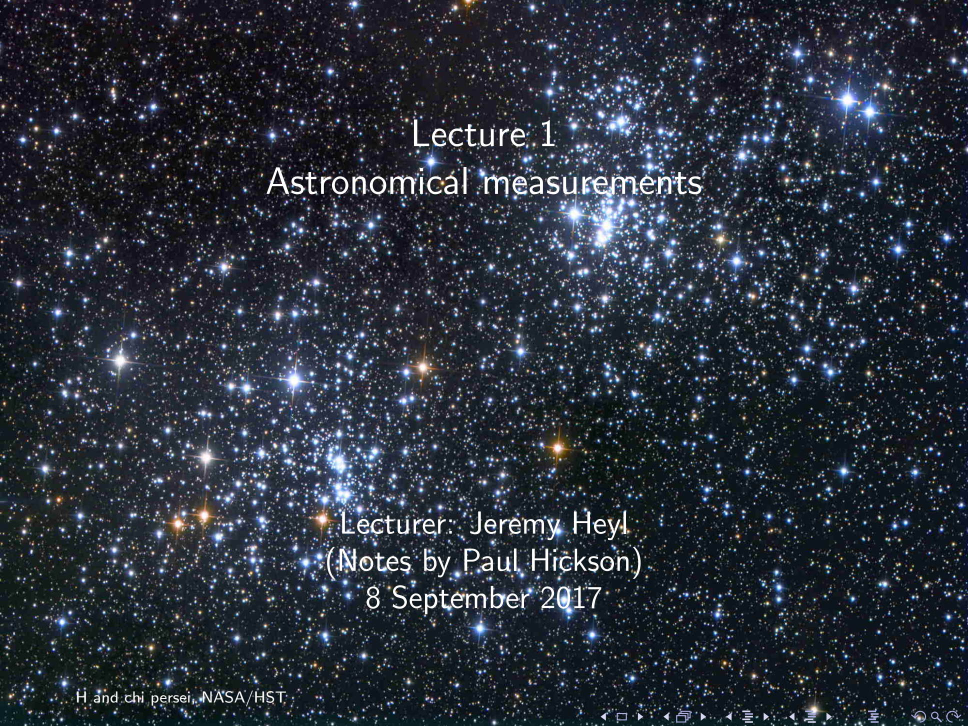
Lecture 1
Astronomical measurements

Lecturer: Jeremy Heyl
(Notes by Paul Hickson)
8 September 2017



H and chi persei, NASA/HST

1 Introduction

Why study galaxies?

- Galaxies are the largest stellar systems in the Universe. They contain the vast majority of luminous matter.
- Galaxies are the primary sites of star formation activity and element production.
- Galaxies trace the large-scale structure of the Universe, and cosmic history over ~ 13 Gyr.
- They are the site of a wide variety of interesting phenomena, including supernovae, gamma-ray bursts, supermassive black holes, and relativistic jets to name a few.

What are the key questions? Here are a few...

- Why are there galaxies?
- Why do they have such varied structure?
- Why are there such large variations in mass and size?
- How do they form and evolve?
- What do they tell us about the Universe?
- What distinguishes a galaxy from its satellites?
- What distinguishes a galaxy from a star cluster?

Essentially everything we know about galaxies comes from observations, which also guide theoretical developments. So we begin with a short review of the techniques, and limitations, of astronomical measurements.

Telescopes

A telescope is a device that collects light from a distant object and focuses it on a detector. Optical, infrared, UV and X-ray telescopes form images of the source directly.

The four 8-m telescopes of the European Southern Observatory Very Large Telescope (VLT) located in Chile (ESO).

Telescopes

Radio telescopes form images indirectly, either by scanning or by aperture synthesis interferometry.

The Very Large Array, located in New Mexico (NOAO).

Luminosity and Flux

Luminosity - $L =$ power radiated, sometimes restricted to a particular wavelength range.

Bolometric luminosity - total power radiated over all wavelengths.

Flux - $F =$ power per unit area of an emitting, or receiving, surface.

