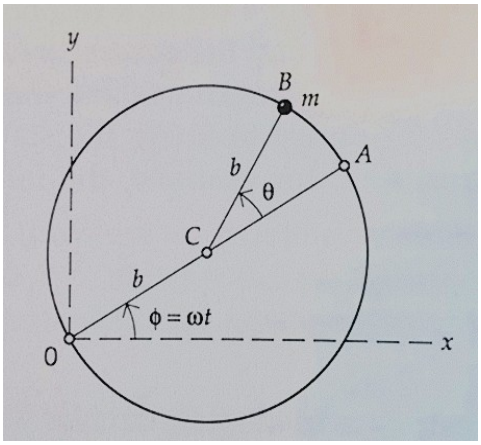
PHYS 306 **Midterm examination** **March 14/2014**

Possibly useful: $\int \frac{dx}{\sqrt{c^2-x^2}} = \arcsin\left(\frac{x}{c}\right)$, $\int_0^{\pi/2} \sin^{2m} x\, dx = \int_0^{\pi/2} \cos^{2m} x\, dx = \frac{\pi}{2} \frac{1\cdot 3\cdot 5 \ldots (2m-1)}{2\cdot 4\cdot 6 \ldots (2m)}$

1. A bead of mass *m* slides freely on a frictionless circular wire of radius *b* that rotates in a horizontal plane about a point on the circular wire with a constant angular velocity ω.
(a) State the number of degrees of freedom and thus identify the generalized coordinate(s).
(b) Use Lagrange's equations to find the equation of motion of the bead. Note that the angle θ is measured in the reference frame of the wire (not in the inertial (*x*,*y*) reference frame).
(c) If the motion results in the value of θ remaining small, then give the period of the oscillation in terms of the various parameters of the problem.

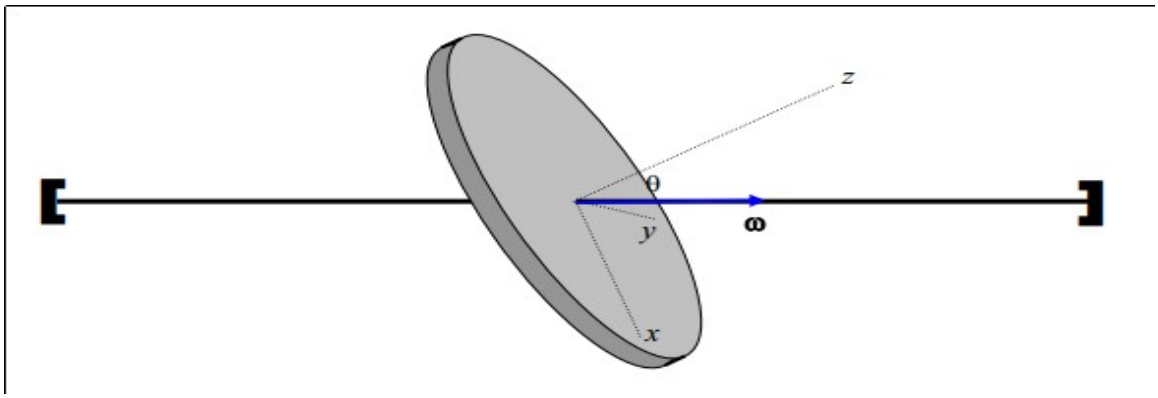


2. A particle of mass m moves in a conservative potential function given by (with a>0) :

$$V(x) = -\frac{m\omega^2}{2}(a^2 - x^2) \quad for \quad |x| \le a, \quad V(x) = 0 \quad for \quad |x| > a$$

(a) If *m* has $|x|<a$ and the total energy is zero, where will the particle be moving fastest? How fast?
(b) For positive energy, describe the motion of the particle when $x>a$.
(c) For the zero-energy case, if one started at $t=0$ with $x=-a$, how long would it take for the particle to reach $x=X$, where $-a<X<a$? As a particular case, how long would it take to reach $x=a$?

3. A thin uniform disk of radius R and mass m is rigidly attached to an axle, but tilted so that the body-fixed principle axes (shown as *x*,*y*,*z*) are inclined, with the *z* axis a polar angle θ from the axle. The axle rotates with angular speed ω . Choose the arbitrary *x* axis so that **ω** is in the x-z plane.



(a) Derive the moments of inertia I_x, I_y, and I_z around the 3 principle axes. (If you just state them because you have it on your formula sheet, you will only receive a small fraction of credit.)
(b) The disk is unbalanced and so there must be a torque which maintains this motion. What are the components of the torque vector? (Hint: consider Euler's equations)