PHYS 306 **Homework 2** Due Monday Feb 24, *5 PM*

In wall box labeled PHYS 306 outside Henn 310

Late Penalty: -30% by 5 PM Feb 25, -60% by 5 PM Feb 26.

1. In tutorial you showed that in spherical polar coordinates, once one evaluated the dependence some of the $\hat{r}$ unit vector on theta and phi, that one could write:

$$\vec{v}=\dot{r}\hat{r}+r\dot{\theta}\hat{\theta}+r\sin\theta\dot{\varphi}\hat{\varphi}$$

a) Show that after evaluating the dependencies of some of the other unit vectors, one obtains

$$\vec{a}=\hat{r}(\ddot{r}-r\dot{\theta}^2-r\dot{\varphi}^2\sin^2\theta)+\hat{\theta}(r\ddot{\theta}+2\dot{r}\dot{\theta}-r\dot{\varphi}^2\sin\theta\cos\theta)+\hat{\varphi}(r\sin\theta\ddot{\varphi}+2\dot{r}\dot{\varphi}\sin\theta+2r\cos\theta\dot{\theta}\dot{\varphi})$$

b) Although this inelegant form of the acceleration might lead one to believe that one is much better off in cartesian coordinates, show that for the case of uniform circular motion and constact radius R in a (i) vertical plane, or (ii) horizontal, thus theta=pi/2, plane, that one returns to the simpler version of plane-polar coordinates (albeit with different angles), and that in both cases the acceleration reduces to the usual v^2/R centripetal acceleration.

2. Starting from the cartesian definition of the kinetic energy T, demonstrate that upon substitution of the tranformation equations from $(x,y,z)\rightarrow(r,\theta,\varphi)$ the algebra collapses to

$$T=\frac{m}{2}(\dot{r}^2+r^2\dot{\theta}^2+r^2\sin^2\theta\dot{\varphi}^2)$$

3. Prelude to nuclear scattering. Consider the problem of firing a proton at a nucelus of atomic number Z>>1 (so that one views the nucleus as immobile and the origin of the coordinate system, only the proton moves). Work in spherical polar coordinates.

a) Write the conservative potential function for the interaction, using your knowledge of the electrostatic force with constant k

b) Write down the system's Lagrangian L.

c) Determine the three Euler-Lagrange equations of motion of the proton.

d) Show that these equations imply that if one picks the polar axis such that $\dot{\varphi}=0$ at the start, then the accelerations will not alter this and phi is always conserved. In this case write the two simpler resulting coupled 2nd-order equations for the r and theta dimensions.