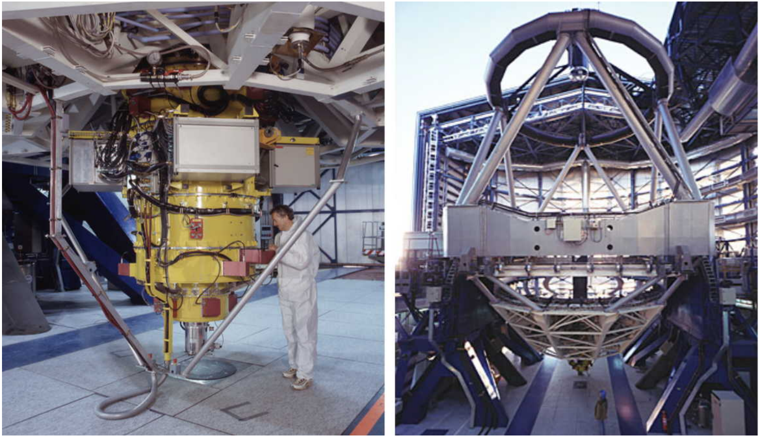
Inverse square law - Flux decreases with distance from the source. For an isotropic emitter, the flux at distance d is

$$F = \frac{L}{4\pi d^2}$$

If we can measure F and d, we can infer the luminosity.

Spectra

Much information can be obtained by measuring the spectrum of the object. Typically one uses a spectrograph employing a diffraction grating to disperse the light.



FORS1 low-dispersion UV spectrograph (left) mounted on UT1 (right), one of the four 8-m telescopes of the VLT (ESO/H. Zodet).

Spectra

Here we are interested in flux per unit wavelength or frequency.

Specific flux - $F_\lambda(\lambda)$, $F_\nu(\nu)$ (also called **flux density**)

These are related. The flux contained in interval $\Delta\lambda$ is the same as that contained in the corresponding interval $\Delta\nu$,

$$F_\lambda(\lambda)\,|\Delta\lambda| = F_\nu(\nu)\,|\Delta\nu|\,.$$

Therefore

$$F_\lambda(\lambda) = \left|\frac{d\lambda}{d\nu}\right|^{-1} F_\nu(\nu) = \frac{\nu^2}{c} F_\nu(\nu),$$

since $\nu\lambda = c$, the speed of light. More simply, $\lambda F_\lambda = \nu F_\nu$.

The total flux is

$$F = \int_0^\infty F_\lambda d\lambda = \int_0^\infty F_\nu d\nu.$$

Intensity

For extended sources like galaxies, we are often interested in the distribution of light within the source. The **intensity** is the power received, per unit solid angle, per unit perpendicular area, as a function of direction $\boldsymbol{\theta} = (\theta, \phi)$.

Intensity - $I(\boldsymbol{\theta})$, flux per unit solid angle.

Integrating the intensity over all directions gives the flux.

$$F = \int_{2\pi} I(\boldsymbol{\theta}) \cos\theta d\Omega \equiv \int_0^{2\pi} d\phi \int_0^{\pi/2} I(\theta, \phi) \cos\theta \sin\theta d\theta,$$

where $d\Omega = \sin\theta d\theta d\phi$ is the element of solid angle. Note that the integral is over 2π steradians, not 4π. The factor of $\cos\theta$ appears because I is power per steradian per unit *perpendicular* area.

Specific intensity - $I_\nu(\boldsymbol{\theta})$, $I_\lambda(\boldsymbol{\theta})$, flux per unit solid angle per unit frequency or wavelength.

As with flux, $\lambda I_\lambda = \nu I_\nu$.

Summary of photometric quantities

Symbol	Units (SI)	Names
L	W	luminosity, power
L_ν, L_λ	WHz^{-1}	specific luminosity
F	Wm^{-2}	flux, irradiance
F_ν, F_λ	Wm^{-2}Hz^{-1}, Wm^{-3}	specific flux, flux density
I	Wm^{-2}sr^{-1}	intensity, radiance
I_ν, I_λ	Wm^{-2}sr^{-1}Hz^{-1}, Wm^{-3}sr^{-1}	specific intensity

Because astrophysical sources are typically faint in terrestrial units, we have defined a new unit of specific flux

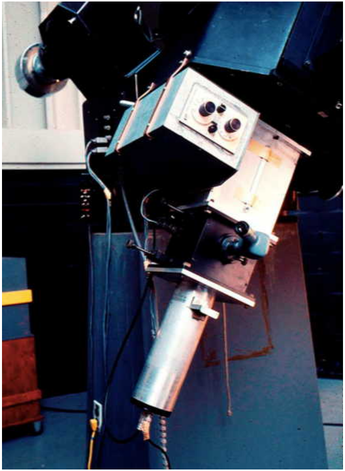
$$1\ \text{Jy} = 10^{-26}\ \text{W m}^{-2}\text{Hz}^{-1} = 10^{-23}\ \text{erg s}^{-1}\text{cm}^{-2}\text{Hz}^{-1}.$$

For example the star Vega has $F_\nu = 3631$ Jy at 555 nm.

