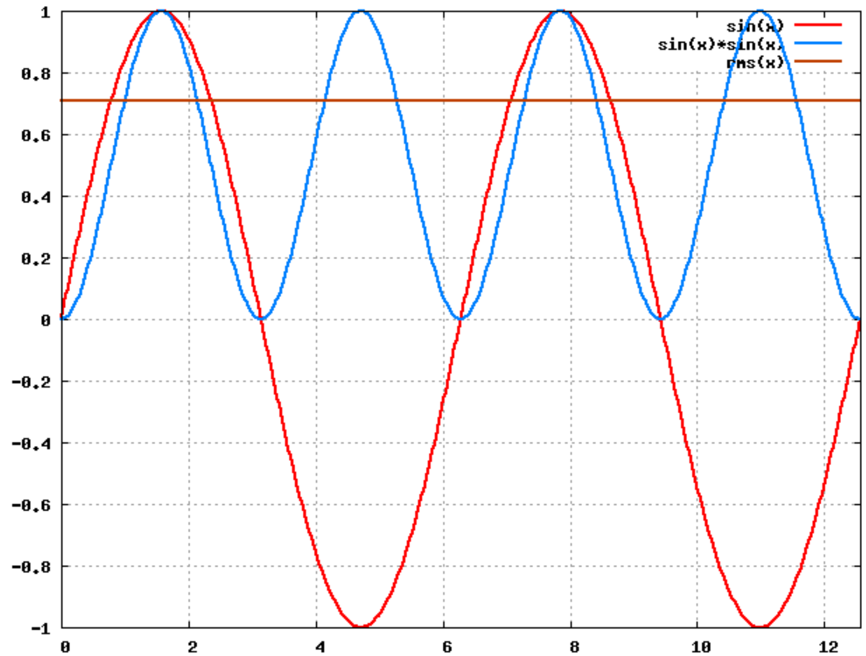
$$< P(t) >= \frac{1}{R}\frac{1}{T}\int_0^T V^2(t)dt$$

$$= \frac{(V_{RMS})^2}{R} = V_{RMS} I_{RMS}$$

Some useful RMSs to know

Sinusoid

$$V(t) = A\sin(\omega t), \quad \text{period} = 2\pi/\omega$$



$$V_{rms}^2 = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} A^2 \sin^2(\omega t)dt$$

$$= A^2 \frac{\omega}{2\pi}\int_0^{2\pi/\omega} \frac{1-\cos 2\omega t}{2} dt$$

(using $\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x$)

$$= A^2 \frac{\omega}{2\pi}\frac{1}{2}[t - \frac{1}{2\omega}\sin 2\omega t]_0^{2\pi/\omega} = A^2 \frac{\omega}{2\pi}\frac{1}{2}\frac{2\pi}{\omega}$$

$$= \frac{1}{2}A^2$$

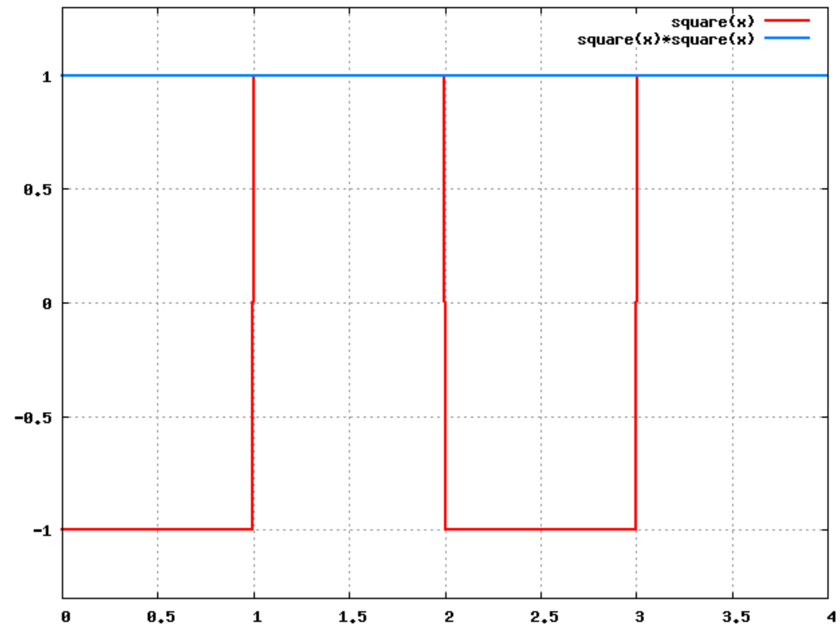
$$V_{RMS} = \frac{1}{\sqrt{2}}A \approx .707A$$

