Nodal Protectorate: A unified model of the ab-plane and c-axis penetration depths in underdoped cuprates

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October 23, 2003

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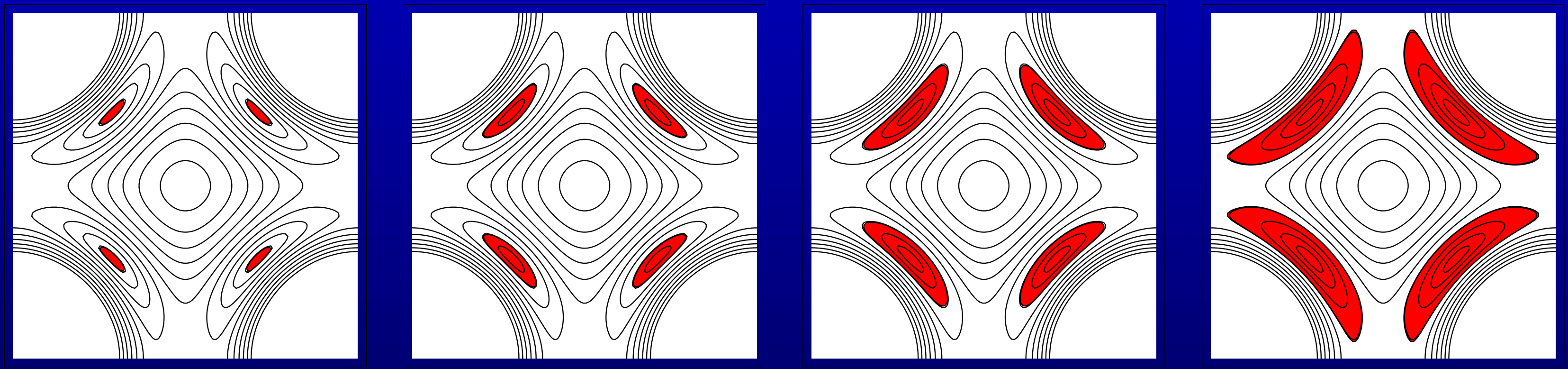
A.Hosseini, D.P. Broun, D.A. Bonn (experiment)

What is the Nodal Protectorate?

Nodal Protectorate consists of regions in the vicinity of nodes containing *coherent* quasiparticle excitations even as the systems teeters on the brink of becoming a Mott insulator.



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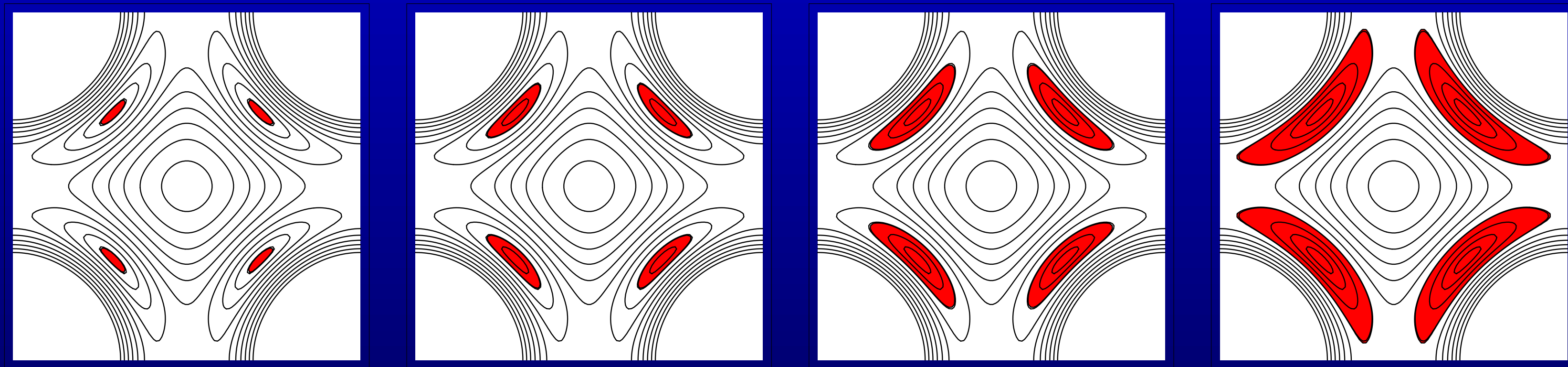

Evidence for these protected regions comes from a host of experiments, most notably thermal conductivity, microwave measurements of the penetration depth, STM, and to lesser extent also ARPES.

