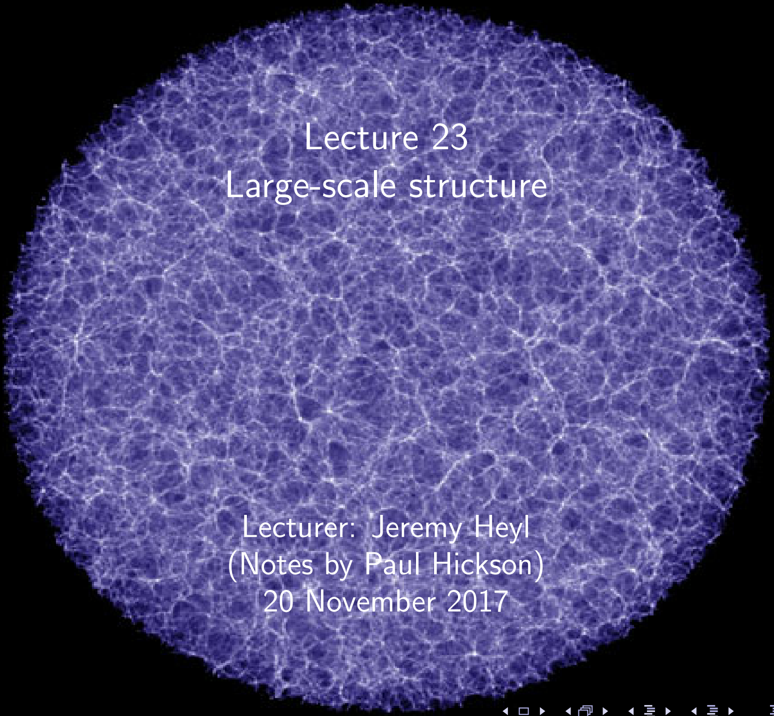
Lecture 23
Large-scale structure



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(Notes by Paul Hickson)
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Our local neighbourhood

The nearest clusters of galaxies are roughly aligned in a plane, nearly perpendicular to the plane of the Milky Way, called the **supergalactic plane**.

Distances to nearby elliptical galaxies can be estimated by the surface-brightness fluctuation technique. Closer galaxies have fewer stars per square arcsec and therefore larger variations in surface brightness within their image.

Most of the light we see in elliptical and S0 galaxies comes from old stars near the tip of the red giant branch, which all have about the same luminosity.

Poisson statistics tells us that the fractional variation in intensity is roughly $1/\sqrt{N}$ where N is the number of stars in the angular area being measured.

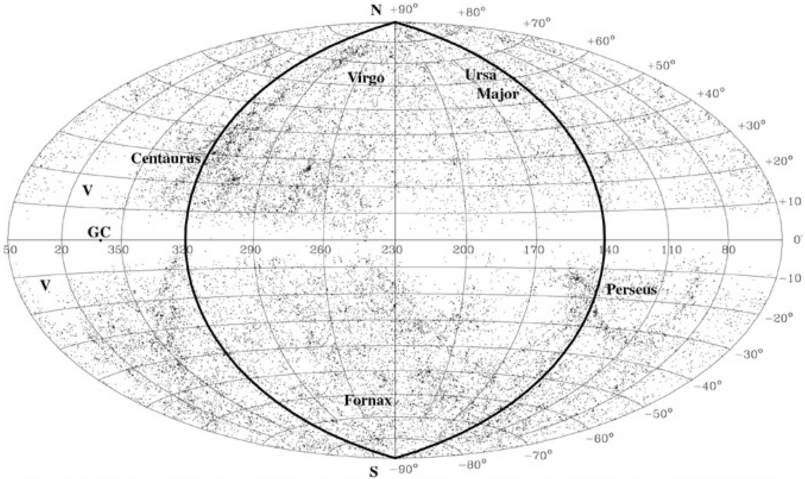

Fig 8.1 (Kolatt & Lahav) 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

Positions of 14650 bright galaxies, in galactic coordinates.. The solid line marks the supergalactic plane.

