PHYS 354 – Midterm 2 Exam Cheat Sheet

Legendre Polynomials

$$P_0(\cos\theta) = 1 \qquad P_1(\cos\theta) = \cos\theta$$
$$P_2(\cos\theta) = \tfrac{3}{2}\cos^2\theta - \tfrac{1}{2} \qquad P_3(\cos\theta) = \tfrac{5}{2}\cos^3\theta - \tfrac{3}{2}\cos\theta$$

Orthogonality of Functions

$$\int_0^a \sin\frac{k\pi y}{a}\sin\frac{n\pi y}{a}\,dy \;=\; \begin{cases} 0, & k \neq n \\ \frac{a}{2}, & k = n \end{cases}$$

$$\int_0^\pi P_l(\cos\theta)P_n(\cos\theta)\sin\theta\;d\theta \;=\; \begin{cases} 0, & l \neq n \\ \frac{2}{2n+1}, & l = n \end{cases}$$

Materials

Bound Charges: $\sigma_b = \vec{P}\cdot\hat{n}$, $\rho_b = -\vec{\bigtriangledown}\cdot\vec{P}$

Multipole Expansion

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int\frac{\rho(\vec{r'})}{r}\,d\tau' \qquad V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\sum_{n=0}^{\infty}\frac{1}{r^{n+1}}\int (r')^n\,P_n(\cos\theta')\,\rho(\vec{r'})\,d\tau'$$
$$\vec{A}(\vec{r}) = \frac{\mu_0 I}{4\pi}\oint\frac{1}{r}\,d\vec{l'} \qquad \vec{A}(\vec{r}) = \frac{\mu_0 I}{4\pi}\sum_{n=0}^{\infty}\frac{1}{r^{n+1}}\oint (r')^n\,P_n(\cos\theta')\,d\vec{l'}$$

Coulomb's Law - Scalar Potential

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int\frac{\rho(\vec{r'})}{r}\,d\tau' \qquad \vec{E} = -\vec{\bigtriangledown}V$$

Biot-Savart Law

$$\vec{B}(\vec{r}) \;=\; \frac{\mu_0 I}{4\pi}\int\frac{d\vec{l'}\times\hat{r}}{r^2} \;=\; \frac{\mu_0}{4\pi}\int\frac{\vec{J}(\vec{r'})\times\hat{r}}{r^2}\,d\tau'$$

Forces

$$\vec{F}_{elec} \;=\; \int\rho\vec{E}\,d\tau \qquad \vec{F}_{mag} \;=\; \int I\,d\vec{l}\times\vec{B}$$

Vector Potential

$$\vec{A}(\vec{r}) \;=\; \frac{\mu_0}{4\pi}\int\frac{\vec{J}(\vec{r'})}{r}\,d\tau' \qquad \vec{B} = \vec{\bigtriangledown}\times\vec{A}$$

Electric and Magnetic Flux

$$\Phi_B \;=\; \int\vec{B}\cdot d\vec{a} \qquad \Phi_E \;=\; \int\vec{E}\cdot d\vec{a}$$