Superfluid density in cuprates, ab-plane

- In the underdoped region experiments show

$$\rho_s^{ab}(x,T) \sim \lambda_{ab}^{-2}(x,T) \simeq ax - bk_B T,$$

with $a \simeq 244$meV and $b \simeq 3.0$ [Lee and Wen, PRL **78**, 4111 (1997)].

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- Problem: models that give correct x-dependence (e.g. RVB-type theories) generally yield strong ($\sim x^2$) dependence of the coefficient b.

Superfluid density in cuprates, c-axis

- Recent UBC Group data on ultrapure YBCO single crystals show c-axis phenomenology that is tantalizingly similar to the ab-plane for doping levels as low as $T_c = 5$K

$$\rho_s^c(x,T) \sim \lambda_c^{-2}(x,T) \simeq A x^{\alpha} - B T^{\alpha}, \quad \text{with } \alpha \simeq 2.4$$

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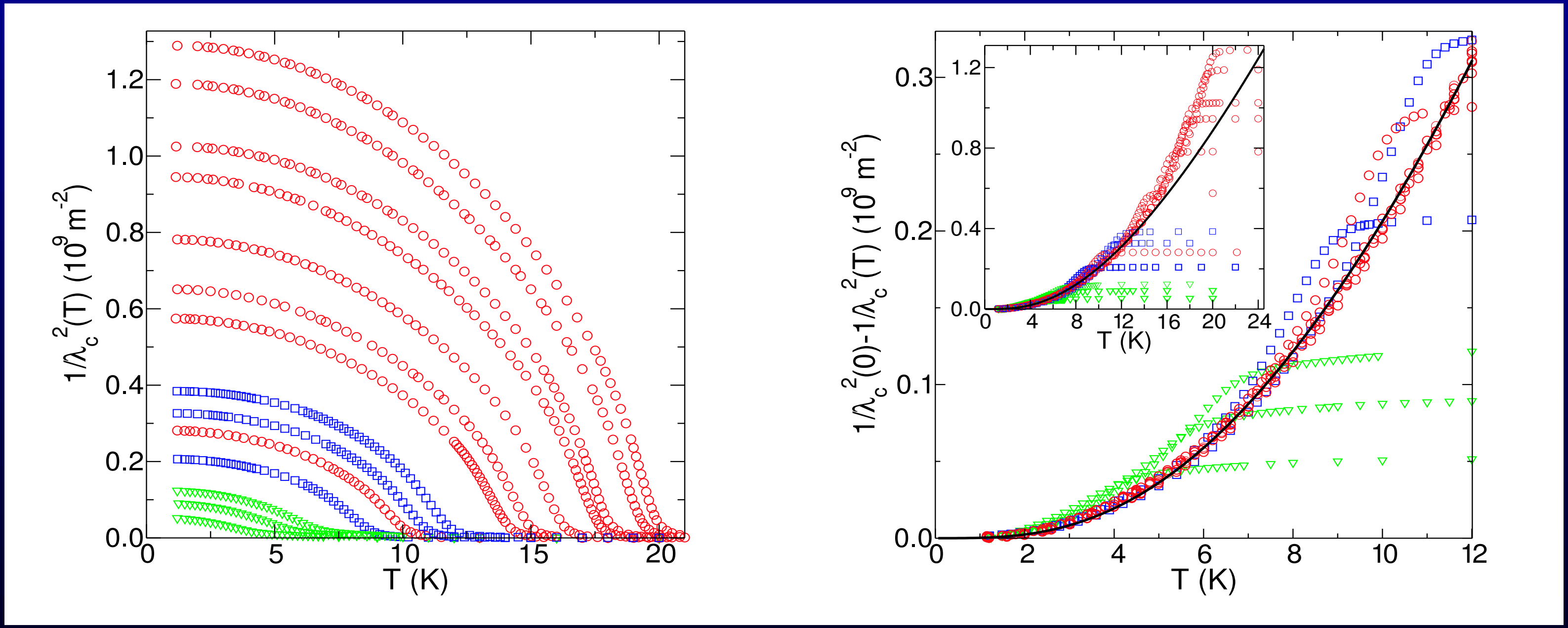

