Lecture 23
Gravitational lensing

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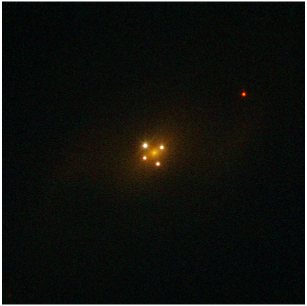
Discovery

The first gravitationally-lensed object discovered was the 'double quasar', Q0957+561, found in 1971. It consists of two star-like images separated by 6 arcsec.

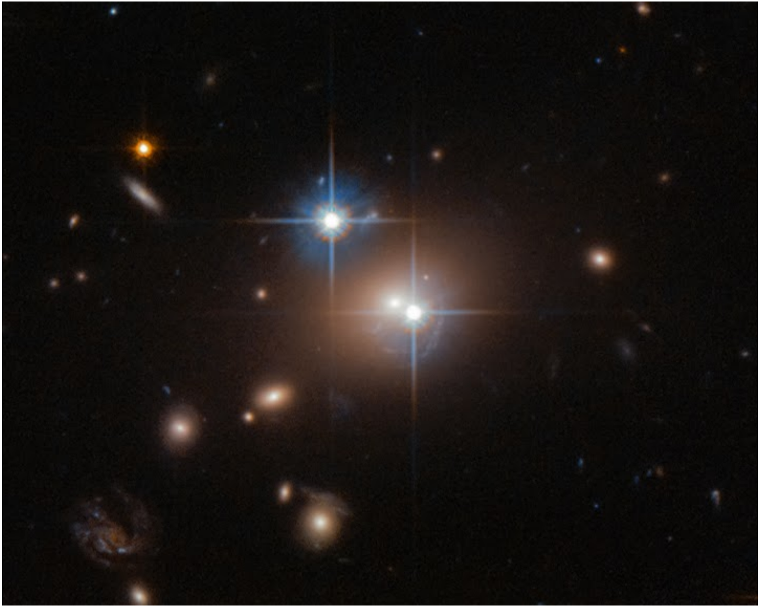
Both were found to have identical spectra - that of a quasar with a redshift of $z = 1.41$. A faint foreground galaxy, with a redshift of 0.355, was found superimposed on one of the images.

It was at first disputed that this was a gravitational lens, as the galaxy did not have sufficient mass to produce such a wide separation. However it was soon found that the galaxy was part of a cluster.

Similar objects were discovered later, such as Q2237+030, the *Einstein Cross*.



Q0957+561, NASA/ESA HST.



Q 0957+561, NASA/ESA HST.

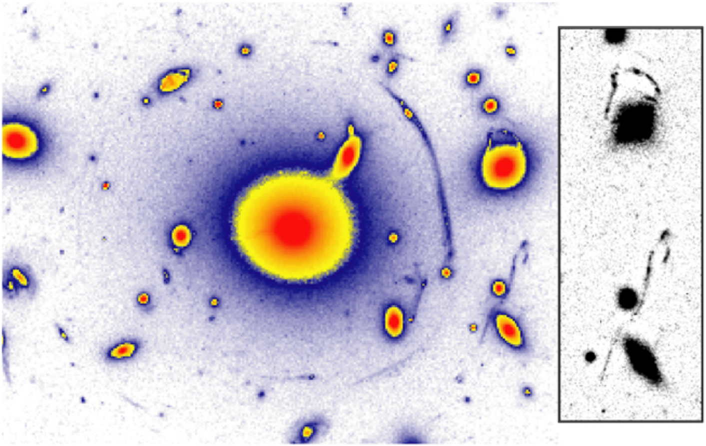


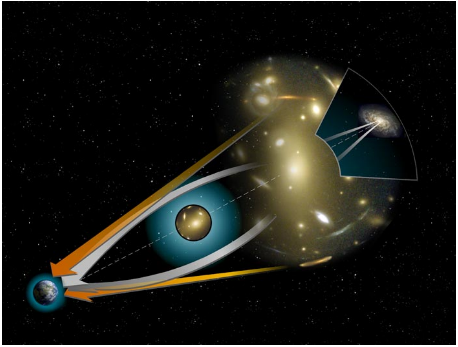
Fig 7.13 (G. Smith) 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

Cluster Abell 383 at $z = 0.188$. The bright arc 16" in radius curving around the central cD galaxy is the image of a background galaxy at $z = 1.01$. The inset shows details of complex arcs where light from another distant galaxy passes close to individual cluster galaxies.

