Lecture 2
Stars

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Distances

Distances to nearby stars can be measured by observing their **parallax** due to the motion of the Earth around the Sun.

$$d = \frac{\text{AU}}{\alpha(\text{radians})} = \frac{\text{pc}}{\alpha(\text{arcsec})}$$

Astronomical unit - AU $= 1.496 \times 10^8$ km.

Parsec - pc $= 3.09 \times 10^{13}$ km (3.26 light-years).

Arsecond - 1 arcsec $= 1/3600$ degree $= 4.848 \times 10^{-6}$ radians.

Small angle approximation - For $\alpha \ll 1$, $\sin\alpha \simeq \tan\alpha \simeq \alpha$ (in radians).

Luminosity

If the distance to a star is known, the luminosity of the star may be estimated by measuring the flux and applying the inverse square law,

$$F = \frac{L}{4\pi d^2}$$

For the Sun, and its neighbours, one finds:

Sun's bolometric luminosity - $L_{\odot} = 3.86 \times 10^{26}$ W.

Range of stellar luminosities - roughly $10^{-4} - 10^{6} L_{\odot}$.

Question: How does one measure the distance to the Sun?

Masses

Masses can often be determined for stars in binary systems, or having observable planets. One needs to measure or infer

- orbital period (P)
- semi-major axis of the orbit (a)
- mass ratio M_1/M_2 (for binary star)

Kepler's third law

$$
\begin{aligned}
P^2 &= \frac{4\pi^2 a^3}{GM} \\
M &= M_1 + M_2 \\
a &= a_1 + a_2
\end{aligned}
$$

Sun's mass - $M_\odot = 2.0 \times 10^{30}$ kg.

Range of stellar masses - roughly $0.075 - 100 M_\odot$

Radii

Sun's radius - $R_{\odot} = 6.963 \times 10^5$ km.

For a few nearby stars, the radii can be measured directly by interferometry.

For others, radii can be estimated spectroscopically by fitting their emitted energy distribution to a **black body** at temperature T.

Stefan-Boltzmann equation

$$L = 4\pi R^2 \sigma_{\mathsf{SB}} T^4$$

Stefan-Boltzmann constant - $\sigma_{\mathsf{SB}} = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}.

Effective temperature - $T_{\mathsf{eff}} =$ temperature of a black body that has the same radius and luminosity as the star.

Sun's effective temperature - $T_{\mathsf{eff}} = 5780$ K.

Weins displacement law - $\lambda_{\mathsf{max}} = 2.9/T$ mm/K

Stellar spectra

Optical spectra of main-sequence stars with roughly solar chemical composition.

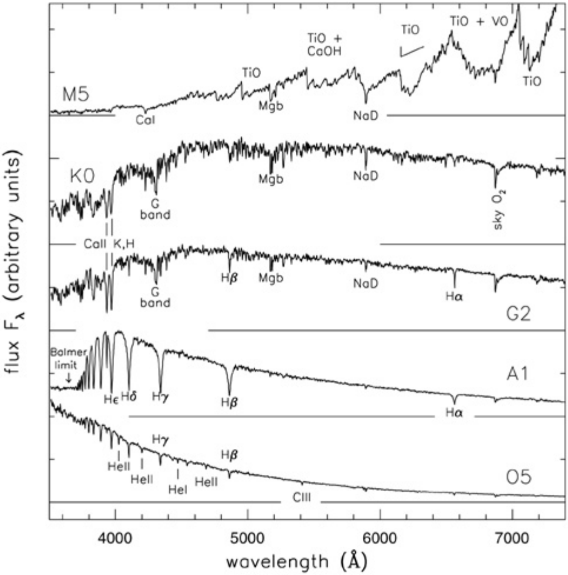


Fig 1.1 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

Spectral classification

O temperatures over 30 000 K, He-II and C-III lines strong, H lines weak.

B lower remperatures than O, stronger H lines, lines of He-I.

A temperatures below 11 000 K , strongest H lines and lines of singly ionized metals.

F weaker H lines than A, and lines of neutral metals.