Data from UBC group [Hosseini *et al.* unpublished]

The model: ab-plane

To preserve the observed linear T-dependence we use BCS d-wave theory with phenomenological charge renormalization factor [Ioffe & Millis, J. Phys. Chem. Solids **63**, 2259 (2002)] to account for doping dependence:

$$\frac{1}{\lambda_{ab}^2(T)} = \frac{e^2\,n}{d} \sum_{\mathbf{k}} Z_{\mathbf{k}}^2 \left(\frac{\partial \epsilon_{\mathbf{k}}}{\partial k_x}\right)^2 \frac{\Delta_{\mathbf{k}}^2}{E_{\mathbf{k}}^2} \left[\frac{1}{E_{\mathbf{k}}} - \frac{\partial}{\partial E_{\mathbf{k}}}\right] \tanh\frac{1}{2}\beta E_{\mathbf{k}},$$

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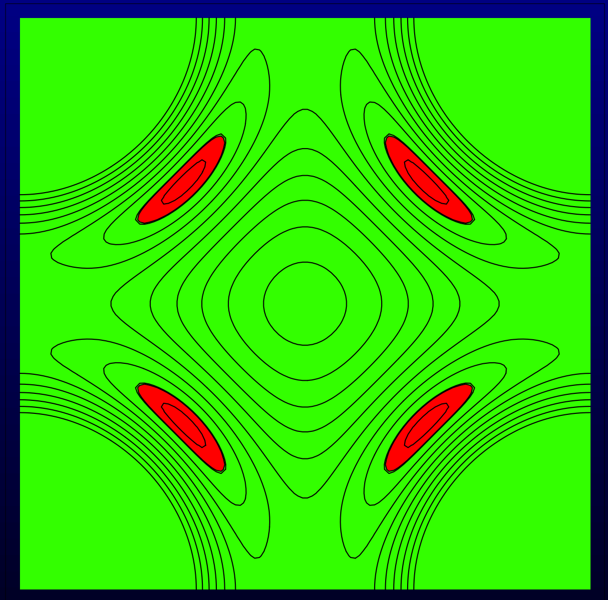
with

$$Z_{\mathbf{k}} \approx \begin{cases} Z_0 & \text{for } E_{\mathbf{k}} < E_{\mathrm{c}}, \\ 0 & \text{for } E_{\mathbf{k}} > E_{\mathrm{c}}. \end{cases}$$



This gives

$$\rho_{ab} \sim Z_0^2 \frac{v_{\mathrm{F}}}{v_\Delta} [E_c - (4 \ln 2) k_B T]$$

in agreement with experiment provided we take $E_c \sim x$.

Temperature dependence: c-axis

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We employ a model of *incoherent* tunneling between Cu-O layers

$$H_{\text{tunn}} = \sum_{m,\sigma} \int d^2 r (t_{\mathrm{r}} c^{\dagger}_{\mathrm{r},m+1,\sigma} c_{\mathrm{r},m,\sigma} + \mathrm{h.c.}),$$

where t_{r} describes random interlayer tunneling with

$$\overline{t_{\mathbf{k}}} = 0, \ \ \overline{t^*_{\mathbf{k}} t_{\mathbf{k}+\mathbf{q}}} = (2\pi)^2 \delta(\mathbf{q}) \mathcal{T}_k^2, \ \text{and} \ \ \mathcal{T}_{\mathbf{k}}^2 = \frac{t_\perp^2}{\pi \Lambda^2} \mathrm{e}^{-k^2/\Lambda^2}.$$

Such “impurity assisted” tunneling is known to lead to weaker-than-linear T-dependence.

