A calculation of required noise and current swings on the SCUBA-2 MUX wafer leads.

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These calculations are done with conservative assumptions, but without adding margin for error. We calculate the levels required to not degrade the unbonded MUX wafer performance. If the levels on a particular line cannot be met, a more detailed systems analysis of the effect on the full system in the presence of minimum TES and photon noise can be conducted. It is likely that some noise levels could be significantly relaxed without undue degradation of overall system NEP.

We use the parameter β to parametrize the amount of degradation. β is the ratio of the MUX noise at a particular level to the noise of the leads (in amplitude). If β=3, the degradation in NEP is approximately 5%. This is the number that is used throughout this calculation.

I first calculate specifications on white noise and signal levels at the wafer. 1/f specifications will follow.

These calculations will need to be verified in testing the MUX wafer prototype over the next months. Some unmeasured parameters will need to be verified.

Address lines

When on (high), locked up at 025 Φ_0,

(1) The predicted flux noise level at 65 mK

$\Phi_n = 0.15\ \mu\Phi_0/\sqrt{\text{Hz}}$ (Table 4.4.2 technology review)

(2) The current responsivity of the SCUBA-2 SQUID given a 1 ohm shunt resistor (the value used in SCUBA-2)

$$I_{\Phi 1} = -\frac{V_{\Phi 1}}{R_{DYN} + R_s}$$ (Eqn. 2, Beyer paper)

or, the current responsivity is the voltage responsivity divided by the sum of the shunt resistor and the dynamic resistance of the SQUID.

The measured value of the loaded current responsivity is:

$I_{\phi 1} = 95\ \mu\text{A}/\Phi_0$ (Fig. 3b, Beyer's paper)

(3) The current noise through the summing coil in each ‘on’ pixel is the product of (1) and (2).

I_{n_on} = 14.25 pA/√Hz.

(4) The current noise through the summing coils from the shunt resistors in the ‘off’ pixels is:

I_{n_off} = 1.9 pA/√Hz.

There are 40 ‘off’ pixels for each on pixel. Their noise adds in quadrature with each other and with (3) giving a total noise of

I_{sum} = 18.6 pA/√Hz.

(5) The current noise through the summing coil due to a current noise in the address line, I_{nadd} , is constrained by

$$\beta I_{nadd} \frac{R_{DYN}}{R_{DYN} + R_s} < I_{sum} \quad \text{(equation for a current divider)}$$

Where we assume that in the ‘off’ pixels, all address-line current noise shunts through the off SQUIDs, rather than the summing coil. This is true for the low frequencies of interest. Putting in (4), and using

$R_s = 1\ \Omega$
$R_{DYN} = 2.5\ \Omega$ (Fig. 3d, Beyer paper)

Gives a constraint on the current noise on the address line of:

I_{nadd} < 8.75 pA/√Hz. To not degrade the pixel.

(6) The requirement on the total current swing is not as well measured. It should be about 500 μA.

(7) Complex impedance of the address line

For modeling purposes, the dynamic complex impedance of the address lines can be estimated. The inductance of the tranformer has not been measured for SCUBA, but the value from the 32-channel multiplexer in deKorte et. al. is:

L_{TR} = 74 nH. (Table 1 deKorte)

Giving a total complex impedance of an ‘on’ line, neglecting strays:

$$Z = 41\frac{R_{DYN}\left(R_s + i\omega L_{TR}\right)}{R_{DYN} + R_s + i\omega L_{TR}} + i\omega L_{stray\_add}$$

Where L_{stray_add} is the stray inductance of the address-line microstrip. This will have to be measured, but it is likely to be of order:

$$L_{stray\_add} \sim 41 \times 1.1\ \text{mm} \times 100\ \text{pH/mm} = 4.5\ \text{nH}.$$

Of course, the bond pads/ wirebonds/ etc. are not included here. The complex impedance of 'off' lines is close to zero.

