Statistics and Parametric Correlations of Coulomb Blockade Peak Fluctuations in Quantum Dots

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We report measurements of mesoscopic fluctuations of Coulomb blockade peaks in a shape-deformable GaAs quantum dot. Distributions of peak heights agree with predicted universal functions for both zero and nonzero magnetic fields. Parametric fluctuations of peak height and position, measured using a two-dimensional sweep over gate voltage and magnetic field, yield autocorrelations of height fluctuations consistent with a predicted Lorentzian-squared form for the unitary ensemble. We discuss the dependence of the correlation field on temperature and coupling to the leads as the dot is opened.

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Confined semiconductor microstructures, or quantum dots, provide a versatile experimental system for studying the crossover from microscopic quantum physics to macroscopic physics [1]. At low temperatures, transport through quantum dots exhibits mesoscopic fluctuations (random but repeatable fluctuations arising from quantum interference) [2] with universal features that are understood to be connected to underlying universalities of quantum chaos [3].

In this Letter, we investigate mesoscopic conductance fluctuations in quantum dots in the Coulomb blockade regime. This regime is of great interest because at low temperatures ($kT \ll \Delta$) transport is mediated by tunneling through a single eigenstate of the dot and so, in principle, provides experimental access to the statistics of wave function fluctuations, something which cannot be obtained in open mesoscopic samples. We find excellent agreement with a statistical theory of Coulomb blockade peak heights [4] in both the orthogonal ($B = 0$) and unitary ($B \neq 0$) ensembles. In addition, we present for the first time parametric correlations of Coulomb blockade peak heights as a function of magnetic field and find good agreement with recent theory [5,6]. We investigate the dependence of the magnetic correlation field B_c on temperature and on the coupling of the leads, extending from strong Coulomb blockade to several open channels per lead. These experiments make use of a novel multigate quantum dot design which allows shape distortion while preserving dot area (Fig. 1, inset).

In open quantum systems, the lifetime broadening $\Gamma \sim \hbar/\tau_{\mathrm{escape}}$ of quasibound eigenstates exceeds the mean level spacing Δ. Interference effects in open systems include universal conductance fluctuations and weak localization, which have been well studied over the last decade [7]. As the system becomes classically isolated and sufficiently cold ($kT, \Gamma < \Delta$) quantum levels become resolved. In addition, conductance in the regime $\Gamma < \Delta$ is suppressed due to charge quantization whenever the energy to add a single electron to the dot, e^2/C (C is the total capacitance of the dot), exceeds the applied

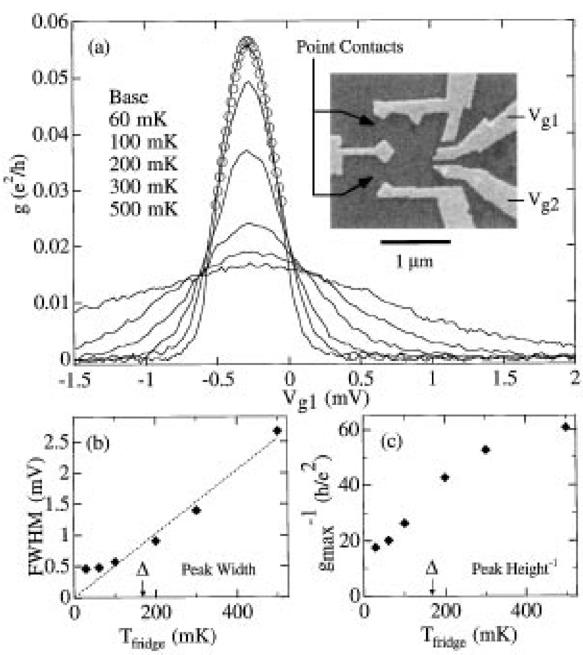

FIG. 1. (a) Temperature dependence of Coulomb peak line shape for the larger dot ($\Delta = 15\ \mu$eV). Circles show fit of base temperature peak by $g/g_{\max} = \cosh^{-2}(\eta eV_g/2kT)$. Inset: Micrograph of the larger dot. V_{g1} and V_{g2} are shape distorting gates. (b) Peak width measured as FWHM of the fit to $\cosh^{-2}$. Linear behavior at high temperatures gives voltage-to-energy scale $\eta = 0.12$. (c) Inverse peak height decreases with temperature for $kT < \Delta$. From saturations at low T in (b) and (c) we estimate the electron temperature in the dot to be 70 ± 20 mK.

bias, V_{bias}, an effect known as the Coulomb blockade [8,9]. When the dot potential is tuned (for instance, via electrostatic gates) so that the number of electrons can fluctuate without energy cost, large conductance peaks are observed. These peaks are nearly periodic in gate voltage, with each peak marking a change in the number of electrons in the dot by one.

