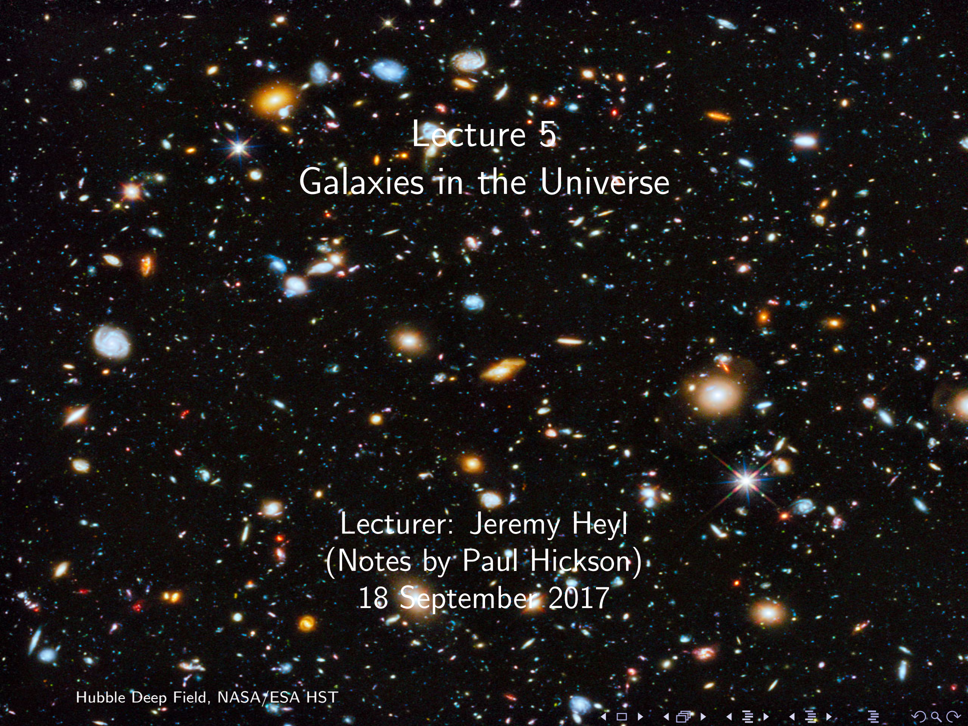
Lecture 5
Galaxies in the Universe

Lecturer: Jeremy Heyl
(Notes by Paul Hickson)
18 September 2017



Hubble Deep Field, NASA/ESA HST

The expansion of the Universe

By the early 20th century, galaxy spectra measured by Vesto Slipher and Milton Humason were showing an excess of redshifts over blueshifts.

Galaxies that appeared to be more distant had higher redshifts than nearby galaxies.

Based on these limited observations, Hubble proposed, in 1929 that galaxies had systematic redshifts that increased with distance.

Since radial velocity V_r is proportional to redshift (for $V_r \ll c$), this can be written as

$$V_r = H_0 d.$$

where H_0 is called the **Hubble constant**.

These systematic radial velocities are in addition to random (**peculiar velocities**) that galaxies have due to the gravitational attraction of their neighbours.

The original Hubble diagram

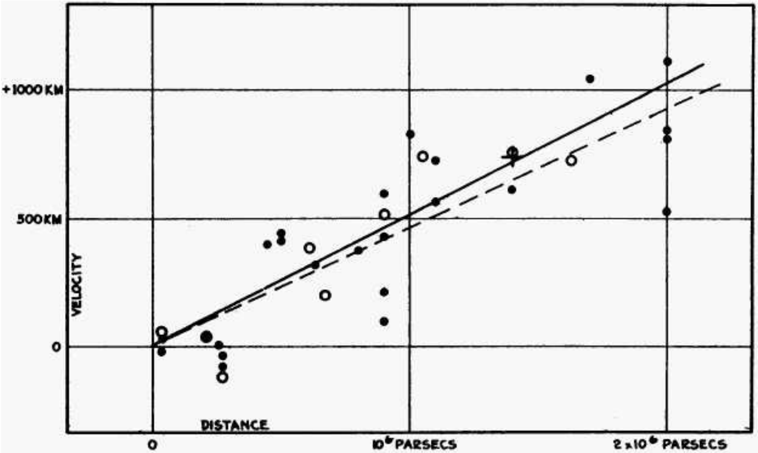


FIGURE 1

Velocity-Distance Relation among Extra-Galactic Nebulae.

Hubble 1929, PNAS, 15, 168.