Lensing by clusters

The first gravitational 'arcs' were discovered accidentally in images taken with the Canada-France-Hawaii telescope on Mauna Kea. There was initially some debate about the nature of these, but the gravitational lens explanation of the arcs was soon established.

Bending light around a massive object from a distant source. The orange arrows show the apparent position of the background source. The white arrows show the path of the light from the true position of the source (Wikimedia).



Gravitational deflection of light

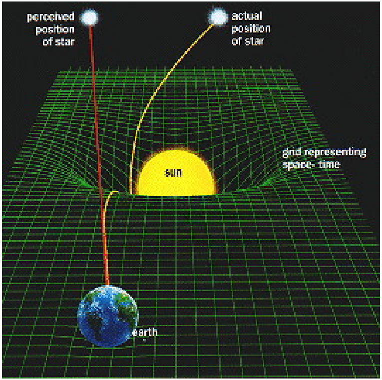
The gravitational field of a point mass is described by the Schwarzschild solution of Einstein's equations, which describes the curvature of spacetime. Light travels along geodesics (shortest distance between two points) in the curved spacetime.

The result is an angular deflection of angle

$$\alpha \simeq \frac{4GM}{bc^2} = \frac{2r_s}{b},$$

where b, the impact parameter, is the distance at closest approach to the mass.

If the lens is far away, we can assume that the bending occurs instanteously.



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Lensing by a point mass

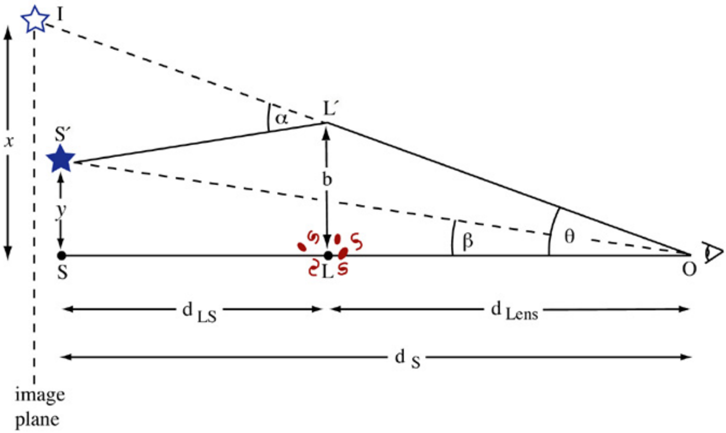


Fig 7.14 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

The gravity of the mass M at L bends light from a distant source at S' toward the observer at O; the source appears instead at position I.

The lens equation

From the previous figure we see that

$$\theta - \beta = \frac{\alpha d_{\mathsf{LS}}}{d_{\mathsf{S}}} = \frac{2r_{\mathsf{s}} d_{\mathsf{LS}}}{\theta d_{\mathsf{L}} d_{\mathsf{S}}} \equiv \frac{\theta_{\mathsf{E}}^2}{\theta}.$$

where

$$\theta_{\mathsf{E}} = \sqrt{\frac{4GM d_{\mathsf{LS}}}{c^2 d_{\mathsf{L}} d_{\mathsf{S}}}}$$

is called the (angular) **Einstein radius**.