G temperatures below 6000 K, strong Ca-II lines and CH band, neutral metal lines stronger than in hotter stars.

K lines of neutral metals and of molecules like TiO.

M temperatures below 4000 K, deep bands of TiO and VO and neutral metal lines

L temperatures below 2500 K, weaker lines of TiO and VO, lines of neutral metals and prominent lines of Sodium.

T temperatures below 1400 K, deep lines of water and methane.

Spectral classification

Subclasses - O0 O1 ... O9 B0 B1 ... etc.

Early type stars - O, B, A.

Late type stars - M, L, T.

Brown Dwarfs - stars L5 or greater cannot sustain hydrogen burning.

Effect of surface gravity

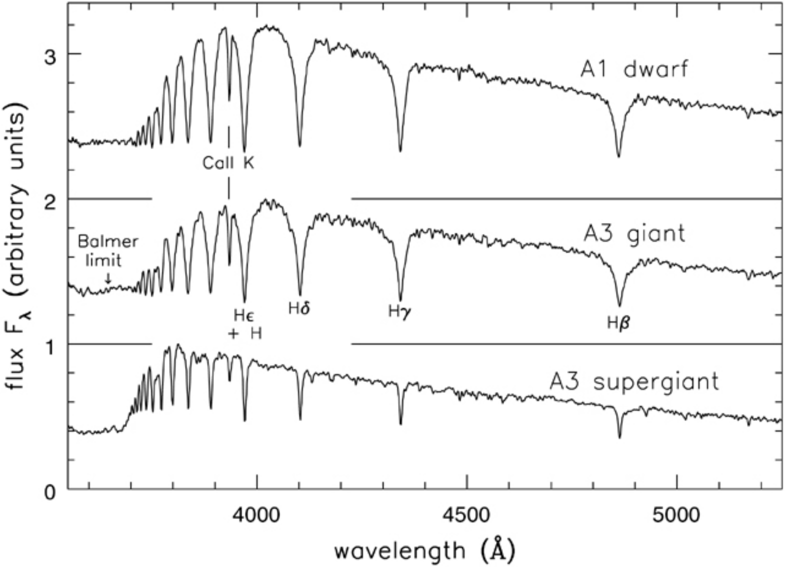


Fig 1.2 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

Effect of surface gravity

Higher g means higher atmospheric pressure which broadens lines due to...

- **Stark effect** - Electric field of nearby electrons and ions split degenerate energy levels, broadening the lines (analogous to the Zeeman effect).
- **Collisional broadening** - collisions reduce effective lifetime of excited state, increasing uncertainty in energy $E = h\nu$ according to Heisenberg's uncertainty principle.

Question: What is the surface gravity of the Sun?

Composition

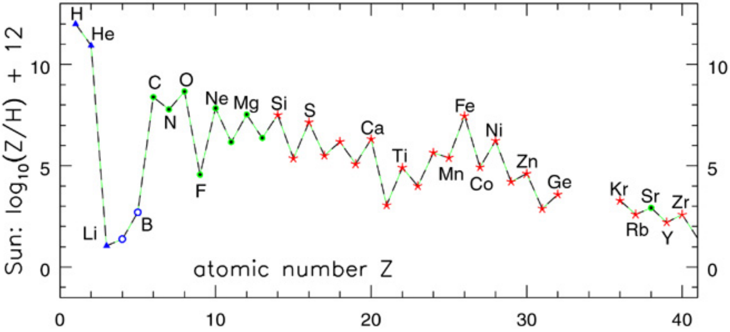


Fig 1.3 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

Fig 1.3. Abundance of elements in the Sun, relative to hydrogen.

Composition

Sun's composition - 72% H, 26% He, 2% others (by mass).

Metals - elements heavier than Helium (also called **heavy elements**).

