

## CHIME Canadian Hydrogen Intensity Mapping Experiment

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### Executive Summary:

CHIME, the Canadian Hydrogen Intensity Mapping Experiment, is a proposed novel radio telescope designed to map the expansion history of the universe over the redshift range  $2.5 \geq z \geq 0.7$ . This purpose-built radio-telescope will observe baryon acoustic oscillations (BAO) by mapping the three-dimensional distribution of neutral hydrogen gas in the universe inferred from redshifted 21-cm radiation. BAO have been very accurately measured at redshift  $z = 1100$  via their impact on the Cosmic Microwave Background. With CHIME, their angular size will be traced through the key epoch when cosmic acceleration appears to turn on, providing measurements capable of mapping the Dark Energy driven transition from deceleration to accelerated expansion.

CHIME will be a very sensitive instrument from the start, and the sensitivity will improve as we integrate over a period of several years. After only three weeks of operation, CHIME would double the cumulative precision of present dark energy experiments! See Fig. 1

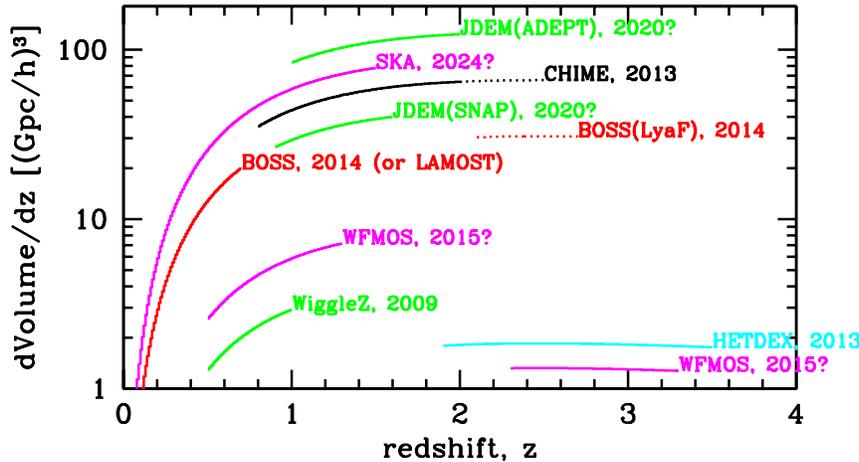


Figure 1: The sensitivity of CHIME after two years operation is compared to other proposed and current experiments. Because it is a dedicated, ground-based experiment CHIME provides sensitivity rapidly and at a very small fraction of the cost of comparably sensitive alternatives.

This measurement requires rapid, wide-area mapping not available from existing facilities. CHIME consists of an array of cylindrical telescopes and has no moving parts or cryogenics. It surveys 1/4 to 1/2 the sky every day with microsecond resolution. Beam formation is accomplished by Fourier transform in a custom digital correlator. It will be built on the campus of the Dominion Radio Astrophysical Observatory (DRAO) near Penticton BC, which is legally protected as a radio-quiet reserve.

Because the signal frequency is the redshift of the 21 cm sources, CHIME will produce three dimensional maps. surveying 200 cubic Gpc with  $\approx 10$  Mpc resolution. This will be the largest volume astronomical spectroscopic redshift survey to date, and covers the optical “redshift desert”  $1.3 < z < 2.5$ , which is otherwise difficult to access from the ground. In addition to its intended use, CHIME will be an excellent transient radio source detector and pulsar timing facility.

## CHIME Science:

**Background:** Cosmology has achieved spectacular success in recent years, entering an era of precision, answering important physical questions, and uncovering new mysteries. One key component of the success is the recent mapping of the cosmic microwave background (CMB) from the early universe, revealing the geometry and material make-up of the universe. These measurements indicate that the universe is filled with a dark energy, whose origin and precise nature are not understood, and whose dynamics appear counter to naive physical expectations, but not to formal general relativity. This dark energy, which can act like a source of antigravity, is inferred from the *acceleration* of the expansion of the universe. This discovery has been heralded as one of the most important discoveries in fundamental science of the past decade and members of the CHIME team are at the very centre of that excitement through their contributions to experimental CMB science and to its theoretical framework.