The equation relating the apparent angular position θ to the true (undeflected) position β is called the **lens equation**

$$\theta^2 - \beta\theta - \theta_{\mathsf{E}}^2 = 0.$$

The lens equation

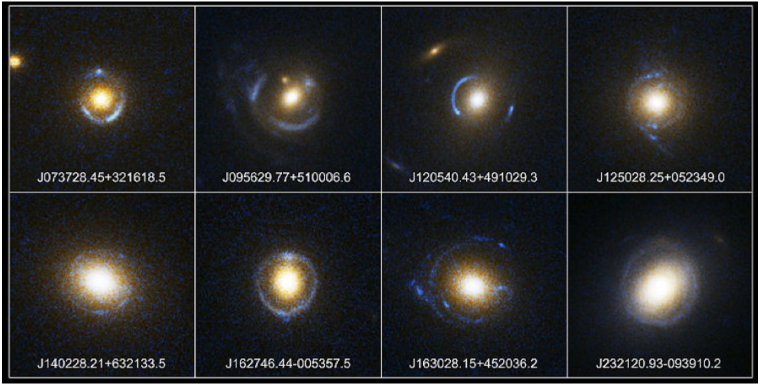
The lens equation is quadratic and generally has two solutions,

$$\theta_{\pm} = \frac{1}{2}\left(\beta \pm \sqrt{\beta^2 + 4\theta_E^2}\right).$$

In the case of perfect alignment, $\beta = 0$ and so $\theta = \theta_E$. We see a ring surrounding the lens, with angular radius θ_E.

In the case that the source is offset from the lens, we see two images, on opposite sides of the mass. If the source has finite size, these appear as arcs.

Einstein rings



Images of Einstein rings, from the Hubble Space Telescope (credit: NASA/ESA)..

Amplification

The light deflection changes the geometry of the source, increasing its solid angle. The intensity is conserved, so the flux increases in proportion to the area. The flux amplification is

$$A = \left| \frac{\theta}{\beta} \frac{d\theta}{d\beta} \right| .$$

Evaluating this using the lens equation,

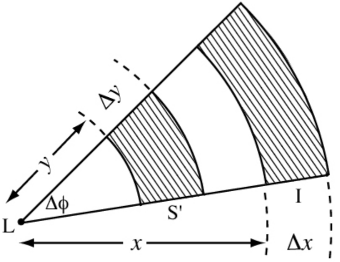


Fig 7.15 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

Magnification of an image by gravitational lensing. (recall that, $x = \theta d_S$ and $y = \beta d_S$.)

$$A_{\pm} = \frac{1}{4} \left(\frac{\beta}{\sqrt{\beta^2 + 4\theta_E^2}} + \frac{\sqrt{\beta^2 + 4\theta_E^2}}{\beta} \pm 2 \right).$$

Deflection by an extended mass

An extended mass can be thought of as the sum of many point masses. First observe that the gravitational deflection of a point mass can be written as the gradient of a potential

$$\alpha(b) = \frac{d\psi_{\mathsf{L}}}{db}, \quad \text{where } \psi_{\mathsf{L}} = \frac{4GM}{c^2} \ln b.$$

For an extended source, we make the potential a two dimensional function of the mass **surface density** $\Sigma(\boldsymbol{x})$.

$$\boldsymbol{\alpha}(\boldsymbol{x}) = \nabla \psi_L(\boldsymbol{x}), \quad \text{where } \psi_L(\boldsymbol{x}) = \frac{4G}{c^2} \int \Sigma(\boldsymbol{x}') \ln \left| \boldsymbol{x} - \boldsymbol{x}' \right| d^2x'.$$

and

$$\Sigma(x, y) = \int \rho(x, y, z) dz.$$

Newton's theorems for gravitational lensing

By analogy with Newtons theorems, one can prove that if the surface density has circular symmetry:

The deflection depends only on the surface density interior to the light ray.

The deflection is equivalent to that of a point mass equal to the mass interior to the ray

$$M(r) = 2\pi \int_0^r \Sigma(r')r'dr'$$

located on the axis.

For a proof of these see Sparke and Gallagher.