Our neighbourhood.

Positions of nearby elliptical galaxies on the supergalactic $X-Y$ plane.

The Milky Way is located at the origin. Shading indicates recession velocity V_r.

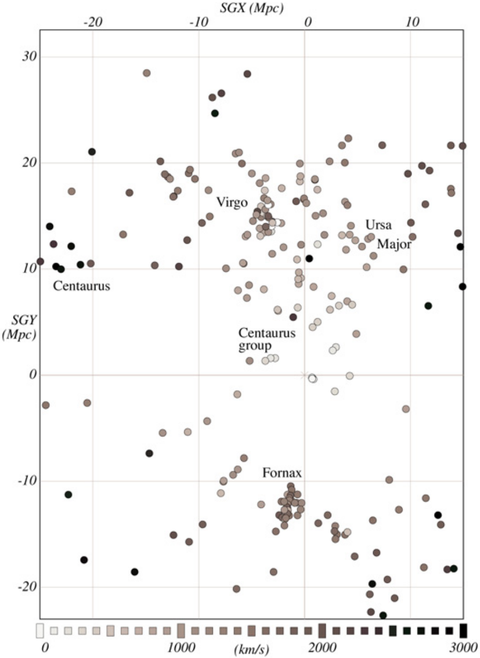

Fig 8.2 (J. Tonry) 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

Beyond our neighbourhood

At distances beyond ~ 20 Mpc the cosmic expansion dominates over peculiar velocities so redshifts can be used to estimate distances via Hubble's law.

Large surveys have now measured redshifts for hundreds of thousands of galaxies. This allows one to plot 'wedge diagrams' which show redshift vs angular position along a strip of sky.

One finds filaments of galaxies connecting clusters, and large voids that are typically $\gtrsim 50$ Mpc across.

At large distances, the density of galaxies appears to drop. This is due to a **selection effect**. Only the brightest galaxies can be seen at those distances and therefore we see fewer per Mpc^{-3}.

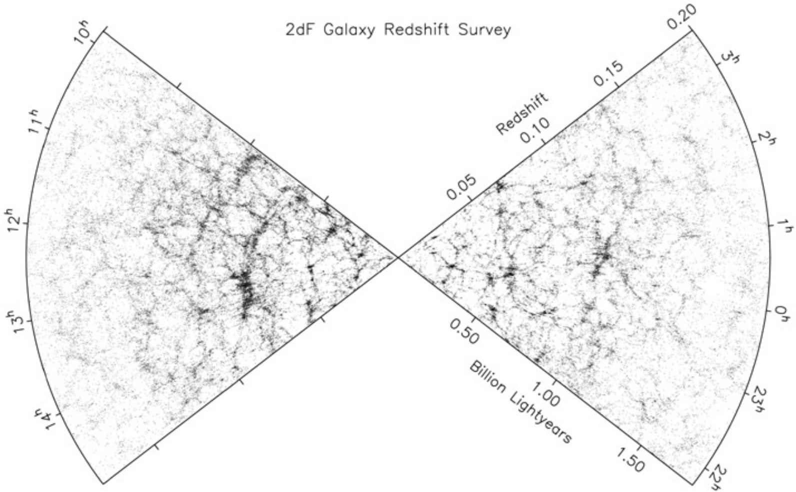

Fig 8.3 (2dF) 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

A wedge diagram of 93 170 galaxies from the 2dF survey with the Anglo-Australian 4-meter telescope, in slices $-4^\circ < \delta < 2^\circ$ in the north (left wedge) and $-32^\circ < \delta < -28^\circ$ in the south (right).