**21-cm Dark Energy Test:** Measurements of the CMB reveal details of acoustic fluctuations in the universe from an early epoch, before dark energy could have any noticeable effect. To quantify the dynamics of the dark energy, one needs to compare those data to maps of the universe at times in its more recent past and at a range of length scales which overlap with scales already measured in the CMB. The same physics that produces intensity fluctuations in the CMB from linear sound waves in the cosmic gas of the early universe also imprints density variations in the matter distribution of the universe at more recent times. These patterns are called Baryon Acoustic Oscillations (BAO) or, more casually, “Cosmic Sound”. The BAO present us with “*a standard ruler*” - an object whose physical size you know, and whose angular size therefore indicates its distance. The BAO characteristic size is dictated by simple physics which we understand well, and which is almost completely independent of cosmology. By measuring the BAO apparent angular scale as a function of redshift, or distance, we are directly observing the expansion history of the universe. CHIME will observe the key redshift range of  $z \approx 0.7$  to 2.5, where the expansion history changes from one dominated by the attractive force of ordinary gravity to one dominated by dark energy.

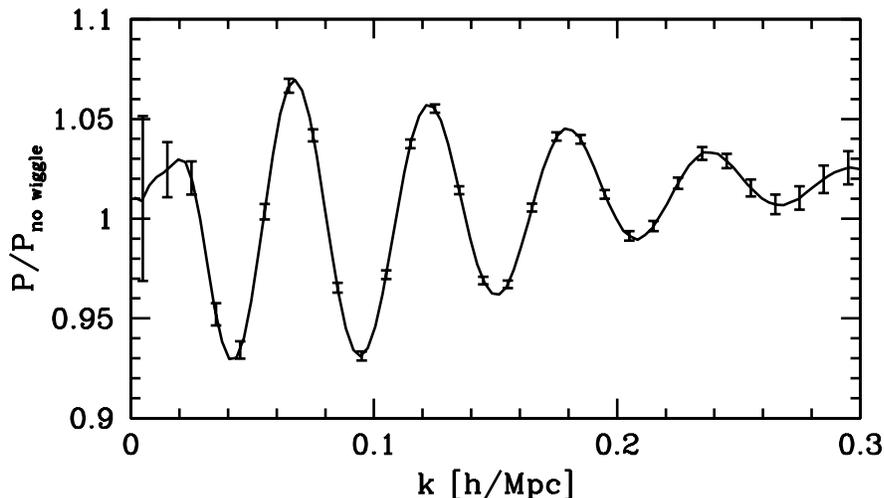


Figure 2: The ratio of the baryon power spectrum with and without Baryon Acoustic Oscillations is shown with the error bars anticipated for a CHIME 2 year survey, and for just one value of redshift. CHIME’s physical size is chosen to provide enough angular resolution to see this structure.

The CMB reveals baryon acoustic oscillations from when the universe was 380,000 years old and first became transparent to light. Our proposed experiment will trace these same oscillations as

they evolved, 2 to 8 billion years later. The measurement of this standard ruler in both the CMB and across these later times is exactly what is required to distinguish among theoretical models of dark energy. With the proposed CHIME telescope we will make one of the first measurements capable of pinning down the timing of the transition from deceleration to acceleration in the expansion of the universe. These data can be combined with existing CMB data from a very early time, such as the WMAP data, and data from the late history of the expansion obtained by optical surveys such as the SDSS and 2dF. With our added BAO-wavelength measurement near redshift  $z=1$  a tight constraint on the redshift of the acceleration/deceleration balance is obtained: our data will permit a measurement of the slope of the dark energy equation of state.

**Three-dimensional mapping of the universe:** Neutral hydrogen atoms emit at a wavelength of 21 cm. The observed frequency of 21-cm radiation, which is redshifted by the expansion of the universe, indicates how long the light has been traveling to reach us, and therefore the distance to its source. A measurement of the intensity and frequency of this radiation in every direction on the sky yields a three-dimensional map of the distribution of hydrogen. The 21-cm emission is faint and the ripples we seek to study are low contrast, so these measurements require a telescope with a very large collecting area which can survey a large fraction of the visible sky.

Until now, three-dimensional maps of the structure in the universe have been obtained from galaxy redshift surveys, which isolate millions of individual galaxies, record spectra for each and determine each redshift. This time-consuming work has been done primarily using optical spectroscopy, and less than 1% of the observable universe has been mapped in this way, concentrating on very near portions, and therefore measuring only recent evolution. Huge dedicated telescopes are being considered to extend such programs to larger volumes of the universe.