We obtain

$$\frac{1}{\lambda_c^2(x,T)} = 8e^2 d \sum_{\mathbf{k},\mathbf{p}} \mathcal{T}^2_{\mathbf{k}-\mathbf{p}} T \sum_{i\omega} F(\mathbf{k},\omega) F(\mathbf{p},\omega).$$

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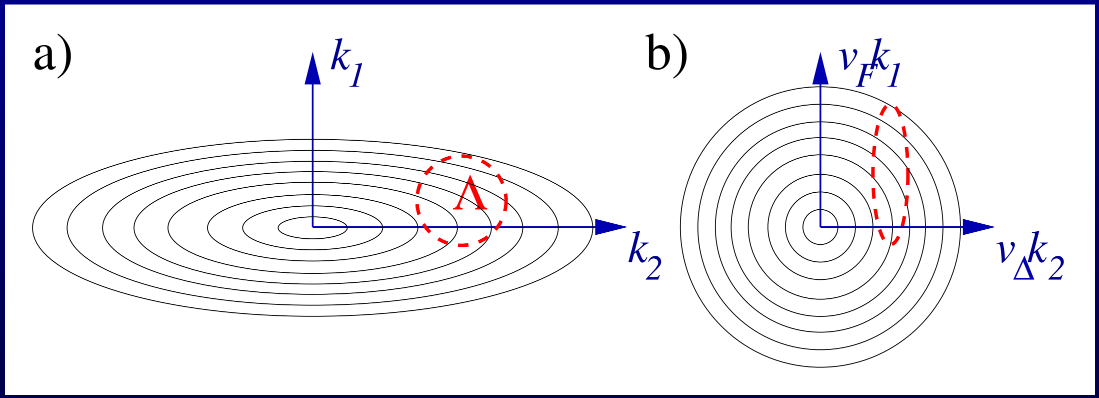
A straightforward scaling analysis leads to the following result

$$\delta\lambda_c(T) \sim \begin{cases} T^3 & \text{for } T \ll v_\Delta \Lambda \ll v_\mathrm{F}\Lambda; \\ T^2 & \text{for } v_\Delta\Lambda \ll T \ll v_\mathrm{F}\Lambda; \\ T & \text{for } v_\Delta\Lambda \ll v_\mathrm{F}\Lambda \ll T. \end{cases}$$



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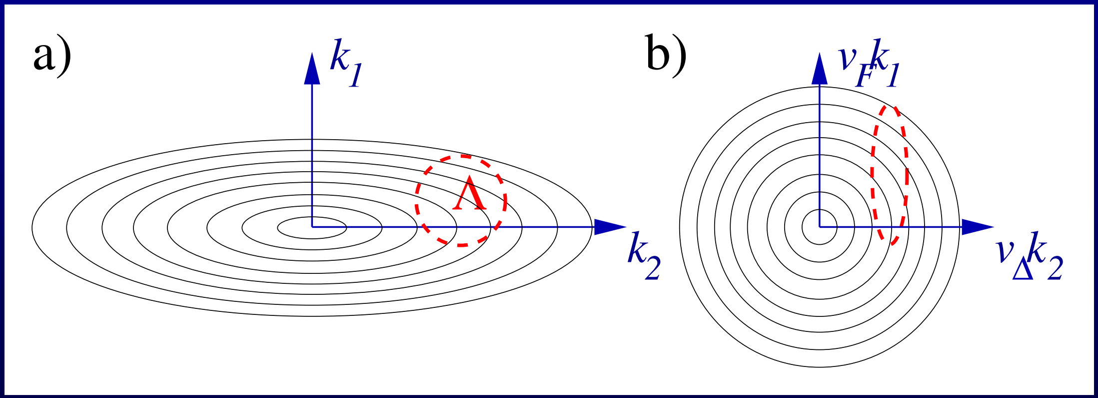

In this model the peculiar $T^{2.4}$ behavior arises as a crossover between T^2 and T^3 dependence in the incoherent tunneling model.