1st Stage Feedback Line

(8) The mutual inductance of the feedback line

From deKorte,

$M_{fb} = 9$ pH

(9) Referring the flux noise to a current noise on the feedback line, we have the constraint:

$$I_{nFB} < \frac{\Phi_n}{\beta M_{fb}}$$

$I_{nFB} < 12$ pA/√Hz, using (1) and (8)

(10) Maximum output swing

The maximum swing is constrained by functional strategies. The project needs to determine the total signal swing over which lock must be maintained, and set the feedback current swing accordingly. If the maximum flux swing is $\Delta\Phi_{\max}$, then the required feedback current swing will be:

$$\Delta I_{FB} < \frac{\Delta\Phi_{\max}}{M_{fb}}.$$

(11) Complex impedance of the 1st-stage feedback line

$$Z = 41 i\omega L_{FB1} + i\omega L_{stray\_FB1} \sim i\omega\left(13\ \text{nH}\right)$$

Where $L_{FB1} \approx 200$ pH is the approximate inductance of the feedback line through both one SQUID and one dummy SQUID, and

$L_{stray\_FB1} \sim 41 \times 1.1\ \mathrm{mm} \times 100\ \mathrm{pH/mm} = 4.5\ \mathrm{nH}.$

Of course, the bond pads/ wirebonds/ etc. are not included here.

2st Stage Bias Line

(12) Flux noise at the second-stage input

The coupling from a current in the summing coil into a flux in the second-stage has not been measured for SCUBA-2. It will only be possible to measure it in testing the full MUX subarray with 40 multiplexed pixels. To start, we can use the value from the 32-channel MUX in the DeKorte paper, with the understanding that it will be changed.

From DeKorte,

$$\frac{\Delta I_{sum}}{\Delta \Phi_{SQ2}} = 105\ \mu\mathrm{A}/\Phi_0.$$

When combined with the noise in the summing coil from (4), we arrive at a flux noise in the second-stage SQUID of $\Phi_{nSQ2} = 0.18\ \mu\Phi_0/\sqrt{\mathrm{Hz}}$. We expect the coupling in SCUBA-2 to be somewhat smaller than in DeKorte, so that the noise in the second-stage SQUID will probably be limited by the second-stage SQUID noise itself. For the purposes of this calculation, we will take the worst-case assumption that the noise is that of the SQUID itself,

$\Phi_{n2} = 0.15\ \mu\Phi_0/\sqrt{\mathrm{Hz}}$.

(13) Current noise at the second-stage input

The flux noise of the SQUID can be referred to a current on the output by using (2), and taking

$R_{s2} = 0.1\ \Omega$	(the second-stage bias resistor)
$R_{DYN} = 2.5\ \Omega$	(from Beyer's paper)
$V_{\phi 1} = 330\ \mu\mathrm{V}/\Phi_0$	(from Beyer's paper)

So that:

$I_{n2} = 22.5\ \mathrm{pA}/\sqrt{\mathrm{Hz}}$.

(14) Current noise on 2nd-stage bias line

The second stage bias current is divided between the second-stage shunt resistor on one side and a series combination of the second-stage SQUID bias and the input coil of the series array on the other side. The current noise of the 2nd stage bias is thus constrained by

$$\beta I_{n\_bias2} \frac{R_{s2}}{R_{DYN} + R_{s2}} < I_{n2} .$$

Thus, the current noise of the second-stage bias must be

$I_{n\_bias2} < 165$ pA/√Hz.

(15) Maximum 2nd-stage bias swing

Since the shunt resistor is about ten times smaller for the second stage as for the first-stage, the maximum swing is about ten times higher, or ~5 mA.

(16) Complex impedance of the 2nd-stage bias line

The complex impedance of the 2nd-stage bias line is a parallel combination of the SQUID 2 bias line in series with the input coil of the series-array SQUID and the significant stray inductance in the leads to the series-array SQUID, and the 0.1 Ω shunt resistor.

$$Z = \frac{R_{s2}\left(R_{DYN} + i\omega L_{SA} + i\omega L_{stray\_bias2}\right)}{R_{s2} + R_{DYN} + i\omega L_{SA} + i\omega L_{stray\_bias2}}$$

where L_{stray_bias2} includes all stray inductance between the series array and the MUX wafer. It will be significant.

2st Stage Feedback Line

(17) The mutual inductance of the 2nd-stage feedback line

From deKorte,

$$M_{FB\_2} < (9 \text{ pH/turn})(2 \text{ turns}) = 18\,\text{pH}$$

(18) Referring the flux noise to a current noise on the feedback line, we have the constraint:

$$I_{n\_FB2} < \frac{\Phi_{n2}}{\beta M_{FB2}}$$

$I_{n\_FB2} < 5.5$ pA/√Hz, using (12) and (17).

(19) Maximum output swing

Thus must be just enough to adjust the 2nd-stage SQUID through a full flux quantum. Make it two flux quanta to be conservative

$$\Delta I_{FB} < \frac{2\Phi_0}{M_{fb}} = 220\ \mu\text{A}..$$

(20) Complex impedance of the 2nd-stage feedback line

$$Z = i\omega L_{FB2} + i\omega L_{stray\_FB2} \sim i\omega\left(450\ \text{pH}\right)$$

Where $L_{FB2} \approx 450\ \text{pH}$ is the approximate inductance of the feedback line through both one SQUID and one dummy SQUID, and the stray inductance is likely to be quite small, since the 2nd-stage SQUID is close to the bond pads. Of course, the bond pads/ wirebonds/ etc. are not included here.