The Hubble constant

The best present estimate is $H_0 \simeq 67.80 \pm 0.77$ km s^{-1} Mpc^{-1}, about an order of magnitude smaller than Hubble's original estimate.

As there is some uncertainty in the exact value, it is common to use the dimensionless parameter h defined by $h = H_0/(100$ km s^{-1} Mpc$^{-1})$.

The current estimate then is that $h \simeq 0.68$.

Warning: This h is not Planck's constant (which the text book writes as h_P). You will need to determine what quantity is meant by this symbol from the context.

Hubble diagram for brightest galaxies in clusters.

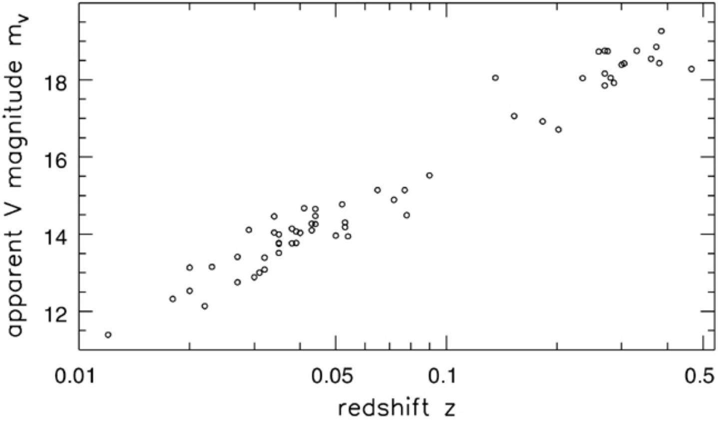


Fig 1.17 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

Gunn & Oke 1975, ApJ, 195, 255.

The age of the Universe

If the expansion velocities of the galaxies have been constant, the distance between them would have been zero at a time

$$t_H = \frac{d}{v} = \frac{d}{H_0 d} = \frac{1}{H_0} \simeq 9.78 h^{-1} \text{ Gyr} \simeq 15 \text{ Gyr}$$

before the present.

One would expect that gravity should slow the expansion as time progresses. This would make H_0 smaller, so t_H would over estimate the age. In fact for a universe dominated by matter (visible and dark), the age is

$$t_0 = \frac{2}{3} t_H \simeq 10 \text{ Gyr}.$$

However, we see stars that are older than this!

This problem is resolved by **dark energy**, which causes the expansion rate to increase with time.

The cosmological scale factor

The simplest models assume that on large scales, the universe is **homogeneous** and **isotropic**.

In these models, the expansion can be represented by a **scale factor** $R(t)$ [$a(t)$ is also commonly used] that increases with time.

All distances increase in proportion to the scale factor, therefore $d(t) \propto R(t)$.

From this it follows that the radial velocity

$$V_r = \frac{d}{R}\frac{dR}{dt} \equiv H(t)d.$$

where

$$H(t) = \frac{1}{R}\frac{dR}{dt}$$

is the **Hubble parameter**. Its present value is the Hubble constant $H_0 = H(t_0)$, where t_0 denotes the present time (starting with $t = 0$ at the start of the expansion).

Cosmological redshift

For nearby galaxies, the nonrelativistic Doppler formula gives a wavelength shift $\Delta\lambda = \lambda_o - \lambda_e$ (observed - emitted wavelength)

$$\frac{\Delta\lambda}{\lambda} = \frac{V_r}{c} = \frac{Hd}{c} = H\Delta t = \frac{1}{R}\frac{dR}{dt}\Delta t,$$

where $\Delta t = d/c$ is the time needed for the light to reach us.

Rearranging this, and taking the limit as $\Delta t \to 0$ gives the equation

$$\frac{1}{\lambda}\frac{d\lambda}{dt} = \frac{1}{R}\frac{dR}{dt}.$$

This equation tells us that $\lambda \propto R$ and therefore

$$1 + z \equiv \frac{\lambda_o}{\lambda_e} = \frac{R(t_0)}{R(t_e)} \equiv \frac{R_0}{R},$$

which is valid for all redshifts.

1.5 The pregalactic era

Extrapolating the expansion back to early times, before the gas in the Universe had formed stars, we find that it must have originally been very hot and dense, and expanding rapidly.

The density of (nonrelativistic) gas is proportional to the inverse cube of the scale factor,

$$\rho_g \propto V^{-1} \propto R(t)^{-3}.$$

The Universe was also filled with blackbody radiation in thermal equilibrium with the gas. This radiation had the Planck spectrum

$$I_\nu = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1},$$

which peaks at a wavelength that is inversely proportional to temperature (Wein's displacement law), $\lambda \propto T^{-1}$.