The 21-cm 3-D mapping technique avoids this painstaking effort and the enormous associated costs. A volume element large enough to contain an acoustic ripple contains a large number of galaxies; their total emission is significantly easier to detect than that of any individual object. We call this strategy of measuring the collective emission of many galaxies without individual detections “Intensity Mapping”. These measurements are most efficiently performed with a telescope whose angular resolution is just sufficient to measure the BAO.

**Foreground subtraction:** CHIME will inevitably observe astrophysical foreground contaminants including both galactic emission, expected to be primarily synchrotron radiation, and broad-band extragalactic emission from point sources. Some foreground contaminants will be significantly brighter than the desired cosmological signal. However, they are expected to have very smooth, power-law spectra, which allows the cosmological signal, which varies rapidly as a function of frequency, to be isolated with enough fidelity to measure BAO.

The smoothness of galactic emission in the CHIME bands is expected both phenomenologically and theoretically as the basic physics of synchrotron emission from fast moving electrons necessitates a smooth broad-band spectrum. CHIME team member Tom Landecker and collaborators have built a global sky model of galactic emission that combines a wide variety of known data and successfully accounts for emission over a broad frequency range with a remarkably simple model [de Oliveira-Costa *et al.* 2008]. Emission on either side of the CHIME bands appears to be well fit by a smooth power law function dominated by synchrotron radiation. Dr. Landecker is a member of the Global Magneto-Ionic Medium Survey (GMIMS) consortium, which is actively

pursuing a program to measure galactic emission at just the wavelengths corresponding to the CHIME bands. Prior to CHIME coming online GMIMS will have a substantial amount of data in the 400 to 900 MHz band that will be invaluable for characterizing and modeling the CHIME foregrounds in detail and developing a real-world foreground-rejection pipeline. With the recent upswing in interest in 21-cm tomography, the foreground subtraction problem has been considered by several groups with encouraging results [Wang et al.2005], [Liu, Tegmark, Zaldarriaga 2008]. Typically these authors have considered lower frequency observations than CHIME with the goal of observing the epoch of re-ionization. However, the ideas and methods used in these studies can be readily adapted to the CHIME band. A simple technique for foreground removal relies on subtracting a smooth function of frequency from the data and is likely to be sufficient.

### **Beyond Cosmology:**

CHIME will map the polarization and intensity of  $1/4 - 1/2$  of the sky with great sensitivity in a poorly explored frequency band. While the experiment has been carefully designed to produce cosmological data, it can do a lot more.

**Radio Transients:** Because of the unprecedented combination of a large collecting area, angular resolution, and 24 hour operation, CHIME will detect and monitor thousands of previously unknown transient sources. The bulk of radio transients which have been discovered to date have been found using the Parkes Multi-beam survey (11 so far). Because of the small number of sources the luminosity function is very poorly constrained, so precise projections of the detection rate are not possible. Our observations will be much deeper than the Parkes survey: CHIME will accumulate data at about 800 times the mapping speed of Parkes, allowing these sources to be studied statistically rather than being treated as exotic occurrences. Because CHIME is a transit instrument, once a source is detected the pulse pattern of the source can be searched for, both in the future and retrospectively, in each days data. Given the recent success of radio transient searches, it is reasonable to anticipate that completely new types of sources will appear in the CHIME data.

**Pulsars:** Because of its large instantaneous field of view, CHIME will provide an exciting new capability to time known pulsars, and eventually to search for new ones. Every pulsar in the Northern hemisphere will spend from 5 minutes to hours within the CHIME field of view. Working from a list of locations and dispersion measures a data set of *daily* timing and scintillation for hundreds of pulsars can easily be produced. For CHIME to act as a search engine, a parallel data stream with much more rapid output than the BAO pipeline requires must be developed by a team working on pulsar searches.

**Galactic Magnetic Fields:** CHIME will be complementary to GMIMS, the Global Magneto-Ionic Medium Survey. The major objective of GMIMS is to improve knowledge and understanding of the Galactic magnetic field. GMIMS will do this by mapping the polarized radio emission from the Milky Way in the frequency band 300 to 1800 MHz. Data in this spectral range will reveal aspects of how magnetic fields regulate star formation and couple the energy released by stellar winds and supernovae to the interstellar medium. CHIME will provide much more sensitive data over the middle part of this frequency band.