Photometry

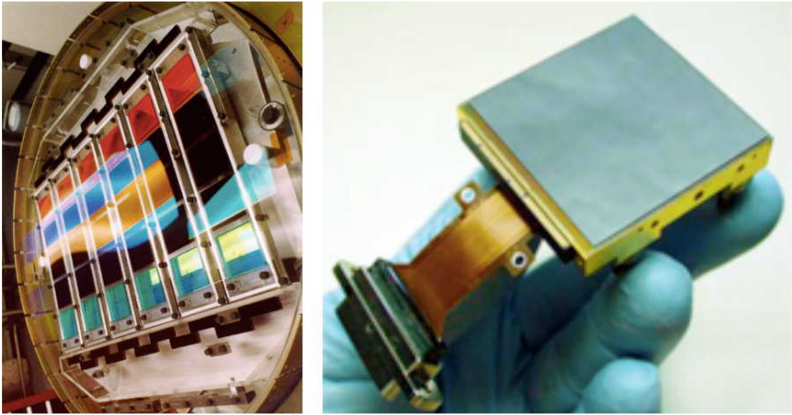
The simplest kind of measurement is that of the brightness of a star. Originally this was done using photoelectric photometers

Early photoelectric photometer attached to a small telescope. The cylinder at the bottom contains a photomultiplier tube that detects photons passing through a small aperture located in the focal plane of the telescope. An eyepiece allows the observer to center a star in the aperture (Delaware Asteroseismic Research Center).



Imaging detectors

Imaging detectors such as CCDs are now used for UV, optical and infrared astronomy. These can image many objects at once



Left: the photometric camera of Sloan Digital Sky Survey, employing 30 4-Mpixel CCD sensors (www.sds.org). Right: a 4-Mpixel HgCdTe array designed for infrared astronomy (Teledyne Imaging Systems)

Magnitudes

Optical astronomers often use **magnitudes**, introduced in 1856 by Norman Pogson. If a star has flux F, defined in some wavelength band, its **apparent magnitude** is given by

$$m = -2.5 \log(F/F_0) + C$$

Where $\log$ means a base-10 logarithm, F_0 is a normalization constant, and C is a constant that generally depends on the band. One can generally choose F_0 so that $C = 0$, but other choices are often convenient.

The factor of 2.5 and negative sign is included for historical reasons, to approximately match the system of magnitudes used to describe the brightnesses of stars by ancient Greeks (Hipparchus).

It follows that the *difference* between the magnitudes of two stars is related to the *ratio* of their fluxes

$$m_1 - m_2 = -2.5 \log\left(\frac{F_1}{F_2}\right)$$

Magnitudes

Magnitudes are convenient because astronomical objects have a huge range of brightness.

Five magnitudes corresponds to a factor of 100 in flux, and 10 magnitudes is a factor of 10^4. One magnitude is about a factor of 2.5.

Because $2.5 \log x \simeq 1.086 \ln x$, a small change in magnitude is about the same as the fractional change in flux,

$$\delta m \simeq \delta \ln(F) \simeq \frac{\delta F}{F}.$$

Thus, a 1% change in flux corresponds to a magnitude change of about 0.01.

Magnitudes

Originally, the constant C was chosen to make the magnitude of the Vega, a bright star of spectral type A0, equal to zero.

Thus

$$m = -2.5 \log \left(\frac{F}{F_{\mathsf{Vega}}} \right).$$

It is easier to measure the relative fluxes than absolute fluxes. (The latter requires accurate calibration of the detector sensitivity, telescope throughput and atmospheric transmission.)

Typically one interleaves measurements of target stars with **standard stars** - stars whose magnitudes have been precisely determined and published.

Vega has now been replaced by a set of A0 stars, in order to improve accuracy. However, the change is generally quite small (a few hundredths of a magnitude).

Surface brightness

One can also represent intensity by a kind of magnitude. The **surface brightness** μ is defined as the magnitude corresponding to the flux received from one square arcsec of the source. Thus

$$\mu = -2.5 \log(I/I_0) + C + 26.57$$

where the numerical constant converts square arcseconds to steradians.