Cosmic microwave background radiation

As the Universe expanded, the wavelengths of the photons increased due to the cosmological redshift, $\lambda \propto R$.

As a result, the temperature corresponding to the peak of the radiation, decreased as the universe expanded, according to

$$T \propto R^{-1}.$$

The energy density of the radiation is given by

$$\rho_{\mathsf{r}} = \frac{4\sigma_{\mathsf{SB}}}{c} T^4 \equiv a_{\mathsf{SB}} T^4,$$

which decreases as the Universe expands

$$\rho_{\mathsf{r}} \propto R^{-4}.$$

The first few seconds

According to General Relativity, the temperature T is related to the time t, measured from the Big Bang, by

$$t = \sqrt{\frac{3c^2}{32\pi G\rho_{\mathsf{r}}}} \simeq 230\ \mathsf{s}\left(\frac{10^9\ \mathsf{K}}{T}\right).$$

In the first few hundred microseconds after the Big Bang, the temperature would have been greater than 10^{13} K.

The typical energy of a photon in the radiation is $\sim kT$. This would have been been greater than the rest mass-energy of a proton, $m_{\mathsf{p}}c^2$.

Photons could produce proton-antiproton pairs (p, $\bar{\mathrm{p}}$)

$$\gamma + \gamma \leftrightarrow \mathsf{p} + \bar{\mathsf{p}}$$

and vice versa. Such reactions allowed the photons and matter to be in thermal equilibrium.

The first few seconds

As the Universe expanded and cooled, the energy of the photons dropped until they could no longer produce proton-antiproton pairs. The protons and antiprotons then annihilated, producing more photons.

There was a small excess of protons over anitprotons, about one part per billion. Because of this there were a few protons left over.

These excess protons formed all the hydrogen and helium in the Universe.

Neutrons and protons are called **baryons** (heavy particles).

The first three minutes

Neutrons and protons can be converted into one another by the weak interactions

$$\begin{aligned} \mathsf{p} + \mathsf{e}^- &\leftrightarrow \mathsf{n} + \nu_\mathsf{e} \\ \mathsf{p} + \bar{\nu}_e &\leftrightarrow \mathsf{n} + \mathsf{e}^+ \\ \mathsf{n} &\leftrightarrow \mathsf{p} + \mathsf{e}^- + \bar{\nu}_\mathsf{e}. \end{aligned}$$

Here ν_e denotes an electron-type neutrino and a bar denotes an antiparticle.

The neutron is slighly more massive than the proton, so more energy is required to make one. The ratio of neutrons to protons in thermal equilibrium is given by the Boltzman equation,

$$n/p = e^{-(m_\mathsf{n} - m_\mathsf{p})c^2/kT}.$$

Nucleosynthesis

As the temperature and density dropped, the reaction time became longer than the age of the Universe, and so the ratio of neutrons to protons "froze out" at a value of about $1/5$. Neutrons began to decay into protons.

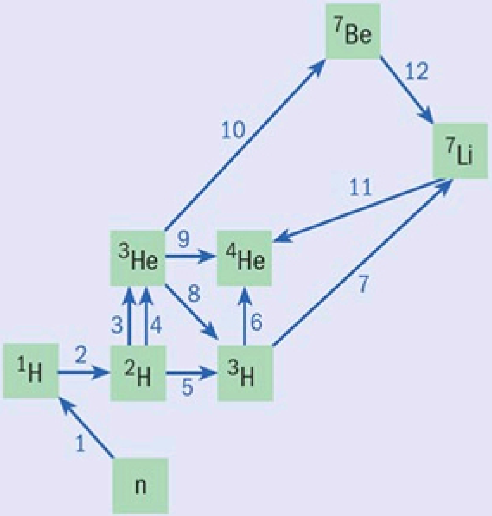
When the temperature dropped below about 0.8 meV, about 1 second after the Big Bang, neutrons and protons could begin to combine to form deuterium, which was then quickly converted to helium. By then the ratio of neutrons to protons had dropped to about $1/7$.

For each helium nucleus, two protons and two neutrons are required. That left 12 free protons for each helium nucleus.

Thus the fraction of helium in the Universe, by mass, is

$$Y = \frac{4}{4 + 12} = 0.25.$$

Big Bang nucleosynthesis.