## The CHIME Instrument :

CHIME is a  $100\text{m} \times 100\text{m}$  hybrid telescope operating from  $\sim 400$  to  $800$  MHz.. CHIME will be built on the radio-protected campus of the Dominion Radio Astrophysical Observatory (DRAO) near Penticton BC. Our approach implements the fastest astronomical survey speeds at a low cost by relying heavily on the commercialization of GHz electronics in the communications and computing industry. CHIME will survey over half of the sky each day as the earth turns and forms its beams digitally and has no moving parts. In this section we outline the baseline CHIME design, but note that our design is evolving as we learn from our prototyping activities and adapt to budgetary constraints and opportunities.

### Choice of CHIME design parameters

**Frequency Coverage:** The sensitivity of a hydrogen intensity survey depends on measuring the history of expansion both before and after dark energy emerges as important at a redshift slightly above  $z = 1$ , corresponding to an observed frequency of  $700$  MHz. The importance of including some data at low redshift (high frequency) is demonstrated in Figure 3. We have designed CHIME to operate over a factor of two in frequency, from  $400$  to  $800$  MHz, with the upper frequency just avoiding interference from cellular telephone transmitters. Even in excellent RF-protected zones where cell phones do not function, signals from cell phone transmission are still present. See Figure 5.

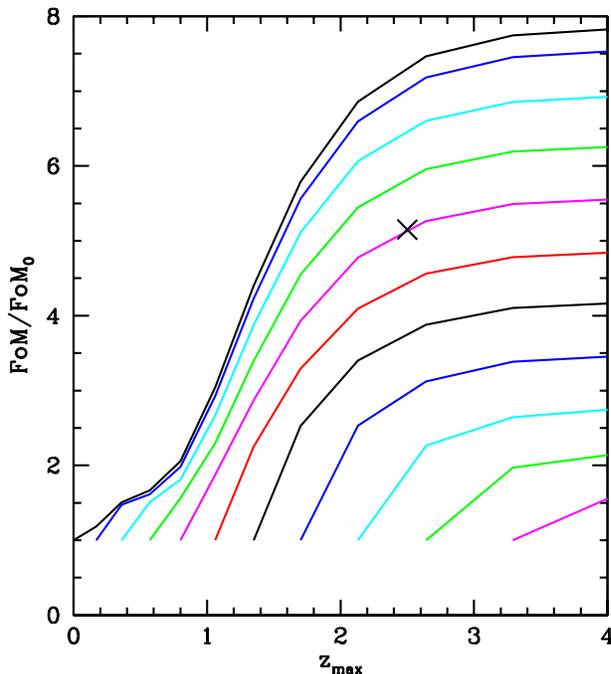


Figure 3: CHIME’s sensitivity depends on the lowest redshift surveyed. The improvement factor for the Dark Energy Task Force *Figure of Merit* over our current knowledge from a BAO survey covering the redshift range from  $z_{min}$  to  $z_{max}$  is plotted as a function of  $z_{max}$  using a line for each choice of  $z_{min}$ . The improvement is limited above  $z_{max} \approx 2.5$ . However the sensitivity can be improved by reaching lower  $z_{min}$ . This pushes CHIME to operate at as high a frequency as possible given the RF environment. The CHIME band of  $400\text{-}800$  MHz ( $2.5 < z < 0.7$ ) is chosen to lie just below the cell phone band and corresponds to the  $\times$  in the figure.

**Resolution:** To constrain the cosmological models to a useful level of precision, maps should have sufficient spatial resolution to see the smallest features and should survey enough volume to detect large scale variations. Features imprinted soon after the Big Bang whose size is smaller than  $10$  Mpc ( $\tilde{30}$  million light-years) today have been erased by gravitational interactions in the intervening time. That is why the fringe amplitude drops a bit at the right hand edge of Figure 2. A  $100\text{m}$ -aperture telescope is sufficient to resolve a co-moving separation of  $10$  Mpc at  $z \approx 1$ .