Transport in strongly blockaded quantum dots has been studied extensively over the last several years and forms the basis of the ultrasensitive single-electron transistor (SET) technology [1,10]. Mesoscopic fluctuations were alluded to in several blockade experiments [11–13] but were not the primary focus of these earlier works. Phase coherence in the Coulomb blockade regime has also been investigated recently [14]. The first systematic experimental study of the distribution of Coulomb blockade peaks has recently been reported by Chang *et al.* [15], and addresses some of the issues reported here.

Statistical theories of both the distribution and correlation of Coulomb blockade peaks have been developed for the low-temperature regime, $\Gamma \ll kT \ll \Delta$. Jalabert, Stone, and Alhassid [4] used random matrix theory (RMT) to derive universal peak height distributions for both systems with time-reversal symmetry (orthogonal ensemble, $B = 0$) and systems with broken time-reversal symmetry (unitary ensemble, $B \neq 0$). Their results have been extended to nonequivalent [16] and multimode [17] leads and to the transition to chaos [18]. These studies consider a quantum dot coupled to left and right reservoirs by tunneling leads with tunneling rates $\Gamma_l/\hbar$ and $\Gamma_r/\hbar$. At low temperatures, $\Gamma \ll kT \ll \Delta$ (where $\Gamma = \Gamma_l + \Gamma_r$), blockade peaks have roughly equal widths $\sim kT$, but heights $g_{\max}$ that depend on the coupling of each lead to the dot [9]

$$g_{\max} = \frac{e^2}{h}\frac{\pi}{2kT}\frac{\Gamma_l\Gamma_r}{\Gamma_l + \Gamma_r} \equiv \frac{e^2}{h}\frac{\pi\overline{\Gamma}}{2kT}\alpha\,, \qquad (1)$$

where $\alpha \equiv \Gamma_l\Gamma_r/\overline{\Gamma}(\Gamma_l + \Gamma_r)$ is a dimensionless peak height. Fluctuations in $g_{\max}$ arise from changes in Γ_l and Γ_r with external parameters such as magnetic field or dot shape and with the number of electrons in the dot. Assuming statistically identical and independent leads ($\overline{\Gamma}_l = \overline{\Gamma}_r = \overline{\Gamma}/2$) and ignoring spin-orbit scattering, one obtains the following universal peak height probability distributions:

$$P_{(B=0)}(\alpha) = \sqrt{2/\pi\alpha}\, e^{-2\alpha}, \qquad (2a)$$

$$P_{(B\neq 0)}(\alpha) = 4\alpha[K_0(2\alpha) + K_1(2\alpha)]e^{-2\alpha}, \qquad (2b)$$

where K_0 and K_1 are modified Bessel functions [4,16].

When a continuous parameter X is used to change the partial widths $\Gamma_{l(r)}$, and hence induce fluctuations in peak heights, one may also consider correlations of $g_{\max}(X)$ [5,6]. We have measured peak height as a function of magnetic field B and obtained autocorrelations $C(\Delta B) = \langle \tilde{g}_{\max}(B)\tilde{g}_{\max}(B + \Delta B)\rangle_B/\mathrm{var}(g_{\max})_B$, where $\tilde{g}_{\max} = g_{\max} - \langle g_{\max}\rangle$ is the deviation from the average peak height. Alhassid and Attias [5] show using RMT that the generic peak height autocorrelation $C(\Delta X)$ in the regime $\Gamma \ll kT \ll \Delta$ is approximately Lorentzian in the orthogonal ensemble and Lorentzian squared in the unitary ensemble. For finite magnetic fields we therefore expect

$$C(\Delta B) = [1 + (\Delta B/B_c)^2]^{-2}. \qquad (3)$$

The correlation field B_c is typically smaller than one flux quantum ϕ_0 through the dot, $B_cA \sim \kappa\phi_0(\tau_{\text{cross}}/\tau_H)^{1/2}$, where $\tau_H = \hbar/\Delta$ is the time scale to resolve individual levels, $\tau_{\text{cross}} \sim \sqrt{A}/v_F$ is the time to cross the dot, and κ depends on the shape of the dot [18,19]. Parametric correlations of density-of-states fluctuations have been measured in vertical transport by Sivan *et al.* [20] and also gave good agreement with theory.