Critical surface density

The lens equation now takes the form

$$\theta - \beta = \frac{1}{\theta}\frac{4GM(b)}{c^2}\frac{d_{\mathsf{LS}}}{d_{\mathsf{L}}d_{\mathsf{S}}}.$$

This can be rearranged, recalling that $b = \theta d_{\mathsf{L}}$, to give,

$$\theta = \beta\left[1 - \frac{\bar{\Sigma}(b)}{\Sigma_{\text{crit}}}\right]^{-1}$$

where

$$\bar{\Sigma}(b) = \frac{M(b)}{\pi b^2}, \quad \text{and } \Sigma_{\text{crit}} = \frac{c^2}{4\pi G}\frac{d_{\mathsf{S}}}{d_{\mathsf{L}}d_{\mathsf{LS}}}.$$

If $\beta = 0$ (perfect alignment), θ will be nonzero only if $\bar{\Sigma}(b) = \Sigma_{\text{crit}}$ for some value of r. One then sees an Einstein ring of radius b/d_{L}.

If the mean surface density is below the critical density for all r, there can be no ring or multiple images.

Mapping cluster dark matter

The positions, shapes, and fluxes of the lensed images depend on the distribution of mass in the cluster.

Points where $d\beta/d\theta = 0$ correspond to very high amplification. They form curves called **caustics** in the source plane and **critical curves** in the image plane.

Gravitational arcs can be used to map the distribution of the mass in clusters of galaxies.

The technique is to compare the predictions of mass density and source distribution models with the observed arc positions and magnitudes.

The results are in general agreement with the mass distribution derived from X-ray.

Lensing by an extended mass

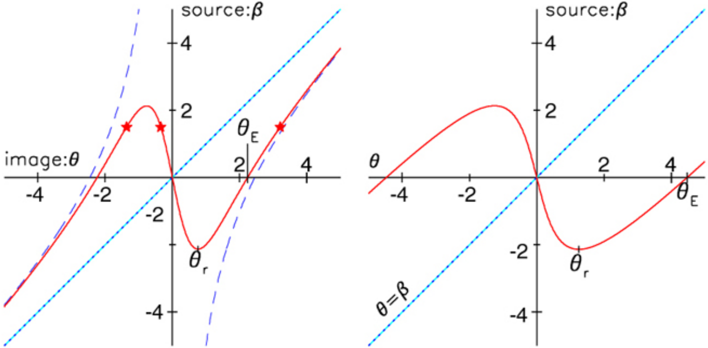


Fig 7.18 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

Positions of a lensed galaxy and its image. Left, for a Plummer sphere with $\Sigma(0) = 6\Sigma_{\text{crit}}$ (solid curve); angles are in units of a_P/d_L. Rays passing far from the centre are bent in almost the same way as by an equal point mass (dashed curve): stars show images of a source at $\beta = 1.5$. Right, for a 'dark halo' potential; angles are in units a_H/d_L. With more mass at large radii the curve approaches the dotted line $\theta = \beta$ (no bending) much more slowly.

Weak lensing

If galaxies are far outside the Einstein radius, or the cluster surface density is below the critical density, the images are weakly magnified and become slightly elongated in the tangential direction.

The difference between the magnification in the tangential and radial directions is related to the **shear**

$$\gamma \equiv \frac{1}{2}\left(\frac{d\beta}{d\theta} - \frac{\beta}{\theta}\right) = \frac{\bar{\Sigma}(b) - \Sigma(b)}{\Sigma_{\text{crit}}}.$$

For any particular galaxy, we do not know the intrinsic shape, but expect that on average galaxies will be randomly oriented. By measuring many galaxies around the cluster and looking for a systematic shear aligned with the cluster centre, one can estimate the mass surface density.

The "Bullet Cluster"

Evidence for dark matter, and against modified gravity theories, comes from an interesting cluster of galaxies, 1E0657-558, a cluster at $z = 0.296$.