The 21-cm hydrogen line is the dominant spectral feature at frequencies less than 1420 MHz and it is an isolated transition. This allows interpretation of the apparent frequency of a source directly as its redshift, without the elaborate spectral fitting required for typical optical redshift measurements. CHIME uses redshifts to map hydrogen in the third dimension, distance. For the co-moving distance measurements to match the instrumental angular resolution a frequency resolution of 1.6 - 4.1 MHz is required in the redshift range of  $0.8 < z < 2.5$ , as shown in Table 1. In practice, CHIME will have a single bandwidth of  $\leq 1.5$  MHz at all redshifts.

Table 1: Summary of distance scales, spatial resolution, minimum frequency resolution, and noise.

$z$	$f_{obs}$ MHz	$d_{co-moving}$ Mpc	$z'$	$\Delta f_{obs}$ MHz	$n$ $h^3(Mpc)^{-3}$
0.8	789	10.55	0.8038	1.66	0.0072
1.	710	13.93	1.0056	1.98	0.0057
2.	473	33.04	2.022	3.45	0.0031
2.5	406	43.46	2.536	4.13	0.0027

**Geometry:** Our operating frequencies allow a hybrid approach: 100m-long parabolic cylinders image in one direction; each has an array of room temperature receivers along its focal line which form images in the other dimension by aperture synthesis.

This hybrid design is driven by two factors. On the one hand, the new possibility to make inexpensive but very sensitive room temperature amplifiers allows population of each focal line with several hundred feeds. At the same time, digital correlation remains expensive. Incorporation of large 1-D cylindrical reflectors means we only need to perform  $N \times 5$  correlations per frequency, while a full 2-D phased array would perform  $N \times N$ , where  $N$ , the number of feeds, is several hundred and scales as telescope size divided by wavelength.

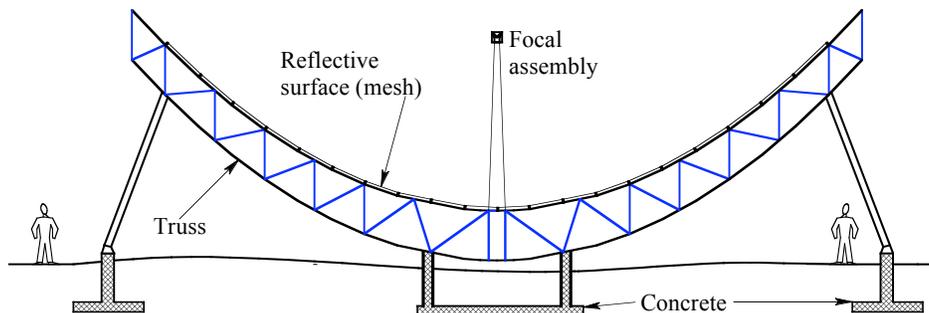


Figure 4: CHIME cylinders will be supported by inexpensive steel frames of the style used in industrial roofing, upon which a mesh will be suspended and adjusted to 1 cm rms accuracy. Such a system uses readily available materials and construction techniques. An enormous advantage for building CHIME quickly and economically is that this structure does not move, so it does not need to be as rigid as a conventional telescope.

An array of cylindrical telescope oriented along the north-south direction offers intriguing advan-

tages for these observations. Each cylinder is a phased array, allowing for beams to be digitally formed from horizon to horizon running North-South. In the East-West direction, the cylindrical surface of the dish concentrates radiation, amplifying the collecting area of each feed. The field-of-view in the E-W direction is defined by the cylinder width and is oriented towards the meridian (local vertical). For an idealized cylindrical telescope, the entire overhead sky is observed daily as the earth's rotation sweeps it through the overhead E-W field-of-view. CHIME has no moving parts.

**FFT Imaging with cylindrical dishes:** An array of receivers is placed at even spacing along the focus of each parabolic cylinder. By making a fast Fourier transform (FFT) of the set of signals from this line feed versus position along the cylinder, a fan of beams along the meridian is created. Hundreds of beams can reasonably be constructed this way. As the Earth turns these beams sweep the sky, allowing daily coverage of about half the celestial sphere. The baseline design of CHIME uses five cylinders as an interferometer array. This means our angular resolution in both the N-S and E-W directions is that of a 100-m aperture telescope while our field of view remains large.