For example, the surface brightness of the night sky in the V band is approximately 21.5. The surface brightness in the B band of the faintest visible features in a galaxy is typically around 26.

Atmospheric extinction

Light is absorbed and scattered as it passes through the atmosphere. As a result, the measured magnitudes require a correction

$$m_{\text{true}} = m_{\text{observed}} - kX$$

where k is a constant called the **extinction coefficient** and X is the **airmass**.

To a good approximation,

$$X = \sec \zeta$$

where ζ is the **zenith angle** (the angle between the line of sight to the star and the **zenith**)

The extinction coefficient can be determined by observing standard stars over a range of zenith angles and performing a least squares fit to equation above.

Photometric bands

More information can be obtained by measuring the fluxes in different wavelength bands.

For each band,

$$F_a = \int_0^\infty F_\lambda(\lambda) T_a(\lambda) d\lambda.$$

where F_λ is the star's specific flux and the function $T_a(\lambda) \in [0, 1]$ describes the transmission of the band a.

The corresponding magnitude is

$$m_a = -2.5 \log \frac{F_a}{F_{\mathsf{Vega},a}}$$

where $F_{\mathsf{Vega},a}$ is the flux measured for Vega in the particular band.

Photometric bands

The effective width or **bandwidth** of the band is defined by

$$\Delta\lambda = \int_0^\infty T_a(\lambda) d\lambda.$$

and the **effective wavelength** by

$$\lambda_{\text{eff}} = \frac{1}{\Delta\lambda} \int_0^\infty T_a(\lambda)\lambda d\lambda.$$

The **spectral resolving power** is defined by

$$R = \frac{\lambda_{\text{eff}}}{\Delta\lambda}$$

Infrared bands are chosen to avoid regions of high atmospheric absorption.

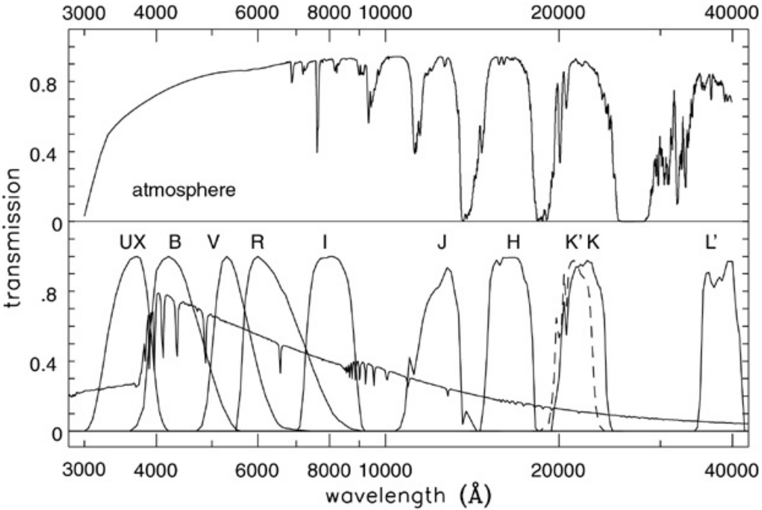


Fig 1.7 (M. Bessell) 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

Fig 1.7. Top: atmospheric transmission. Bottom: common photometric bands, and the spectrum of an A0 star.

Common photometric Bands

Band	$\lambda_{\text{eff}}(um)$	R	Flux (Jy) for $m = 0$	$m_{\text{AB}} - m$
U	0.36	6.7	1810	0.756
B	0.44	4.5	4260	-0.174
g	0.52	7.1	3730	-0.029
V	0.55	6.3	3640	-0.003
R	0.64	4.3	3080	0.179
r	0.67	6.3	4490	-0.231
I	0.79	6.3	2550	0.384
i	0.79	6.3	4760	-0.294
z	0.91	7.7	4810	-0.305
J	1.26	6.3	1600	0.890
H	1.60	4.3	1080	1.316
K	2.22	4.3	640	1.885
L	3.40	4.9	312	2.665
M	5.00	4.2	183	3.244

Flux-based magnitudes

Recently, flux-based magnitudes have been introduced. These are defined by

$$m_a = -2.5 \log \langle F_a/F_0 \rangle + C.$$

where

$$\langle F_a \rangle = \frac{1}{\Delta\lambda} \int_0^\infty F_\lambda(\lambda) T_a(\lambda) d\lambda$$

is an average flux over the band.