Origins - H and He (and a small amount of deuterium, ^{3}He and lithium) were produced in the big bang. The rest were made from these by stars or supernovae.

$$[A/B] \equiv \log_{10} \left\{ \frac{\text{number of atoms of A/number of atoms of B}}{\text{this ratio for the Sun}} \right\}.$$

Stellar evolution

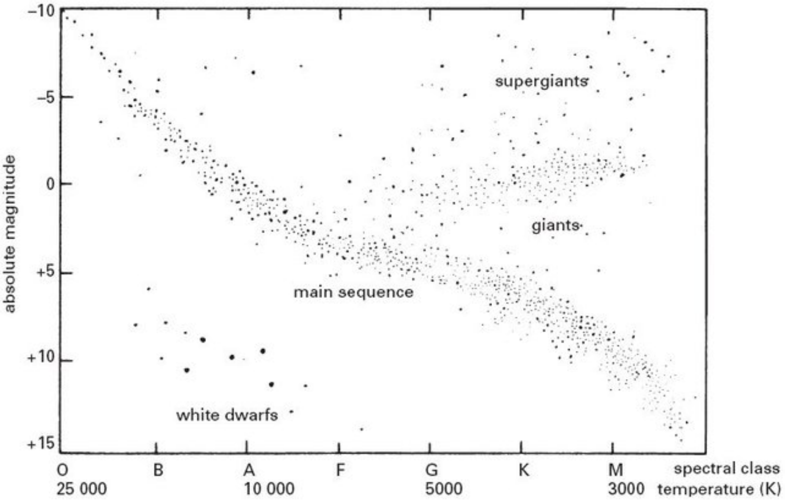
Herzpring Russell diagram - a plot of luminosity vs surface temperature for stars.

Most stars are found close to a line extending from lower right (low T and L) to upper left (high T and L) called the **main sequence**.

Here stars generate energy by fusion of hydrogen in their cores.

The position of a star in the main sequence depends primarily on its mass. This is the Vogt-Russell theorem.

Hertzsprung-Russell Diagram



Hertzsprung–Russell diagram

Mass-radius and mass-luminosity relations

On the **main sequence** one finds,

$$R \simeq R_\odot \left(\frac{M}{M_\odot}\right)^{0.7}$$

$$L \simeq L_\odot \left(\frac{M}{M_\odot}\right)^{5} \qquad M \lesssim M_\odot$$

$$L \simeq L_\odot \left(\frac{M}{M_\odot}\right)^{3.9} \qquad M_\odot \lesssim M \lesssim 10M_\odot$$

$$L \simeq 50L_\odot \left(\frac{M}{M_\odot}\right)^{2.2} \qquad M \gtrsim 10M_\odot$$

Giants and supergiants - R up to $\sim 1000\ R_\odot$.

White dwarfs - $R \simeq 0.01R_\odot \sim R_\oplus = 6371$ km.

Neutron stars - $R \sim 20$ km.

How long does a star spend on the main sequence?

How long does a star spend on the main sequence?

Roughly, this is the time needed to burn all the Hydrogen in the core.

The hydrogen core mass is proportional to the mass of the star.

The time required to consume it is inversely proportional to the consumption rate

$$\tau_{\mathsf{ms}} = \frac{\Delta M}{dM/dt}$$

The consumption rate is proportional to the luminosity.

Therefore,

$$\tau_{\mathsf{ms}} \propto \frac{M}{L}.$$

How long does a star spend on the main sequence?

Taking $L \propto M^{3.5}$ an average mass luminosity relationship,

$$\tau_{\text{ms}} \propto \frac{M}{L} = CM^{-2.5}.$$

where C is some proportionality constant.

Divide this by the same equation for the Sun,

$$\frac{\tau_{\text{ms}}}{\tau_{\text{ms}\odot}} = \left(\frac{M}{M_\odot}\right)^{-2.5},$$

Taking $t_{\text{ms}\odot} \simeq 10$ Gyr, we find

$$\tau_{\text{ms}} \simeq 10 \text{ Gyr} \left(\frac{M}{M_\odot}\right)^{-2.5} \simeq 10 \text{ Gyr} \left(\frac{L}{L_\odot}\right)^{-5/7}.$$

The lifetime ranges from a few tens of millions of years for massive stars to longer than the age of the Universe for stars of mass $M \lesssim 2M_\odot$. (See Table 1.1 in the textbook.)

