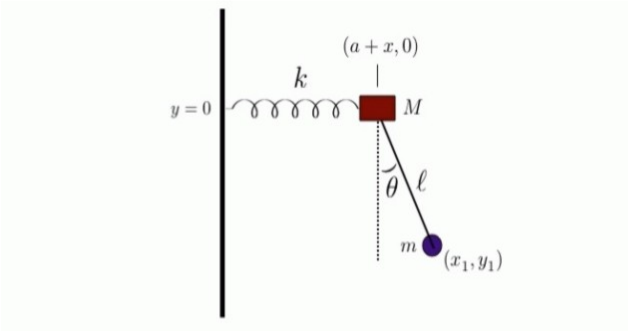
PHYS 306 **Homework 3** Due Thursday March 6, *5 PM*

In wall box labeled PHYS 306 outside Henn 310

Late Penalty: -30% by 5 PM Mar 7,

-60% by 5 PM Mar10



1. Consider the system above, consisting of a mass *M* attached to a spring (with spring constant *k*) which moves on a horizontal table; the equilibrium extension of the spring is *a* with *x* being the distance beyond that (although *x* can be negative). Below the mass *M*, a negligible-mass rigid rod of length *l* is connected to a bob of mass *m*, which is attached to *M* through a slot in the table on which *M* moves (horizontally) with negligible friction. Thus, use x and theta as generalized coordinates. Derive the two second-order equations of motion using the Lagrangian method. Also give the two generalized momenta; are either of these conserved? Show that if $M \to \infty$ one recovers the regular rigid-rod pendulum equation of motion.

2. A particle of mass *m* is constrained to lie along a frictionless, horizontal plane subject to the force $F(x) = -kx + \frac{kx^3}{A^2}$.

It is projected from $x = 0$ to the right along the +x direction with initial kinetic energy T_o where k and A are positive constants. Find

(a) the potential energy function $V(x)$ for this force;
(b) the kinetic energy as a function of the particle's position, given T_o
(c) the total energy as a function of the particle's position;
(d) Sketch the potential, kinetic, and total energy functions, for the case *k=A=1*
(e) the turning points of the motion and the condition the total energy of the particle must satisfy if its motion is to exhibit turning points, as a general function of *k* and *A*.

3. Determine the oscillation period, as a function of energy E, when a particle of mass m moves in a field with potential energy $V(x) = -V_o / \cosh^2(\alpha x)$, for $-V_o < E < 0$ with V_o positive.

(a) First show that, with $s = \sinh(\alpha x)$ and determining the appropriate s_{max} ,the period satisfies

$$T = \frac{2\sqrt{2m}}{\alpha\sqrt{E}} \int_0^{s_{max}} \frac{ds}{\sqrt{s^2 + \left(\frac{E + U_o}{E}\right)}}$$

(b) Using the following three relations,

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln\left|x + \sqrt{x^2 + a^2}\right| \quad ; \quad \sinh(ar\cosh x) = \sqrt{x^2 - 1} \text{ (for } |x| > 1) \quad ; \quad \ln(\sqrt{-1}) = \frac{\pi}{2}\sqrt{-1}$$

show that this can be reduced to $T = \frac{\pi}{\alpha}\sqrt{\frac{2m}{|E|}}$