The quantum dots [Fig. 1(a)] were defined by electron beam lithography on a GaAs/AlGaAs heterostructure using Cr/Au gates 800 Å above the two-dimensional electron gas (2DEG). A mobility of 10^6 cm^2/V s and density of 3.5×10^{11} cm^{-2} give a Fermi wavelength of $\lambda_F = 41$ nm and a transport mean free path of 9 μm, so that transport within the dot is ballistic. The dots were coupled to bulk 2DEG by individually adjustable point contact leads. In addition, small changes to the shape and area could be controlled nearly independently from the dot conductance by varying voltages V_{g1} and V_{g2} on two shape-distorting gates. We report measurements on two similar dots with areas (and mean level spacings $\Delta = 2\pi\hbar^2/m^*A$) of 0.32 μm^2 ($\Delta \sim 22$ μeV) and 0.47 μm^2 ($\Delta \sim 15$ μeV), assuming $\sim$100 nm depletion around the gates. Conductance was measured in a dilution refrigerator ($T_{\text{base}} = 30$ mK) using a lock-in amplifier with an ac voltage bias of 5 μV rms at 11 Hz.

Figure 1 shows the temperature dependence of a typical Coulomb blockade peak for the larger dot. For $\Gamma \ll kT \ll \Delta$ one expects a thermally broadened peak shape in gate voltage (relative to peak center), $g/g_{\max} = \cosh^{-2}(\eta eV_g/2kT)$ and $g_{\max} \propto T^{-1}$ as given in Eq. (1). For $kT > \Delta$ this smoothly crosses over to $g/g_{\max} \approx \cosh^{-2}(\eta eV_g/2.5kT)$ and a temperature independent $g_{\max}$ [9]. Figure 1(a) shows a fit of the base temperature peak by the $\cosh^{-2}$ form, and Fig. 1(b) shows the FWHM extracted from similar fits at each temperature. Above 200 mK, we find FWHM $\propto T$, which we use to find the scaling factor $\eta = C_{\text{gate}}/C \sim 0.12$ (0.090) for the larger (smaller) dot to convert gate voltage V_{g1} to dot energy. Using this value of η, the Coulomb peak separation of $\sim$5 mV gives a charging energy e^2/C of 620 μeV, or $C = 260$ aF. The electron temperature in the dot was estimated to be 70 ± 20 mK based on saturation of peak width [Fig. 1(b)] and deviation from $g_{\max} \propto T^{-1}$ [Fig. 1(c)]. This gives $kT_{\text{base}} \sim \Delta/2$. From Eq. (1), the typical peak height $g_{\max} \sim 0.05e^2/h$ gives a broadening

Γ below 0.1Δ. These values place the experiment squarely in the regime $\Gamma < kT < \Delta$; however, the strong limit $\Gamma \ll kT \ll \Delta$ assumed by theory is not met, so theoretical results must be applied cautiously.

Peak height statistics and correlations were found by sweeping one shape-distorting gate voltage, V_{g1}, repeatedly over one or several peaks with either the other gate voltage, V_{g2}, or B incremented at the end of each sweep, yielding a two-dimensional raster pattern of conductance [21]. Figure 2 shows peak height distributions for both $B = 0$ and $B \neq 0$ that are in good agreement with Eq. (2). For each ensemble, statistics from a number of runs were combined after first transforming each data set to the scaled variable α using a two-parameter fit by Eq. (2). Histograms of the combined data are shown as probability distributions by normalizing total areas to unity.

Rastering over V_{g1} and V_{g2} yields Coulomb blockade ridges along lines of constant dot area [Fig. 2(b), inset]. Because of shape distortion, ridge heights fluctuate through $\sim$3 correlation lengths over the accessible range of gate voltages, $\sim$0.3 V. The capability to distort shape greatly extends the available data set (particularly at $B = 0$) as originally suggested by Bruus and Stone [18]. V_{g1} can be swept over $\sim$40 peaks before the conductance of the point contacts changes. We find that neighboring peak heights are correlated, but correlations vanish after $\sim$4 peak spacings at $T_{\rm base}$. At higher temperatures the correlation between neighboring peaks increases. Each distribution in Fig. 2 represents $\sim$600 peaks, of which we estimate $\sim$90 to be statistically independent.