To view half of the sky from the DRAO (latitude 49 degrees), CHIME requires a field of view in the north-south direction extending approximately 45 degrees either way from straight overhead. At 800 MHz, this field of view is Nyquist sampled without aliasing by digitizing the signals at 390 places spaced evenly along the 100 metre focal line of each cylinder.

**CHIME Site:** CHIME will be built and operated at the Dominion Radio Astrophysical Observatory (DRAO), a Canadian government facility near Penticton BC. The DRAO site is extremely well shielded, by surrounding hills, from locally generated radio-frequency interference (RFI). The Observatory owns most of the land within the ring of hills and there are stringent zoning controls on the remainder to maintain federal protection of this RFI environment. Observatory equipment is thoroughly shielded, and ongoing monitoring rapidly locates sources of on-site interference. The radio interference within the CHIME 400-800 MHz window at the DRAO is likely to be the best at any radio observatory in North America. See Figure 5

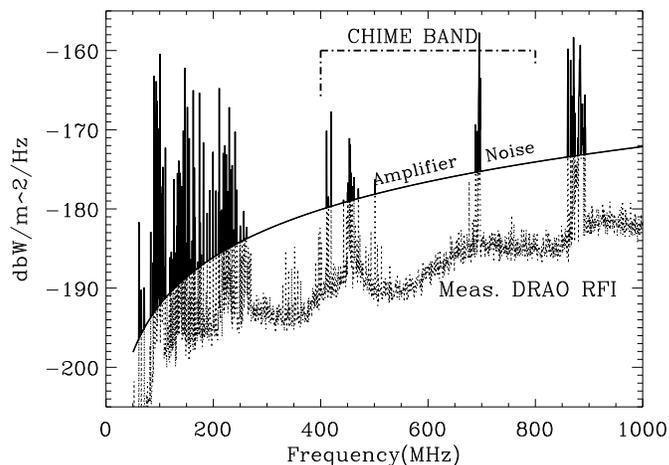


Figure 5: Measurement of Radio Frequency Interference at the CHIME site at DRAO, December 2009. The data are collected at 10kHz resolution and show the TV and cell phone bands that sandwich the CHIME window. The smooth curve shows the noise floor of a  $T_{sys}=40K$  amplifier with  $1\mu s$  integration time and confirms that low resolution ( $\leq 3$  bit) A/D conversion will suffice.

**Signal Processing:** Image formation proceeds as follows. The antennas along each focal line are broadband and sensitive to both linear polarizations. Signals from each antenna are amplified and digitized. These broadband digital signals are separated by the digitizer circuit into many

narrow frequency channels,  $\Delta B \approx 1$  MHz, which together span the whole input band. For each polarization and for each  $\Delta B$ , the data from every antenna on a cylinder are sent to a single computer which correlates them all by performing a fast Fourier transform. There will be five cylinders for CHIME, and the transforms from each of them will be correlated to form a two-dimensional image. This all happens roughly once every microsecond, but the output of the second level of correlation can be averaged for a longer time, perhaps one second, allowing much less data to be recorded than is accumulated.

**Low Noise Amplifiers:** CHIME will take advantage of recent rapid improvements in the noise figure of room temperature radio-frequency amplifiers which today cost as little as \$3 each. We have assumed a 50K noise temperature in all of our sensitivity estimates. However, we have prototyped low noise room temperature amplifier circuits suitable for CHIME which exceed this performance requirement by a wide margin. See Figure 6.