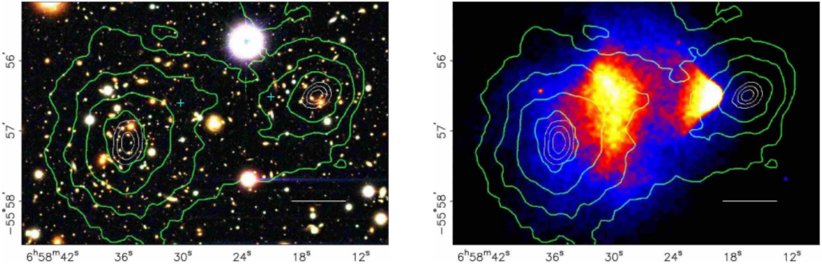
Optical observations reveal two distinct concentrations of galaxies. But X-ray observations show that the hot gas, which contains most of the baryonic matter, lies *between* these two concentrations.

The interpretation is that these are two clusters that have recently collided. The galaxies passed between each other, but the gas was swept out of both clusters.

Weak lensing studies show that the mass distribution coincides with the galaxy distribution, and not with the gas.

This would appear to rule out "modified gravity" models in favour of non-interacting dark matter.

The "Bullet Cluster"



Images of the Bullet Cluster. On the left the colour image from the Magellan telescope. On the right is the Chandra Xray image. The green contours in both images are the weak lensing convergence map (Clowe et al. 2006, ApJL, 648, L109).

These show that the mass distribution follows the galaxies, and not the gas (which contains most of the baryons). This is counter to any "modified gravity" models in which gravity arises from baryons, but with a modified force law.

Microlensing

Small masses such as stars and planets can also produce gravitational lensing. In this case the Einstein radius is too small to be resolved, so multiple images cannot be observed.

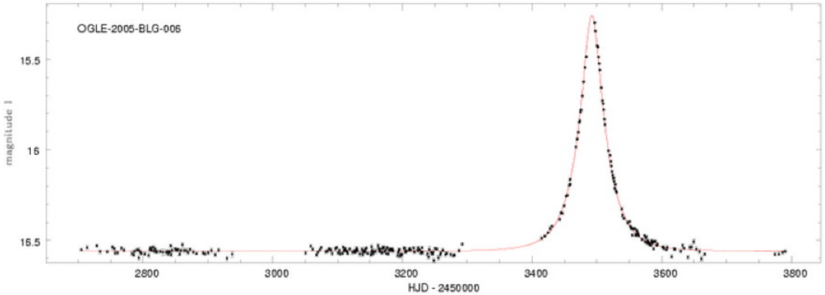
However, one can detect the amplification. This is **microlensing**.

If the mass is moving with respect to the line of sight to the source, the amplification varies with time as the impact parameter changes. By monitoring the light curve of the source, one can measure the mass and velocity of the lensing object.

This technique has been used to search for massive compact objects in the halo of the Galaxy (MACHO's), by monitoring millions of background stars.

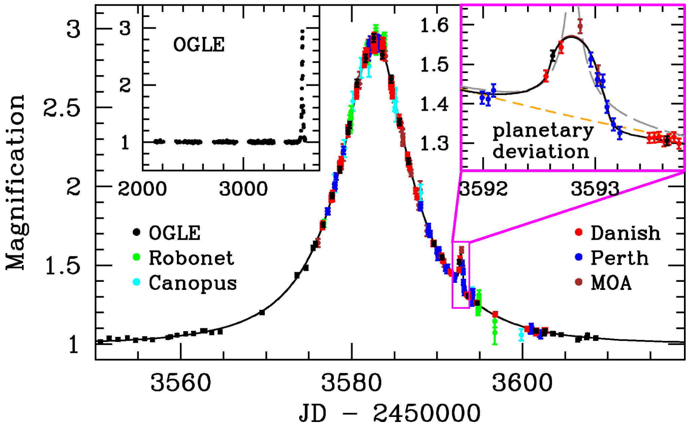
It has also resulted in the detection of planets orbiting the star that is the primary lens.

Microlensing in the Galactic halo



Light curve of a star showing a gravitational microlens event (OGLE team).

Microlensing by a planet



Light curve of the microlens event OGLE-2005-BLG-390Lb where the planet caused a second maximum. The horizontal axis covers a period of 70 days; the magnified part in the upper right corner covers only 2 days (PLANET group).