As illustrated in Fig. 3, sweeps over V_{g1} and B allow fluctuations in peak height and position to be measured as a quasicontinuous function of an external parameter, in this case magnetic field. The superposition of peak heights [Fig. 3(a)] at $\pm B$ demonstrates that even the fine structure is repeatable. Large peak height fluctuations are observed $[\sqrt{\mathrm{var}(g_{\max})} = 0.01e^2/h, \langle g_{\max}\rangle = 0.019e^2/h]$, with the amplitude dropping essentially to zero in several places. Fluctuations in the peak position are also seen [Fig. 3(b)]. Presumably these are a direct reflection of the parametric motion of the energy level (or levels) participating in tunneling. Using the scaling factor $\eta = 0.090$, the rms peak excursion is 0.55Δ.

Autocorrelation functions $C(\Delta B)$ (defined above) are shown in Fig. 4(a). At base temperature, $C(\Delta B)$ agrees well with the form of Eq. (3) for small ΔB ($\chi^2 = 0.004$ up to 9 mT, compared to $\chi^2 = 0.014$ for a Lorentzian), but systematically dips below zero at larger ΔB contrary to the RMT prediction. Fits by Eq. (3) give $B_c = 8.1 \pm 0.5$ mT at base temperature and values for B_c that increase with both temperature [Fig. 4(a), inset] and coupling of the dot to the leads [Fig. 4(b)]. Notice in Fig. 4(b) that B_c changes only modestly from the strong blockade regime ($\Gamma \ll \Delta$) to the open regime, where the number of modes per lead $N \sim \pi\Gamma/\Delta$ exceeds 3, and where all traces of Coulomb blockade have vanished. Speculating on the dependence of B_c on T and Γ, one may simply be able to replace τ_H in the expression for

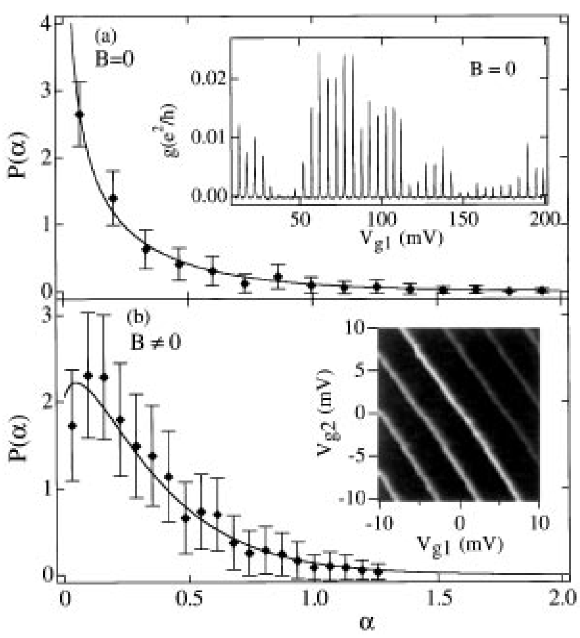

FIG. 2. Distribution of Coulomb peak conductances, scaled to dimensionless conductances α [Eq. (2)] for (a) the orthogonal ($B = 0$) ensemble and (b) the unitary ensemble ($B \neq 0$). Insets: (a) Example of a set of peaks from which the distribution was obtained. (b) Grayscale plot of small region of gate-gate sweep used to derive these distributions, showing Coulomb "ridges" along lines of constant area (lighter = higher conductance). Error bars assume $\sim$90 statistically independent samples (see text).