These differ from previous magnitudes in that the zero-point constant C is the same for all bands.

It has the value $C = -18.6$ if $F_0 = 1\ \mathrm{Wm^{-2}\mu m^{-1}}$, and -21.1 if $F_0 = 1\ \mathrm{ergcm^{-2}\AA^{-1}}$.

The constant was chosen to make the flux-based magnitude agree with the V magnitude for the V band.

AB magnitudes

There is also a kind of magnitude that is a *continuous function of frequency or wavelength*. It is called the **AB magnitude** and is defined by

$$m_{\mathsf{AB}}(\nu) = -2.5 \log \frac{F_\nu(\nu)}{1\ \mathrm{Wm^{-2}Hz^{-1}}} - 56.1.$$

The zero point was chosen so that the AB magnitude of Vega at a wavelength of 555 nm is zero.

An equivalent definition is

$$m_{\mathsf{AB}}(\nu) = -2.5 \log \left[\frac{F_\nu(\nu)}{3631 \mathsf{Jy}} \right]$$

where 3631 Jy is F_ν of Vega at 555 nm. AB magnitudes differ from magnitudes (such as UBVRI, etc) that are based on the flux density of Vega, which depends on wavelength.

Colours

Colours are defined by the differences between magnitudes of the same object in different bands.

For example, the magnitudes m_{U}, m_{B}, m_{V} in the Johnson UBV bands are usually denoted by the letters U, B and V. The $U - B$ colour is then

$$U - B = m_{\mathsf{U}} - m_{\mathsf{B}} = -2.5 \log \left(\frac{F_{\mathsf{U}}}{F_{\mathsf{B}}} \right) + C_{\mathsf{U}} - C_{\mathsf{B}}.$$

By definition, an A0 star like Vega will have colours that are very close to zero for all combinations of bands.

It is much easier to measure colours of stars in a cluster than it is to measure their spectra. One needs only to take well-calibrated images of the star cluster in several bands.

Absolute magnitudes

The apparent magnitude of a star depends both on the luminosity and the distance to the star.

If the distances a number of stars is known, one can remove the distance dependence, by "moving" the stars to a common distance, using the inverse square law to compute the new flux.

By convention, a distance of 10 pc is used. The result is called the **absolute magnitude** M of the star. We see that it is related to the apparent magnitude m by

$$M = m - 5 \log \left(\frac{d}{10 \text{ pc}} \right)$$

or more simply

$$m - M = 5 \log d - 5.$$

where d is the distance to the star in parsecs.

Absolute magnitudes

Since the distance dependence has been removed, absolute magnitudes are a logarithmic measure of luminosity, typically in some wavelength band.

Corresponding to the total luminosity L, we have the bolometric absolute magnitude,

$$M_{\mathrm{bol}} = -2.5 \log L + 80.50$$

where L is in Watts.

A **bolometric correction** (BC) is an estimate of the difference between the magnitude in the V band, and the bolometric magnitude of a star (which depends on spectral type),

$$M_{\mathrm{bol}} = M_{\mathrm{V}} - BC.$$

The bolometric magnitude of the Sun is $M_{\mathrm{bol}\,\odot} = 4.75$, and its bolometric correction is about 0.07.

Photometric quantities and magnitudes

Symbol	Name	Symbol	Name
L	luminosity	M	absolute magnitude
F	flux	m	apparent magnitude
F_ν, F_λ	specific flux	m_{AB}	AB magnitude
I	intensity	μ	surface brightness

Atmospheric emission

In both photometry and spectroscopy, the performance of a telescope is generally limited by light from the night sky, which is usually much brighter than the objects being studied.

This light is produced by several mechanisms:

- atomic transitions from atoms excited by collisions (eg. aurora)
- molecular vibrational and rotational transitions.
- scattering of moon and starlight (Rayleigh scattering)
- thermal (black-body) emission
- light pollution

There are also some important extraterrestrial sources of background light. These also affect telescopes in space.