Fig 1.4. Evolutionary tracks in the HR diagram.

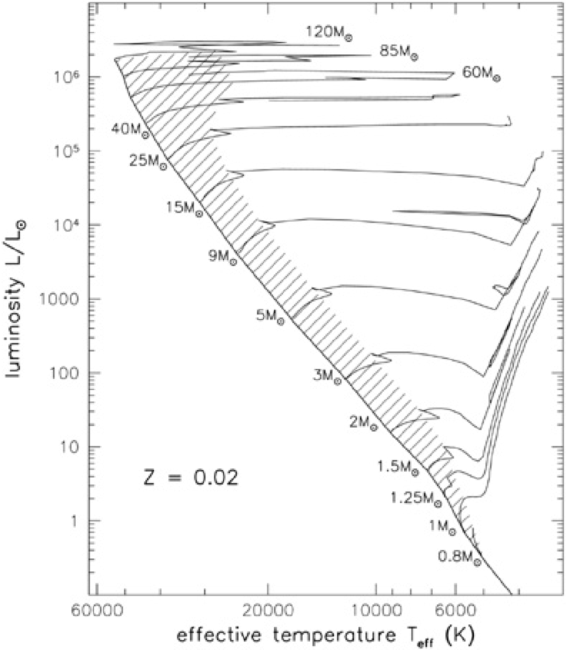


Fig 1.4 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

Post-main-sequence evolution

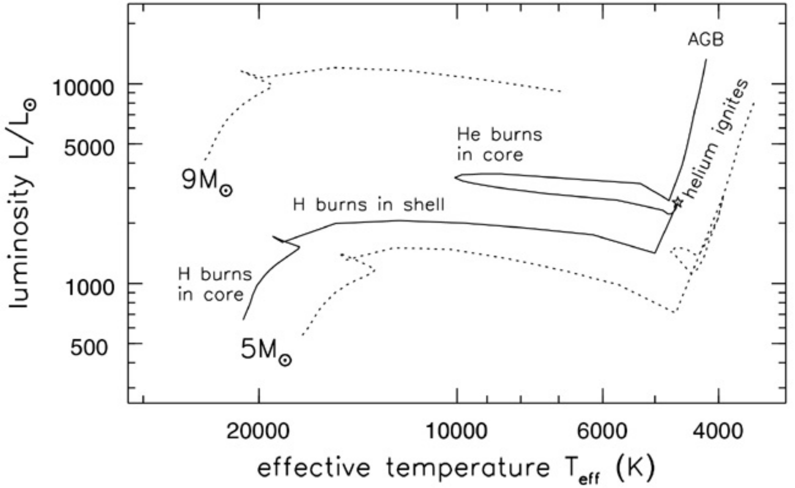


Fig 1.5 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

Fig 1.5. Evolutionary tracks in the HR diagram.

Post-main-sequence evolution

Subgiant - A star that is brighter than a normal star of the same spectral type on the main sequence, but not as bright as a giant. It is an intermediate phase between the main sequence and the giant branch for stars of low to intermediate mass. In these stars the He core is contracting and the envelope is expanding.

Giant - H is burning in a shell surrounding the He core, producing more energy and a higher rate. The star has an extended envelope. The He core continues to contract and eventually ignites via the triple-alpha reaction.

Helium flash - rapid ignition of He in degenerate core of a low-mass red giant. The core becomes non-degenerate after about 100 s.

Post-main-sequence evolution

Degenerate gas - The Pauli exclusion principle forbids electrons from occupying the same quantum state, so at high density they are forced into high-momentum states. If the resulting "quantum-mechanical" pressure exceeds the thermal pressure, the gas is degenerate,

$$\frac{h^2 n_e^{5/3}}{20\ m_e} > \frac{\rho k T}{\mu m_H}$$

Tip of the red giant branch - Luminosity here is set by the mass of the He core at the time of the helium flash. This is roughly constant for stars older than $2 - 3$ Gyr (and therefore less than $\sim 2\ M_\odot$), so can be used as a distance indicator for nearby galaxies.