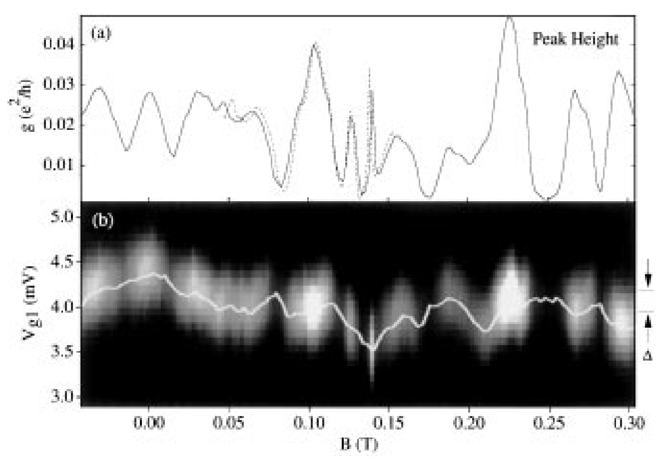

FIG. 3. Fluctuations in Coulomb blockade peak height and position as a function of magnetic field at $T_{\rm base}$ (a) Peak height $g_{\max}$ at $+B$ (solid) and $-B$ (dashed) magnetic field show symmetry and repeatability of fine structure in the data. (b) Grayscale plot of gate voltage–magnetic field raster (lighter = higher conductance). Peak heights in (a) are taken at peak center, marked by white curve in (b). Mean level spacing $\Delta = 22$ μeV indicated at right, after scaling by $\eta = 0.09$ for the smaller dot.

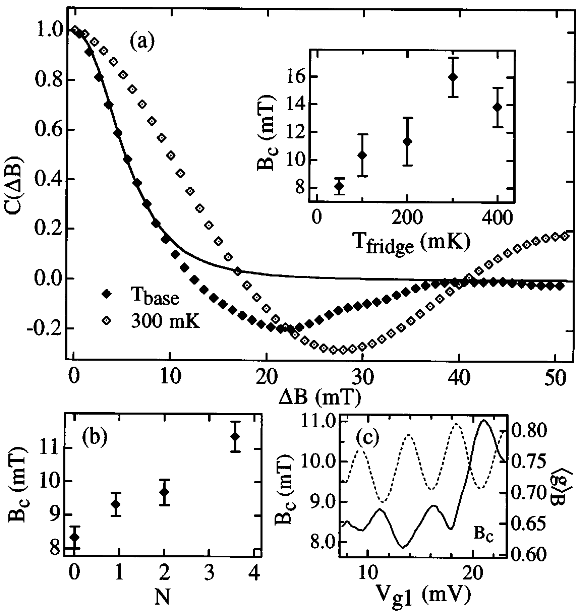

FIG. 4. (a) The magnetic field autocorrelation $C(\Delta B)$ of peak height at $T_{\rm base}$ (solid) and 300 mK (open), along with a fit of Eq. (3) (solid curve) giving $B_c = 8.1 \pm 0.5$ mT. Data at $T_{\rm base}$ are averaged over seven mutually uncorrelated peaks; 300 mK data are averaged over three peaks. Inset: Temperature dependence of the correlation field B_c from fits by Eq. (3). (b) Dependence of B_c on the number of modes, N, in the leads $[g_{\rm lead} = (2e^2/h)N]$ as the dot is opened. (c) Field-averaged conductance $\langle g \rangle_B$ (dashed) and correlation field B_c (solid) oscillate out of phase in the weak blockade regime. Regions of stronger blockade (minima of $\langle g \rangle_B$) have larger B_c.

B_c by the smallest relevant time scale—$\hbar/\Delta$, $\hbar/kT$, or $\hbar/\Gamma$—implying a square-root dependence on the largest of these energy scales, which appears consistent with the data. Phase breaking may also contribute to $B_c(T)$ by increasing broadening $\Gamma_{\rm tot} = \Gamma + \Gamma_\varphi(T)$ [22]. However, theoretical estimates [23] and data from open dots [24] suggest $\Gamma_\varphi(T) < \Delta$. Further investigation is needed to settle this question.

Finally, we observe that for weak Coulomb blockade ($\langle g \rangle > 0.1$) B_c is modulated (in gate voltage) along with the field-averaged conductance $\langle g \rangle_B$, with the maxima of one aligned with the minima of the other, as shown in Fig. 4(c). This can be viewed in terms of a number-phase uncertainty relation if at the minima of $\langle g \rangle_B$, where number uncertainty is reduced (strongest blockade), the corresponding increase in phase uncertainty acts to increase B_c, in analogy to the effect of dephasing on B_c in open dots [24,25]. Note that a semiclassical explanation would predict an effect opposite to that observed: lower conductance implies greater dwell time which would decrease B_c. A satisfactory quantum treatment of this effect remains to be worked out.

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