Detector Bias Line

(21) To determine the detector bias line specifications, we need to make some assumptions about the detector performance. We can choose several different bias points and wavelengths. The most challenging "downselected" performance numbers from Table 4.2.1.1 from the review document are:

$R_{b\_TES} = 25\ \text{m}\Omega$ (TES bias resistance)

$I_{n\_TES} = 35\ \text{pA}/\sqrt{\text{Hz}}$, (TES current noise)

where the TES noise is the out of band value for the 850 μm pixel. Recall also that the shunt resistor has a design value of:

$$R_{s\_TES} = 5\ \text{m}\Omega\,.$$

(22) The applied current noise is current divided between the shunt resistor and the TES. The constraint on the detector bias line noise level is thus:

$$\beta I_{TES\_bias\_line} \frac{R_{s\_TES}}{R_{b\_TES} + R_{s\_TES}} < I_{n\_TES}\,.$$

Which gives a maximum acceptable TES bias line noise of:

$$I_{TES\_bias\_line} < 14\ \text{pA}/\sqrt{\text{Hz}}.$$

(22) The maximum required current swing is determined by the maximum power dissipation that will be required on the TES. This question is one of observational strategy. From table 3 in the review document, the goal for TES power handling is 10 pW. Then the power dissipation in the TES at the bias point is:

$$P = \left(\frac{I_{TES\_bias\_line} R_{s\_TES}}{R_{b\_TES} + R_{s\_TES}} \right)^2 R_{b\_TES} = 10 \text{ pW}$$

which gives a TES bias line current of:

$$I_{TES\_bias\_line} \approx 40 \ \mu\text{A} .$$

HOWEVER, this is one of the least determined SCUBA-2 parameters. *I would want a LOT of engineering margin on this – a factor of 10 would be nice.*

Heater Line

(23) To determine the heater line specifications, we must use the following parameters:

$R_{HTR} = 3\,\Omega$	(from Fig 3.4.9.2 in the review document)
$P_{MAX} = 230$ pW	(maximum required power from Table 3)
$NEP = 3.5 \times 10^{-17} \ W/\sqrt{Hz}$	(best goal NEP from Table 3)

(24) The current noise can be derived from the NEP:

$$I_{n\_HTR} = \sqrt{NEP / \beta^2 R_{HTR}} \approx 1.1 \text{ nA}/\sqrt{\text{Hz}} .$$

(25) The current swing can be derived from the maximum bias power:

$$\Delta I_{HTR} = \sqrt{P_{MAX} / R_{HTR}} \approx 8.8 \ \mu\text{A} .$$

Note that heater lines are also VERY subject to RF pickup! Shielding, filtering and possible low-temperature shunting should be done very carefully.

1/f Noise on all lines

I will leave the constraints on the 1/f noise for the reader to compute. Suffice it to say that for the detector line and heater line, the specifications derived (22 and 24) must be maintained all the way to the desired noise corner of ~ 50 mHz. The specifications for all the other lines are significantly aided by the fact that the mutual inductance to the first-stage SQUID is significantly over-engineered in order to meet the constraints on aliased noise from the larger SQUID bandwidth when multiplexing.

To compute these levels, the reader should take the output current noise from the TES (from 23), and convert it to a flux noise in the first-stage SQUID using:

$$M_1 = 480 \text{ pH}$$

from Table 4.3 of the review document. This value, $8.4\ \mu\Phi_0/\sqrt{\text{Hz}}$ in the first-stage SQUID, is the flux noise level that must be reached when it is referred to all of the above lines (save the detector and heater bias) *at the target noise corner of ~ 50 mHz.*

Note that the dark SQUID should mitigate some of these noise sources. These include the second-stage bias and flux bias lines, which should have significantly improved 1/f. There should be some mitigation of 1/f due to the 1st-stage feedback line, which is common, but the degree of 1/f cancellation will be limited by the variation in pixel-to-pixel flux gain. The 1/f from the address lines will not be mitigated at all, since they are not common to the pixels in a column. The heater and detector bias line 1/f will also be mitigated, since they are common, but again, this will be limited by variation in pixel-to-pixel power responsivity, differential resistance, and flux gain.