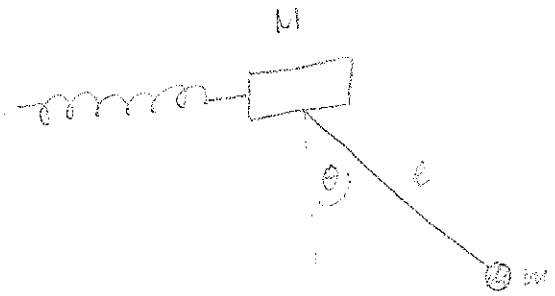
phys 306 HW 7

(1)



$$T = \frac{1}{2} M \dot{x}^2 + \frac{1}{2} m \left(\dot{x} + \ell \dot{\theta} \cos\theta\right)^2 + \frac{1}{2} m \left(\ell \dot{\theta} \sin\theta\right)^2$$

$$= \frac{1}{2} (M+m) \dot{x}^2 + m \dot{x} \ell \dot{\theta} \cos\theta + \frac{1}{2} m \ell^2 \dot{\theta}^2$$

$$V = -mg\ell\cos\theta + \frac{1}{2} k x^2 \quad ; \quad \mathcal{L} = T - V$$

eom's

(1) $\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) = \frac{\partial \mathcal{L}}{\partial x}$

$\Leftarrow$ $(M+m)\ddot{x} + m\ell\ddot{\theta}\cos\theta - m\ell\dot{\theta}^2\sin\theta = -kx$

(2) $\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}}\right) = \frac{\partial \mathcal{L}}{\partial \theta}$

$\Leftarrow$ $\frac{d}{dt}\left[m\dot{x}\ell\cos\theta + m\ell^2\dot{\theta}\right] = -m\dot{x}\ell\dot{\theta}\sin\theta - mg\ell\sin\theta$

$\Leftarrow$ $m\ddot{x}\ell\cos\theta - m\dot{x}\dot{\theta}\ell\sin\theta + m\ell^2\ddot{\theta} = -m\dot{x}\ell\dot{\theta}\sin\theta - mg\ell\sin\theta$

$\Leftarrow$ $m\ddot{x}\ell\cos\theta + m\ell^2\ddot{\theta} + mg\ell\sin\theta = 0$

generalized momenta:

$$P_x = \frac{\partial L}{\partial \dot{x}} = (M+m)\dot{x} + m\ell\dot{\theta}\cos\theta \quad \text{1p}$$

not conserved since $\frac{\partial L}{\partial x} \neq 0$.

$$P_\theta = \frac{\partial L}{\partial \dot{\theta}} = m\dot{x}\ell\cos\theta + m\ell^2\dot{\theta} \quad \text{1p}$$

not conserved since $\frac{\partial L}{\partial \theta} \neq 0$. 1p

$M \to \infty$ limit:

(1) $$\left(1+\frac{m}{M}\right)\ddot{x} + \frac{m}{M}\ell\ddot{\theta}\cos\theta - \frac{m}{M}\ell\dot{\theta}^2\sin\theta = -\frac{k}{M}x$$

$\xrightarrow{M\to\infty}$ $\ddot{x} = 0$

since $\ddot{x} = 0$ for $M \to \infty$ 1p

(2) $m\ell^2\ddot{\theta} + mg\ell\sin\theta = 0$

eqn for regular pendulum.