Some useful RMSs to know

Square Wave

$$V(t) = \begin{cases} V_{\max}, & 0 < x < 1 \\ -V_{\max}, & 1 < x < 2 \end{cases}, \quad \text{period} = 2$$

$$V_{rms}^2 = \frac{1}{2}\int_0^2 V_{\max}{}^2 dt = V_{\max}{}^2$$

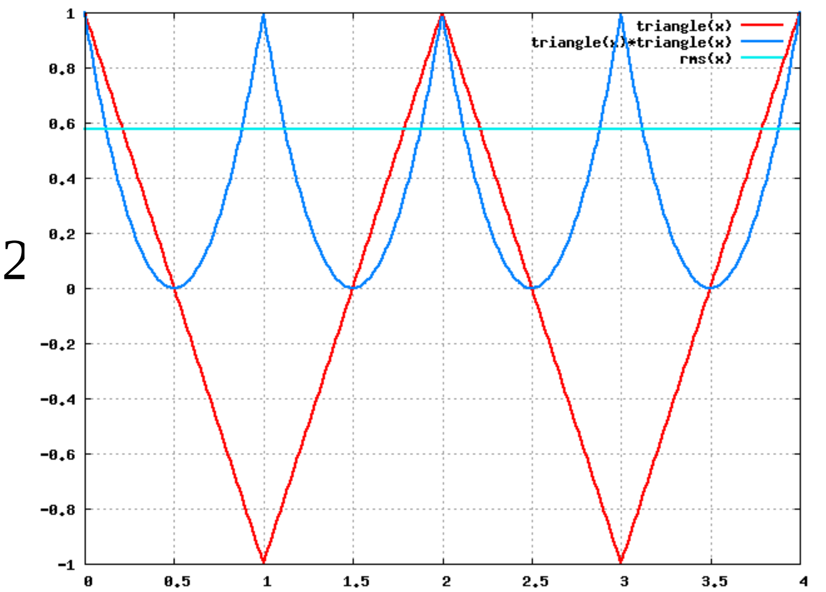


$$V_{RMS} = V_{\max}$$

Some useful RMSs to know

Triangle Wave

$$V(t) = \begin{cases} V_{\max}(1-2x), & 0 < x < 1 \\ V_{\max}(-3+2x), & 1 < x < 2 \end{cases}, \quad \text{period} = 2$$



$$V_{rms}^2 = \frac{1}{2} 2 \int_0^1 V_{\max}{}^2 (1-2t)^2\, dt$$

$$= V_{\max}^2 \frac{-1}{6}(1-2t)^3 \Big|_0^1$$

$$= V_{\max}^2 \left(\frac{1}{6} + \frac{1}{6}\right)$$

$$= \frac{1}{3} V_{\max}^2$$

$$V_{RMS} = \frac{1}{\sqrt{3}} V_{\max}$$

RMS and DC offset

DC Offset

Suppose we add a DC offset to a pure AC signal $V(t) = V_{AC}(t) + V_{DC}$

$$V_{rms}^2 = \frac{1}{T}\int_0^T (V_{AC}(t) + V_{DC})^2 dt$$

$$= \frac{1}{T}\int_0^T (V_{AC}^2(t) + 2V_{DC}V_{AC}(t) + V_{DC}^2)dt$$

$$= V_{AC-rms}^2 + 2V_{DC} * 0 + V_{DC}^2$$

$$= V_{AC-rms}^2 + V_{DC}^2$$

$$V_{RMS} = \sqrt{V_{AC-rms}^2 + V_{DC}^2}$$