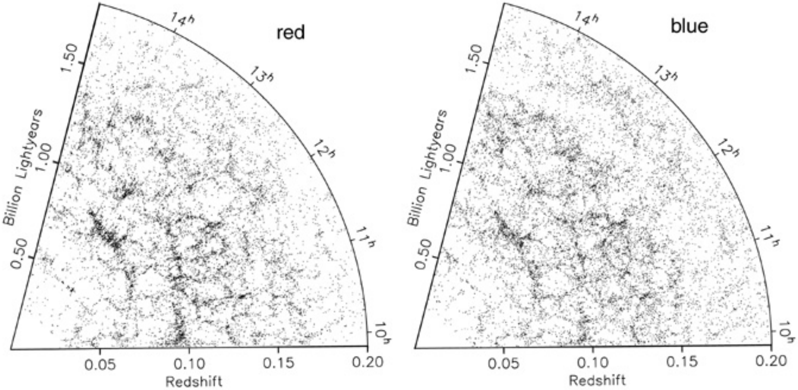

Fig 8.5 (2dF) 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

About 27000 red galaxies (left) with spectra like those of elliptical galaxies, and the same number of star-forming blue galaxies (right), in a slice $-32^\circ < \delta < -28^\circ$ from the 2dF survey. These are luminous galaxies, with $-21 < M(B_J) < -19$. The elliptical and S0 galaxies cluster more strongly than the spiral-like systems.

Correlation function

It is very useful to have an objective measure of the degree of clustering of galaxies, without having to decide whether or not they belong to particular clusters. This is provided by the **correlation function** $\xi(r)$.

If ΔV_1 and ΔV_2 are two small volumes, the probability of finding a galaxy in ΔV_1 *and* a galaxy in ΔV_2 can be written as

$$\Delta P = n^2[1 + \xi(r_{12})]\Delta V_1 \Delta V_2,$$

where n is the average galaxy number density and r_{12} is the distance between the two volumes.

The correlation function is a measure of the *excess probability* due to galaxy clustering, over and above the Poisson probability.

It is presumed to depend only on the distance r_{12}, and not the orientation. In other words, the clustering is **isotropic**.

Correlation function

Over a wide range of scales r, the correlation function is well-represented by a power law,

$$\xi(r) = \left(\frac{r}{r_0}\right)^{-\gamma}$$

with $\gamma \simeq 1.77$.

The constant r_0 represents the distance at which the probability is twice the Poisson probability.

The value of r_0 depends on the type of galaxies being studied. It is smaller for early-type galaxies than spirals, indicating that the former are more strongly clustered.

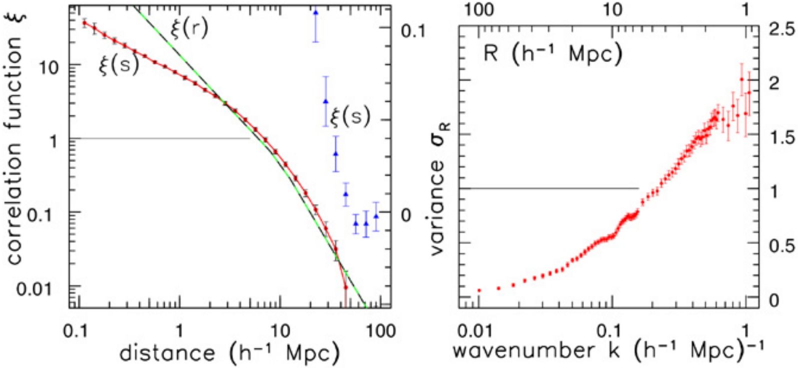

Fig 8.6 (Maddox, Cole) 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

Left, the correlation function $\xi(s)$ for the 2dF galaxies, at small (circles, left logarithmic scale) and large (triangles, right linear scale) separations; vertical bars show uncertainties. $\xi(s)$ is calculated assuming that Hubbles law holds exactly, but proper motions reduce $\xi(s)$ on scales $r \lesssim 1$ Mpc and infall to the walls makes clustering look stronger on scales near r_0. The dashed line shows $\xi(r)$, corrected for these effects. Right, the variance σ_R describing how much the average density varies between regions of size R.

