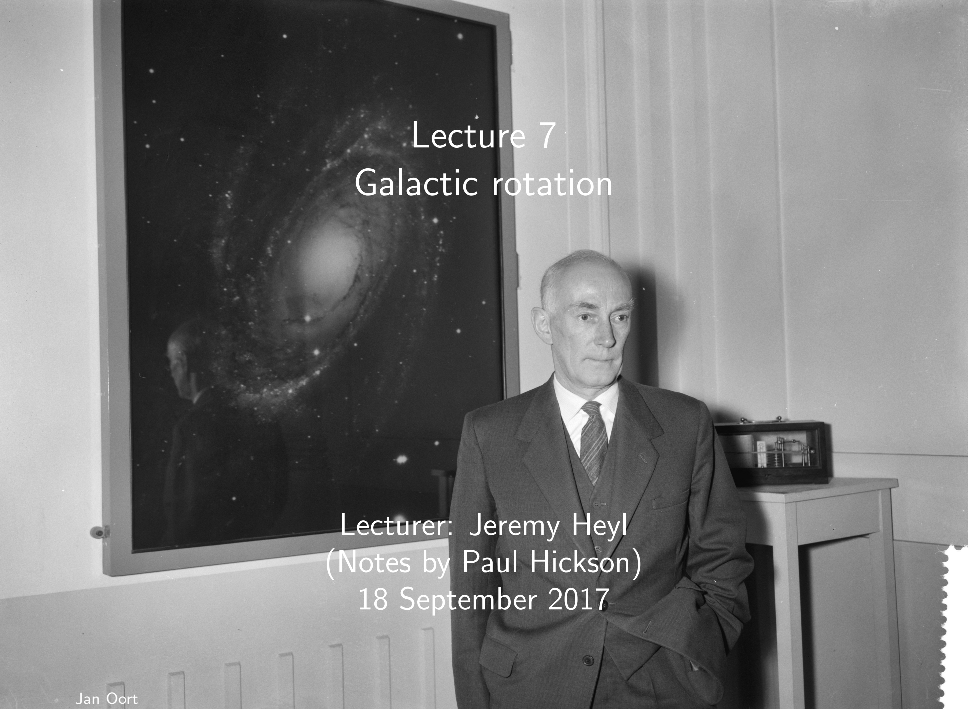
Lecture 7
Galactic rotation



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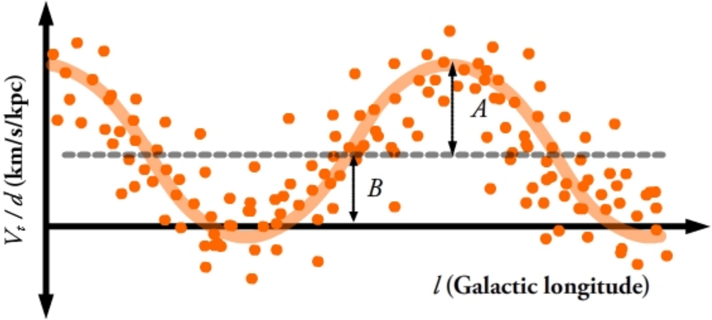
History

Galactic rotation was discovered, from the inside, looking out, by measurement of proper motions of nearby stars. This angular motion depends on the *difference* between the motion of the star and the motion of the Sun.

It was observed that their was a systematic component to the proper motion that varied with galactic longitude l as $\mu \propto \cos(2l)$.

The reason for this was explained by Jan Oort in 1927.

Tangential velocities vs galactic longitude.



Wikipedia

Relative motions of stars with respect to the Sun.

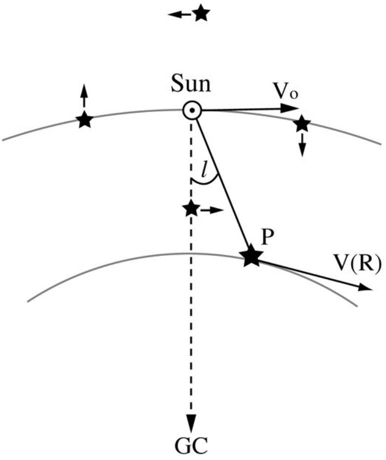


Fig 2.18 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

Relative motions of stars with respect to the Sun.

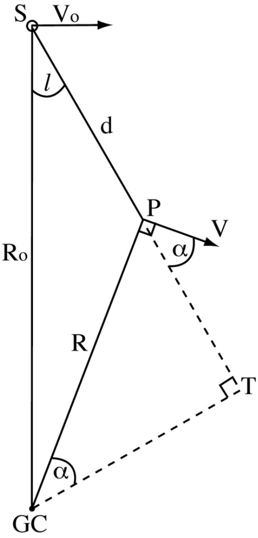


Fig 2.19 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

Oort's analysis

Oort assumed that stars in the disk were moving in circular orbits around the galactic centre (GC), with a velocity $V(R)$ that depends only on R, the radius of the orbit.

Denote the Sun's velocity and radius by V_0 and R_0 and define $\omega = V/R$, $\omega_0 = V_0/R_0$.