1 $n \longrightarrow {}^1H + e^- + \bar{\nu}$

2 ${}^1H + n \longrightarrow {}^2H + \gamma$

3 ${}^2H + {}^1H \longrightarrow {}^3He + \gamma$

4 ${}^2H + {}^2H \longrightarrow {}^3He + n$

5 ${}^2H + {}^2H \longrightarrow {}^3H + {}^1H$

6 ${}^2H + {}^3H \longrightarrow {}^4He + n$

7 ${}^3H + {}^4He \longrightarrow {}^7Li + \gamma$

8 ${}^3He + n \longrightarrow {}^3H + {}^1H$

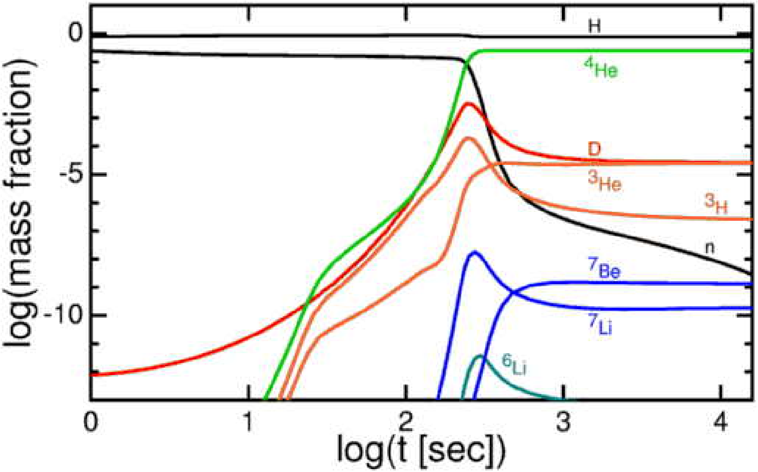
9 ${}^3He + {}^2H \longrightarrow {}^4He + {}^1H$

10 ${}^3He + {}^4He \longrightarrow {}^7Be + \gamma$

11 ${}^7Li + {}^1H \longrightarrow {}^4He + {}^4He$

12 ${}^7Be + n \longrightarrow {}^7Li + {}^1H$

Big Bang nucleosynthesis.



ESA/Planck

Nucleosynthesis

The mass fraction of light elements produced by cosmological nucleosynthesis (duterium, helium-3 and lithium) is sensitively dependent on the ratio of baryons to photons.

The expansion rate of the Universe depends on the density of mass/energy ρ,

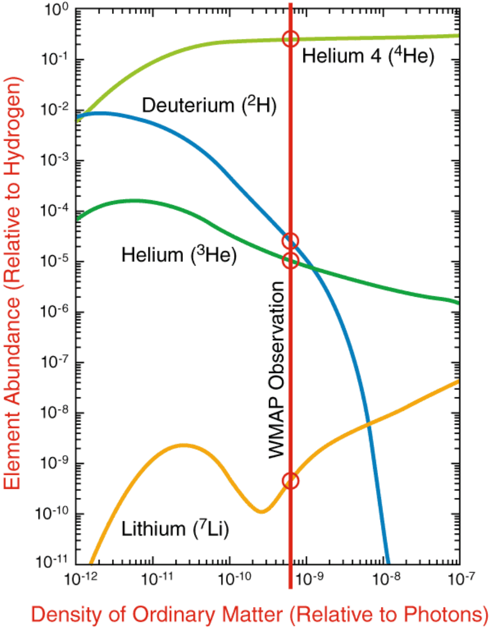
$$\left(\frac{1}{R}\frac{dR}{dt}\right)^2 \simeq \frac{8\pi G}{3}\rho.$$

Observations indicate that the amount of baryonic matter determined from the abundances of light elements is insufficient to explain the current and past expansion rate.

For this, dark matter is needed, and for the later expansion rate, dark energy.

Nucleosynthesis also predicts that the number of neutrino species is three, in agreement with observations (electron, mu and tau neutrinos). This comes primarily from the Helium abundance.

Light element abundances vs baryon/photon ratio



The hot plasma

At temperatures above $\sim 10^5$ K, the energy density of the radiation was greater than that of matter, and dominated the evolution of the Universe.

When the temperature dropped below this, dark matter was free to begin to collapse under its own gravity.

However, ordinary (baryonic) matter and radiation were closely coupled due to scattering with the free electrons. This prevented baryonic matter from collapsing under its own gravity or falling into the clumping dark matter.

After about 300,000 years, the temperature had dropped to a low enough value, about 3000 K, to allow ions and electrons to combine to form atoms.

Recombination and the CMB

Once the gas became neutral, photons were no longer coupled to matter and could propagate freely throughout the Universe. We see them today (redshifted) as the cosmic microwave background radiation.

The gas was then longer constrained by the pressure of the radiation and could collapse to form the first stars and galaxies. These first stars formed about 400 Myrs after the Big Bang.