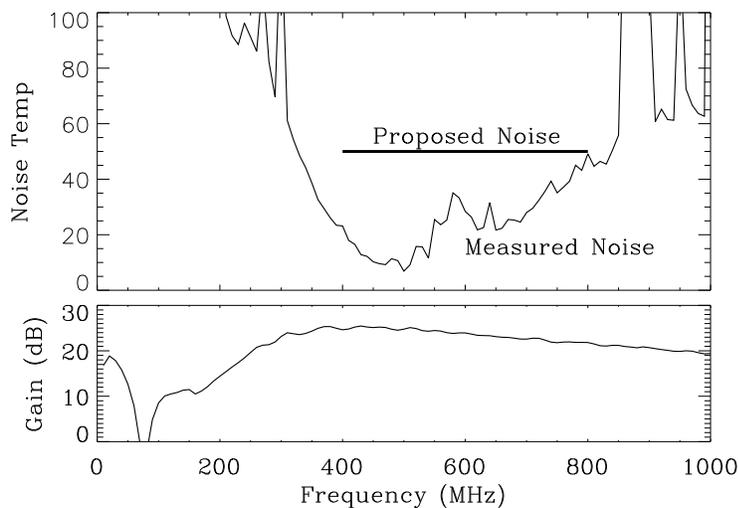


Figure 6: Gain and noise temperature are shown for a prototype CHIME amplifier we have built which greatly exceeds our requirements across the full CHIME band. The amplifier is based on room-temperature commercial components originally developed for the cell phone market, and no fine tuning is required after assembly. The solid line shows the noise performance of 50K which we have assumed in all of the CHIME sensitivity estimates.

**Digitization:** The analog signals from the summed amplifiers will be converted to digital signals with purpose-built digitizer/channelizer electronics boards, sampling 3 bits at  $\approx 2$  GSps, and then channelized into 400 bands, each 1 MHz wide. Three-bit data conversion is more than sufficient given the excellent RFI environment at the DRAO, shown in Fig. 5. A state-of-the-art field programmable gate array (FPGA) will handle the challenging real-time channelization using special purpose firmware for the prototype stage.

**Data flow and network:** The channelized data will be assembled into Ethernet packages and sent to the second stage, spatial domain FFT-correlators via a 10 Gb Ethernet network. The data rate at the output of the channelizer electronics, after discarding out-of-band channels and accounting for bit-growth in the FFT is 4 Gb per second, per digitizer.

For any of the 1MHz-wide bands, signals from every digitizer are brought to one computing node, where a linear image is formed by digital fast Fourier transform of these signals. The final step in image formation is correlation between the five arrays. Performing this FFT and correlation simultaneously for both polarizations and every frequency band once per microsecond is an enormous task.

The internal data rate of CHIME is gigantic. 4000 digitized channels stream 800 million samples

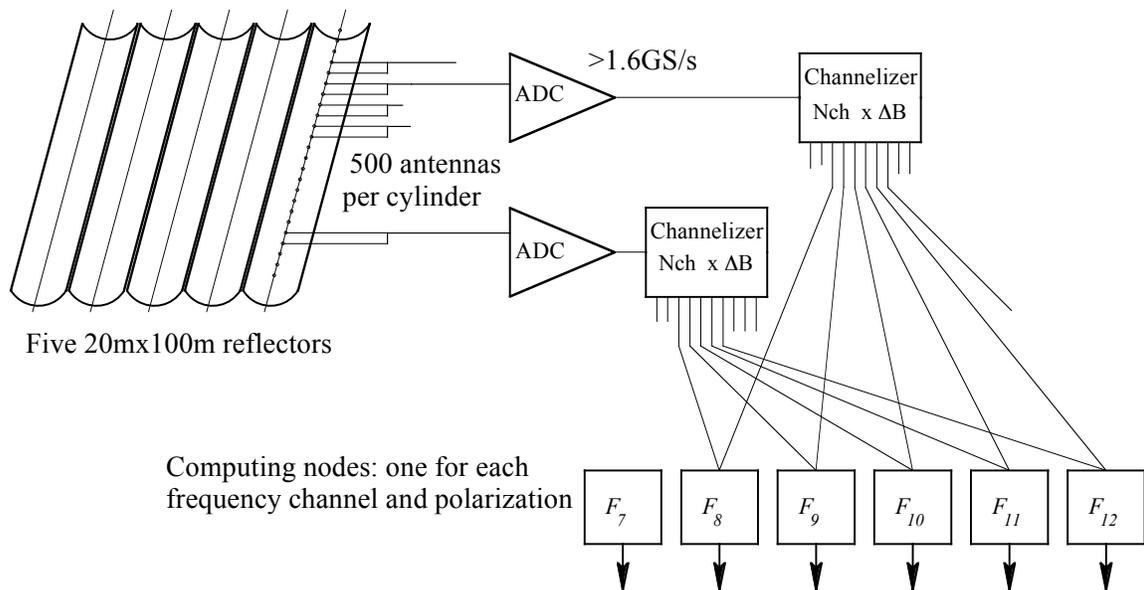


Figure 7: CHIME signal path. For each linear polarization, signals from adjacent antennas are optionally summed before digitization. 390 digitized signals per cylinder are required to Nyquist sample half the sky. The digitized signal within the CHIME band is split into  $N_{chan} = 250 - 400$  channels with bandwidth  $\Delta B \leq 1.5 \text{ MHz}$  and sent to a computing node which forms a 1D image by Fourier transform with respect to location along the cylinder and correlation between cylinders.

per second each, and all these data need to be routed through a switch to arrive at the appropriate computing node which forms a coherent image. The raw data rate of 3.2 Terasamples per second presents a major switching challenge. Summing adjacent channels at the cost of sky coverage or reducing our spectral bandwidth are options for reducing this huge rate at the cost of initial sensitivity.

