PHYS 306 **Homework 5** Due Wednesday March 26, *5 PM*

In wall box labeled PHYS 306 outside Henn 310

Late Penalty: -30% by 5 PM Mar 27,

-60% by 5 PM Mar 28.

1. In class we derived the equations of motion of a projectile moving above the spinning Earth, where the origin was placed at the launch point (at geocentric latitude lambda), as three second-order differential equations for the *(x', y', z')* coordinates in the non-intertial reference frame of the launch point.

$$\ddot{x}' = -2\omega(\dot{z}' \cos\lambda - \dot{y}' \sin\lambda)$$
$$\ddot{y}' = -2\omega\dot{x}'\sin\lambda$$
$$\ddot{z}' = -g + 2\omega\dot{x}'\cos\lambda$$

a) Integrate these once to get the differential equations for the velocities and identify the integration constants.
b) The spin rate omega of the Earth is quite small so, neglecting terms of order omega squared, generate a second order differential equation for *x'* where you have eliminated the *y'* and *z'* velocities. Then integrate it to obtain the solution for *x'(t)*. Hint: you will see *t*-cubed terms!
c) Obtain the solutions for y'(t) and z'(t), continuing to neglect omega-squared terms.
d) Assume a projectile is launched straight up with initial velocity vector $\mathbf{v}_{0}'$ in the rotating frame. Show that in the non-spinning limit you recover usual equations. Then assume that the initial speed vector is 'straight up' with a speed of 1.1 km/s (one tenth of the escape speed) from Vancouver (whose latitude is 49.25 degrees North). Use g=9.81 m/s^2. How far in (x',y') will the projectile land from the launch point? (Use the convention as in class that east is positive and north is positive)

2. Playing catch with the nephew on a Merry-go-Round.

You are playing catch with your nephew using a baseball. The two of you are at the oppositve extremities of a merry-go-round of radius R (that is, the line between you passes through the center, which can be the origin of your coordinate system), which is turning with angular frequency ω. In a macabre setting, feet are glued to the merry-go-round so don't worry about people moving.

Take your initial position to be at *x=-R* and your nephew at *x=+R*, and let's save ink by dropping the primes (that is, all quantities are in the rotating non-intertial reference frame; use a subscript 'inert' if you want to refer to a quantity in the non-rotating frame).

You throw the ball with a fast speed $v_0 \gg \omega R$ at the instaneous location of your nephew (that is, directly across the center line), how far off the line between you and your nephew will the ball be when it passes him? For the purposes of this problem, neglect the vertical motion of the ball (that is, neglect gravity and the downwards *z* acceleration it causes; all motion is in the *x,y* plane).
(a) Without restricting to this particular initial condition or using any approximations, derive the equations of motion of the ball in this rotating reference frame.
(b) Without any approximations, what are the apparent accelerations in *x* and *y* of the ball at *t*=0? After stating that, then apply the 'fast v' limit (which will essentially allow you to neglect terms of order ω^2, but not $v_0\omega$). Based on this, derive approximate solutions for x(t) and y(t) and compute the deflection (that is, the value of y when x crosses x=+R where your nephew is).
(c) Say you can throw a ball at 8 m/s and the 2-meter radius merry-go-round is turning at 0.2 radians per second. Verify that your approximations are valid. If your nephew can reach out 40 cm to catch the ball, is the ball within reach when it passes him?