Doping dependence: c-axis

At $T = 0$, all integrals are cut off by $E_c \sim x$ and because of the linear Dirac spectrum one obtains the same crossover behavior in x:

$$\lambda_c^{-2}(x,0) \sim \begin{cases} x^5 & \text{for } E_c \ll v_\Delta \Lambda \ll v_{\mathrm{F}} \Lambda; \\ x^2 & \text{for } v_\Delta \Lambda \ll E_c \ll v_{\mathrm{F}} \Lambda; \\ x & \text{for } v_\Delta \Lambda \ll v_{\mathrm{F}} \Lambda \ll E_c. \end{cases}$$

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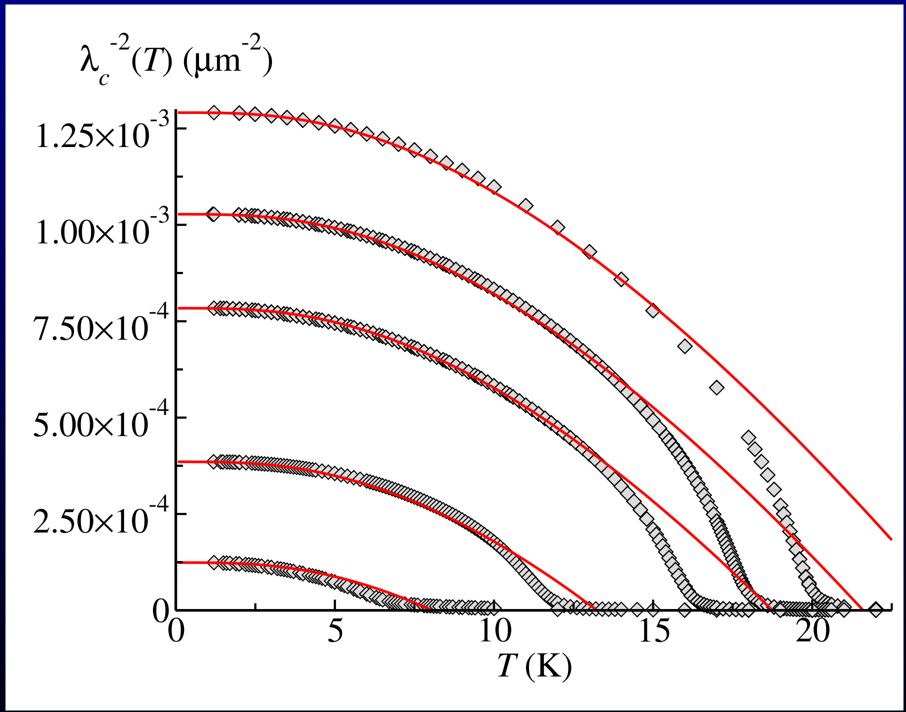
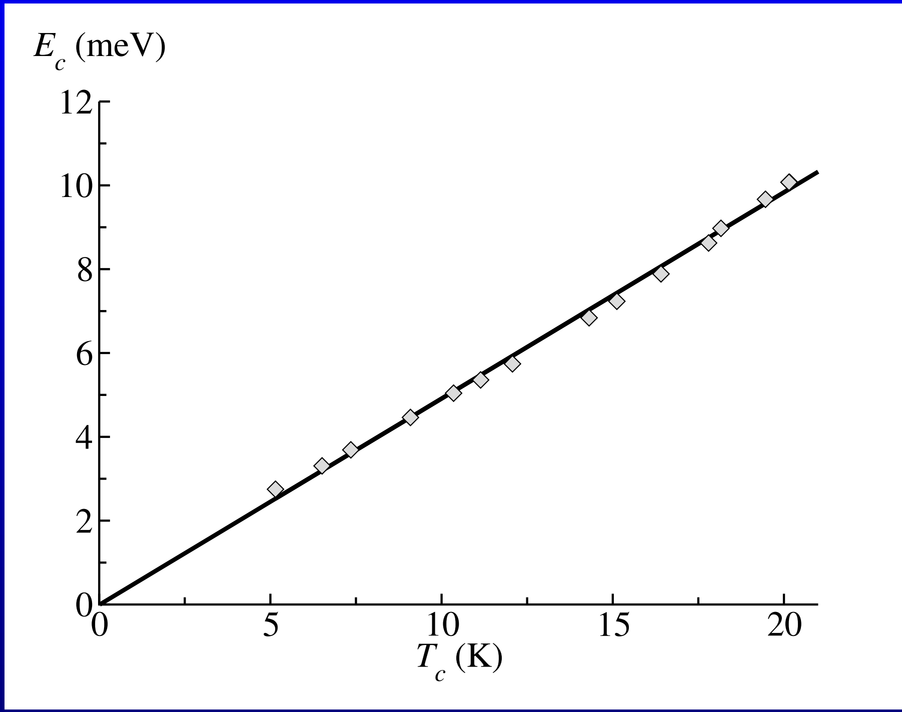
Numerical evaluation of $\lambda_c^{-2}(x,T)$ confirms the above scaling and gives excellent agreement with experiment.