After correlation, the final information in the image is modest: each day, a data cube of 1 million spatial pixels times  $\sim 400$  frequency channels will be produced, which in floating point storage takes less than 4 gigabytes to store, equivalent to one DVD per day. This is much less data than many modern telescopes produce.

**Beam shapes and Calibration :** The foregrounds must be removed to a part in 10,000 to be below the thermal noise in the maps. The basic approach is to take the maps made at each frequency and re-combine them to form a spectrum along the line of sight of each map pixel. Removing galactic signals from each line of sight requires careful beam calibration at each frequency. Since CHIME is a filled aperture in one dimension and close packed in the other, it is somewhat spared from a common problem in interferometry of missing baselines and “dirty” beams. Nevertheless, understanding the spectral response of an instrument to one part in 10,000 is challenging. Being a drift-scan survey, we can exploit several aspects of symmetry to achieve this goal. The raw data is a 3-D data cube, where each frequency is the sky map convolved by an a priori unknown beam. We will use a PCA analysis from EoR studies (arxiv.org 0807.1056),

Table 2: Design parameters of the baseline CHIME instrument.

Observing Frequency	800 to 400 MHz
Observing Wavelength	37 to 75 cm
Redshift	$z \approx 0.8$ to 2.5
System Noise Temperature	$\leq 50\text{K}$
Beam size	$0.26^\circ$ to $0.53^\circ$
Field of View, N-S	$90^\circ$ †
Field of View, E-W	$1.3^\circ$ to $2.5^\circ$
Size of Each Cylinder	100m $\times$ 20m
Number of Cylinders	5
Collecting Area	10,000 m <sup>2</sup>
Dual polarization Antenna Spacing	19. cm
Number of Antennas per Cylinder	536.
Number of Digitizers per Cylinder	380 †
Bandwidth of Channeled outputs	$\sim 1.5$ MHz

† Depends on choice of summing neighboring antennas

and determine the two dimensional beam response at each frequency. It may seem at first that there is an equal number of degrees of freedom in the beam model as there is in the sky map. But the beam is small, and a stationary function on the sky, so the problem is overdetermined by the ratio of beam size to total sky size, which is a factor of at least 100. We thus perform a “self-calibration” of the system. The continuous scan of the sky with this transit telescope keeps the instrument stable, enabling an accurate characterization of its response. This method has been tested successfully with the Green Bank Telescope (GBT) in drift scan mode, and the team has made HI intensity detections at  $z \sim 1$ . The GBT has a similar collecting area as CHIME, but only a single beam instead of hundreds, so the 3 year survey on CHIME would take a millenium on GBT.

The fully digital interferometric analysis of signals allows modern gain and phase calibrations, using techniques such as phase closure, common in interferometers. For large numbers of input channels, this has been pioneered by LOFAR and MWA. The information revolution allows CHIME to have precise beam calibrations, which had not been possible with cylinders in the past.

**Complementarity to SKA:** We are confident that pursuing both CHIME and a strong Canadian role in the Square Kilometer Array (SKA) enables both projects and makes them more successful. CHIME is a narrowly focused experiment which has a rapid schedule and modest requirements for technical innovation. However, a few devices we are developing, particularly amplifier and A/D technologies, overlap well with anticipated needs, and present development work, of the SKA. Very importantly, with its short time scale to exciting scientific results, CHIME is already drawing a new cadre of young Canadian students and researchers into Radio Astronomy and instrumentation, a group that Canada will surely need as the SKA effort matures.

**Conclusion:** CHIME is a transformative instrument which can be built now and for only a modest cost. The combination of a solid design, an excellent team and an outstanding site provide Canada with an opportunity to make a large impact in experimental cosmology.

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