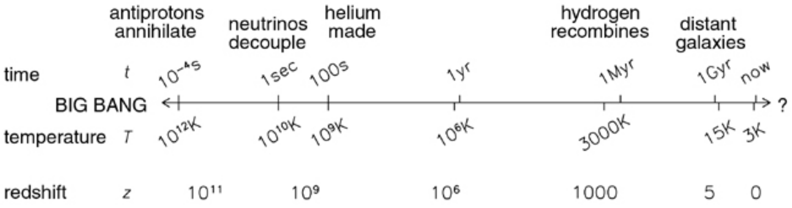
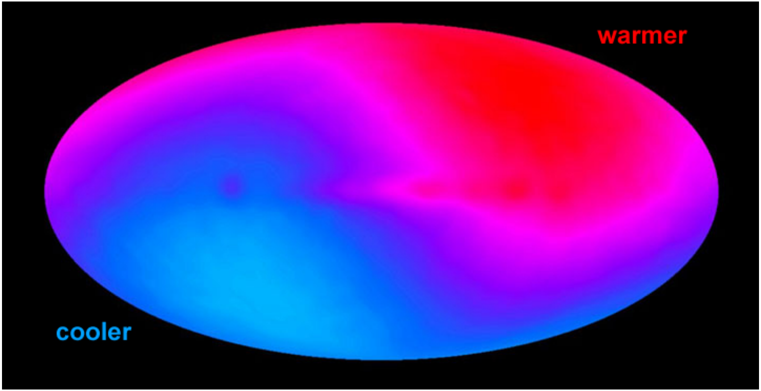


Fig 1.18 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

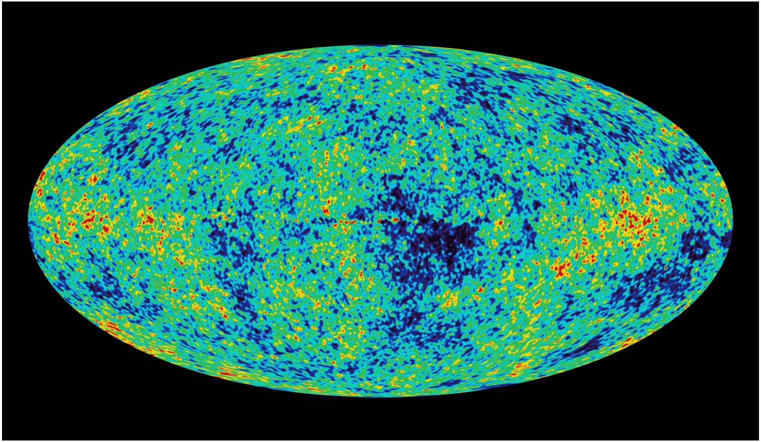
CMB dipole anisotropy



ESA/Planck

A small ($\sim 10^{-3}$) anisotropy in the temperature of the CMB is observed. This is caused by the Doppler shift due to our motion through the Universe, at about 370 km/s.

CMB residual anisotropy



WMAP

Smaller ($\sim 10^{-5}$) anisotropies are seen. These were produced by interaction of the photons with matter as it collapsed to form the first galaxies and clusters.

Extragalactic background radiation

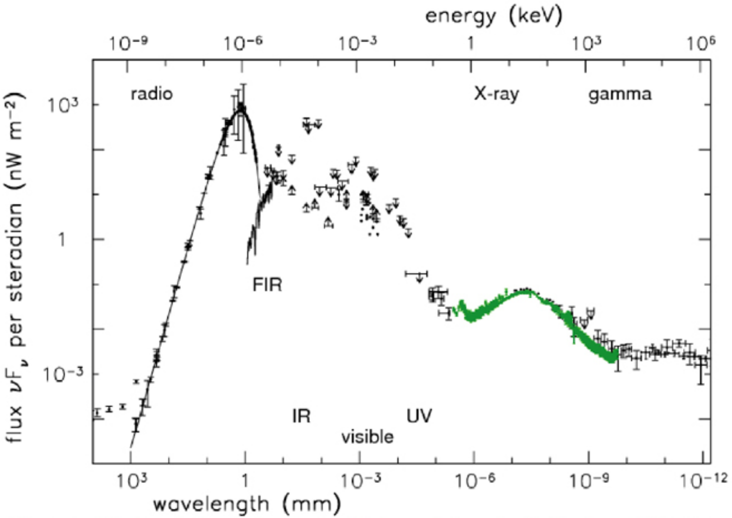


Fig 1.19 (D. Scott) 'Galaxies in the Universe' Sparke/Gallagher CUP 2007