Referring to Fig 2.19, we see that the observed radial velocity of the star will be

$$V_r = V\cos\alpha - V_0\cos(\pi/2 - l) = V\cos\alpha - V_0\sin l.$$

The angle α can be eliminated using the sine rule,

$$\frac{\sin(\alpha + \pi/2)}{R_0} = \frac{\cos\alpha}{R_0} = \frac{\sin l}{R}.$$

Therefore,

$$V_r = R_0\left(\frac{V}{R} - \frac{V_0}{R_0}\right)\sin l = R_0(\omega - \omega_0)\sin l.$$

Oort's analysis

R can be found from the cosine rule

$$R^2 = R_0^2 + d^2 - 2dR_0 \cos l.$$

If the star is not too distant, $d \ll R_0$ and we can ignore the d^2 term. We get

$$R^2 - R_0^2 = (R + R_0)(R - R_0) \simeq 2R_0(R - R_0) \simeq -2dR_0 \cos l$$

so

$$R - R_0 \simeq -d \cos l$$

Oort's analysis

Now expand $\omega(R)$ in a Taylor series about the point R_0,

$$\omega(R) = \omega_0 + (R - R_0)\left[\omega'\right]_{R_0} + \frac{1}{2}(R - R_0)^2\left[\omega''\right]_{R_0} + \cdots$$

where the prime denotes a derivative with respect to R.

If the distance to the star is small compared to the distance over which w' changes significantly, we can ignore the second and higher-order terms in the expansion. Thus

$$\omega - \omega_0 \simeq (R - R_0)\left[\omega'\right]_{R_0} \simeq -d\left[\omega'\right]_{R_0}\cos l.$$

Putting this into our equation for the radial velocity and recalling that $2\sin l\cos l = \sin(2l)$ gives us Oort's first formula,

$$V_r = Ad\sin(2l),$$

where $A = R_0\left[\omega'\right]_{R_0}/2$.

Oort's analysis

Now consider the tangential component of the relative velocity,

$$V_t = V \sin\alpha - V_0 \cos l$$

From the figure we see that $R_0 \cos l = R \sin\alpha + d$. Therefore,

$$V_t = R_0 \left(\frac{V}{R} - \frac{V_0}{R_0} \right) \cos l - \frac{V}{R} d = R_0(\omega - \omega_0) \cos l - \omega d.$$

Using our previous result for ω this becomes

$$V_t = dR_0 \left[\omega' \right]_{R_0} \cos^2 l - \omega_0 d - d^2 \left[\omega' \right]_{R_0}.$$

For stars near the Sun the last term can be ignored and we get Oort's second formula,

$$V_t = d[A \cos(2l) + B],$$

where $B = A - \omega_0$.

Oort's analysis

A and B are called **Oort constants** and there values are estimated to be

$$A = 14.8 \pm 0.8 \text{ km s}^{-1} \text{ kpc}^{-1}$$
$$B = -12.4 \pm 0.6 \text{ km s}^{-1} \text{ kpc}^{-1}.$$

They measure, respectively, the local shear (departure from rigid-body rotation) and vorticity (angular momentum gradient) in the disk.

From the definitions, we see that

$$A - B = \omega_0 = V_0/R_0.$$

Putting in $R_0 = 7.6$ kpc, we find that $V_0 \simeq 207$ km/s.

21 cm Rotation curves

The disk, interior to the Sun, is heavily obscured by dust. However radio waves easily penetrate this. Radio observations can probe the gas component and measure the radial velocity from the 21-cm emission line of neutral hydrogen.

The 21-cm spectrum is the sum of all the hydrogen emission along the line of sight, and therefore includes emission from clouds moving a different speeds as they orbit the galaxy.

We do not know the distance to any of the sources of emission, but can use the **tangent-point method** to measure $V(R)$.

The angular velocity $\omega(R)$ increases as R decreases, so the maximum radial velocity occurs for clouds that are at the tangent point, where the line of sight is closest to the GC. At that point we have

$$R = R_0 \sin l, \quad \text{and} \quad V(R) = V_r + V_0 \sin l.$$

Rotation curve for the Milky Way.

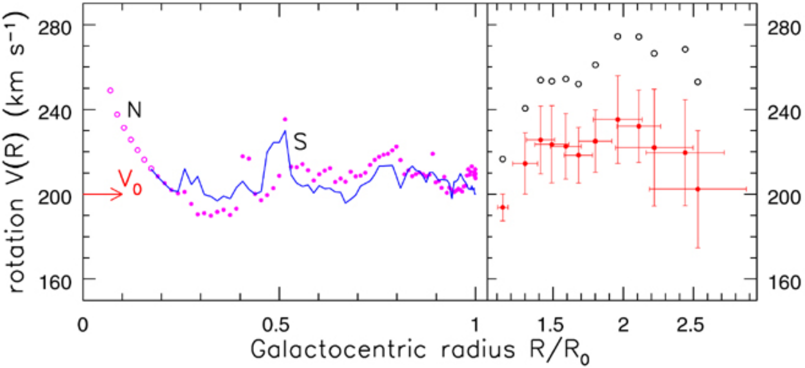


Fig 2.21 (Burton, Honma) 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

Dots: $l > 270^\circ$, line: $l < 90^\circ$, purple circles: galactic bar. For $R > R_0$, black circles assume $V_0 = 200$ km/s, filled circles $V_0 = 220$ km/s (Burton & Honma).

Dark matter in the Galaxy

If the mass distribution in the galaxy is spherically symmetric, the mass interior to radius R is given by

$$M(R) = \frac{V^2 R}{G}$$

In units of pc, Myr and $M_\odot$, Newton's constant has the value $G = 4.5 \times 10^{-3}$.

Since the rotation velocity is observed to be nearly constant, the mass must increase roughly in proportion to radius, $M(R) \propto R$.

However, the density of stars fall off rapidly, so the amount of stellar mass does not increase as fast, and is insufficient to explain the flat, and high, rotation velocity. Additional mass is required, which we do not see.

It is presumed to be some form of **dark matter**.