Horizontal branch and red clump - Stars burning He in their core. The red clump contains stars with mass $\sim 1 - 2\ M_\odot$

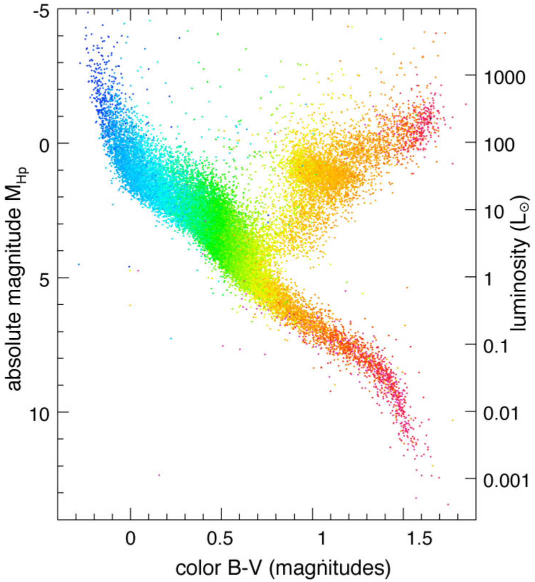


Fig 2.2 (F. van Leeuwen) 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

Fig 2.2. Colour-magnitude diagam for stars near the Sun.

Post-main-sequence evolution

Asymptotic giant branch (AGB) - Stars burning He and H in shells. Instabilities eject the outer layers of the star.

Planetary nebula - gas ejected from AGB star now excited by UV radiation from the exposed core.

White dwarf - The core cannot generate nuclear energy and slowly cools as it radiates thermal energy.

Intermediate-mass stars - $2 - 8\ M_\odot$. These stars eject most of their mass, leaving a carbon-oxygen white dwarf.

Cepheid variables - F and G stars in the **instability strip** of the horizontal branch. They pulsate with a period that depends on their luminosity (higher luminosity, longer period). This makes them useful as distance indicators.

Post-main-sequence evolution

Massive stars - $M > 8\ M_\odot$, spend less than ~ 20 Myr as giants then explode as supernovae.

Wolf-Rayet star - exposed core of a massive star ($20+\ M_\odot$) with strong emission lines of He, C, N produced by a fast stellar wind.

Nucleosynthesis - Massive stars are the main source of elements heavier than nitrogen.

s-process - production of heavy elements by neutron capture in the cores of stars. Intermediate-mass AGB stars dredge up these elements and eject them into the interstellar medium through a stellar **superwind**.

Very massive stars eject most of their mass and may never become a supergiant.

Supernovae

Type I supernova - weak or no H lines in the spectrum.

Ia - Si II line at 6150 Å
Ib - He I line at 5876 Å
Ic - weak or no He

Type II supernova - strong H lines in the spectrum.

IIb - spectrum evolves to become line Ib
II-P - no narrow lines, plateau in light curve
II-L - no narrow lines, linear decrease in light
IIn - some narrow lines

Core-collapse supernovae - $10 - 40\ M_{\odot}$ stars produce iron cores. Fe is the most stable element and cannot be "burned" further. The core collapses forming a neutron star or black hole and a type II, Ib or Ic supernova results.

Wolf-Rayet stars collapse to produce type Ic supernovae.

Novae and supernovae

Mass-transfer binaries - If one of the stars in a close binary system becomes a red giant, it can transfer mass to its companion. If the companion is a white dwarf, a nova or Type Ia supernova may occur.

Nova - recurring thermonuclear explosion on the surface of a white dwarf due to transfer of H from a companion.

Type 1a supernova - If a white dwarf accreting matter exceeds the Chandrasekhar limit, about 1.4 $M_\odot$, it collapses triggering a thermonuclear explosion that totally disrupts the star.

Type 1a supernovae can outshine an entire galaxy. They have peak luminosities in the range $2 \times 10^9 L_\odot \lesssim L <\lesssim 2 \times 10^{10} L_\odot$.

The luminosity is correlated with the decay time of the supernovae, which makes them good **standard candles** for estimating distances to galaxies as far as 10^{10} light years away.