- zodiacal light
- diffuse starlight and emission from interstellar gas and dust

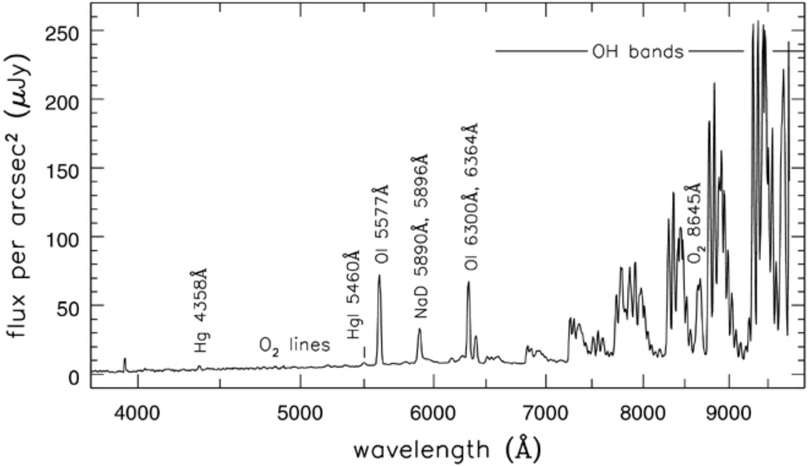


Fig 1.6 (Benn & Ellison) 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

Fig 1.6. Night sky emission at the Observatorio del Roque del los Muchachos (ORM), La Palma, Canary Islands.

Photon statistics

In order to understand why "background" light is a problem, we need to understand a principal source of noise in astronomical measurements - photon statictics.

Light takes the form of discrete photons, whose arrival times are **random**.

If we expect to detect, say, 100 photons from a star in a certain time interval, we will sometimes see 99, sometimes 103, etc. Since the exact number is unpredictable, this is a source of noise that results in an error in our measurement.

The number of photons detected can be written as $N = \bar{N} + \delta N$, where $\bar{N}$ is the expected number (determined from a long term average) and δN is a fluctuating number.

Photon statistics

Random arrivals of discrete objects such as photons, or electrons, are described by **Poisson statistics**. One can show that for many identical measurements, the **variance** (the mean square difference from the mean value) in the number of detected events is equal to the expected number of events,

$$\mathrm{Var}(N) = \bar{N}$$

The **standard deviation** σ is the square root of the variance,

$$\sigma(N) = \sqrt{\bar{N}} \simeq \sqrt{N}.$$

This is good estimate of the *typical value* of the fluctuation δN.

Signal-to-noise ratio

The quality of a measurement can be described by its **signal-to-noise ratio**. The larger this number, the better. Typically, it needs to be at least 5 before anyone will believe the result.

The **relative error** or uncertainty is the reciprocal of the signal-to-noise ratio.

From the preceding discussion, we see that the signal-to-noise ratio depends on the number of photons detected,

$$s/n = \frac{N}{\sigma(N)} = \frac{N}{\sqrt{N}} = \sqrt{N}.$$

Thus, we need to detect 100 photons to get a s/n ratio of 10, and 10^4 photons for a s/n ratio of 100 (1% uncertainty).

Effect of sky background

Suppose now that we want to measure the flux of a star. We use a small aperture mask that allows the light from the selected star to fall on a detector, but blocks most (but not all!) of the background light. We record the number of photons, N_1 detected in some fixed exposure time.

Now we repeat the measurement, with the star not in the aperture, in order to estimate the amount of background light. We measure N_2 photons.

Our best estimate of the number of photons received from the star in this time is then $N_1 - N_2$.

What is the relative uncertainty in our measurement of the star's flux?

Effect of sky background

The signal in this case is $N = N_1 - N_2$. But what is the noise? We made two measurements, and each will have photon noise. By the usual rules for combining independent noisy measurements,

$$\text{Var}(N) = \text{Var}(N_1) + \text{Var}(N_2) = N_1 + N_2 = N + 2N_2.$$

So the signal-to-noise ratio is

$$s/n = \frac{N}{\sqrt{N + 2N_2}}$$

and the relative error is

$$\frac{\delta F}{F} = \sqrt{\frac{1 + 2N_2/N}{N}}$$

We see that the background light (N_2 photons) has increased the error. If $N_2 \gg N$, it is the dominant source of noise.

Resolution

The resolution of a telescope is limited by the wave nature of light. Because of this a point source, imaged with a perfect telescope of aperture diameter D has an angular diameter (defined as the full width at half maximum intensity: FWHM) of

$$\varepsilon = 0.976\frac{\lambda}{D} \text{ radians.}$$

If D is in metres and λ in um,

$$\varepsilon = 0.20\frac{\lambda}{D} \text{ arcsec.}$$

Ground-based telescopes are limited by atmospheric **seeing** to a typical resolution of about 0.6 arcsec, unless **adaptive optics** is used to compensate for the atmospheric turbulence.