Parameters extracted from the fits:

- Disorder corelation length
  $\hbar\Lambda^{-1} \simeq 120$Å
- Tunneling matrix element
  $t_\perp = 26\text{meV} \ll t$
- The nodal protectorate cutoff scale
  $E_c \simeq 0.49 T_c/\text{K} + 0.01(\text{meV})$ in
  agreement with Uemura scaling.



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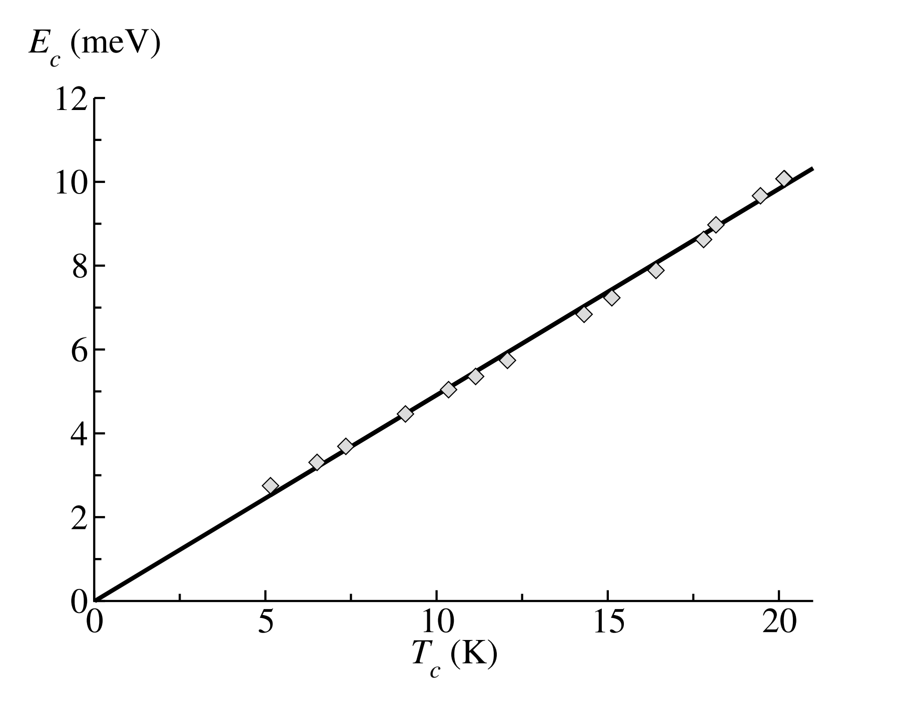

Since $\rho_s^{ab}(x,0) \sim E_c$ our model predicts that Uemura scaling for the ab-plane superfluid density will continue to hold down to very low doping.

This is a non-trivial prediction testable in future experiments.

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- This phenomenology puts severe constraints on microscopic models of underdoped cuprates.