The power spectrum of clustering

Another useful parameter is the **power spectrum**, equal to the Fourier transform of the correlation function

$$P(k) = \int \xi(r) e^{-i\boldsymbol{k}\cdot\boldsymbol{r}} d^3r = 4\pi \int_0^\infty \xi(r) \frac{\sin(kr)}{kr} r^2 dr.$$

The power spectrum is a measure of the strength of clustering as a function of wavenumber k.

The function $\sin(kr)/kr$ becomes small and oscillates when $kr > \pi$. So $P(k)$ measures the average clustering within a scale $R \sim \pi/k$. Small values of k correspond to large scales and vice versa.

The power spectrum can be predicted by cosmological models and then compared to the data derived from large-scale galaxy redshift surveys.

Density fluctuations

One can represent the mass density in the Universe by a smooth function $\rho(\boldsymbol{x})$. This can be written in terms of a dimensionless density parameter $\delta(\boldsymbol{x})$ defined by

$$\rho(\boldsymbol{x}) = \bar{\rho}[1 + \delta(\boldsymbol{x})],$$

where $\bar{\rho}$ is the average density.

Now define $\delta_R(\boldsymbol{x})$ which is the average value of δ within a sphere of radius R centred at $\boldsymbol{x}$.

$\delta_R(\boldsymbol{x})$ is a fluctuating function of $\boldsymbol{x}$. It's variance is denoted by σ_R^2.

σ_R^2 is related to the power spectrum. One can show that

$$\sigma_R^2 \simeq \frac{k^3 P(k)}{2\pi^2} \equiv \Delta_k^2, \quad \text{where } k \simeq 1/R.$$

Density fluctuations

It is convenient to represent $P(k)$ as a power law, $P(k) \propto k^n$.

Equivalently, $P(k) = P_0(k/k_0)^n$, where P_0 is a normalization constant (the power on a scale k_0).

Present cosmological models predict that initially, $n \sim 1$. However, physical processes in the early Universe modify the spectrum so that on small scales ($\lesssim 50$ Mpc), $n \sim -3$.

If $n = -3$, σ_R is independent of scale, so the density is equally lumpy on all scales.

Cosmologists commonly use σ_8, the RMS density fluctuation within a comoving radius of 8 Mpc, as a measure of the strength of clustering (i.e. the normalization of the power spectrum).

Observations indicate that $\sigma_8 \sim 0.8$. Thus the density evolution is roughly linear on scales larger than this.

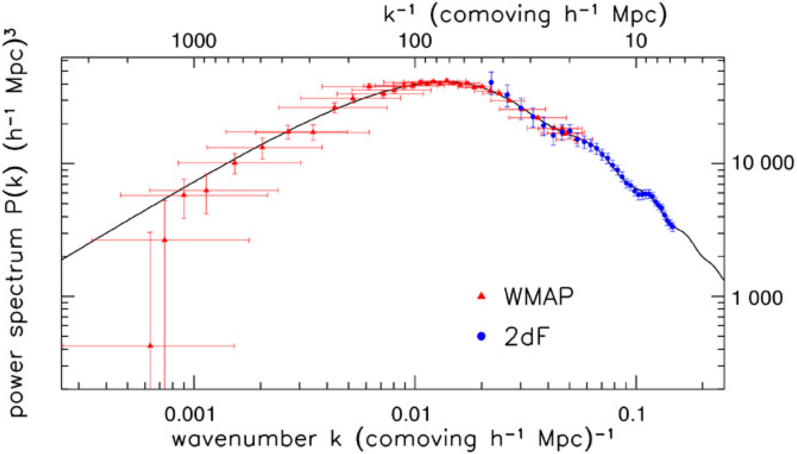

Fig 8.17 (A. Sanchez) 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

Data from WMAP (triangles) and the 2dF galaxy survey (dots) are combined to trace the power spectrum $P(k)$. The smooth curve shows the prediction from a flat ($k = 0$) model similar to the benchmark cosmology. The wiggle at $k \simeq 0.1$ is an acoustic peak on a scale of ~ 10 Mpc, too small to be measured by WMAP.