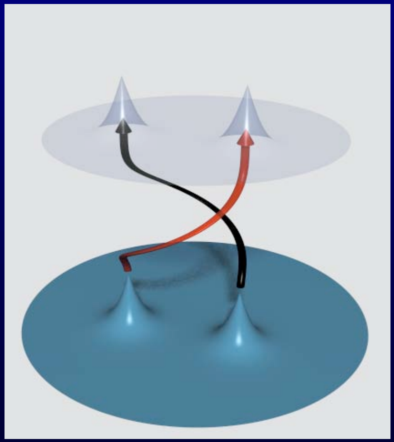
Anyons, fractional charges, and topological order in a weakly interacting system

M. Franz
University of British Columbia
franz@physics.ubc.ca

February 16, 2007

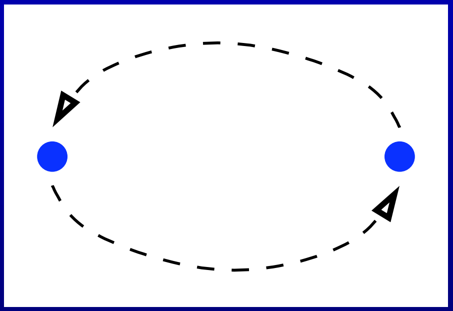


In collaboration with: C. Weeks, G. Rosenberg, B. Seradjeh

Particle statistics

In 3 space dimensions indistinguishable particles can be bosons or fermions,

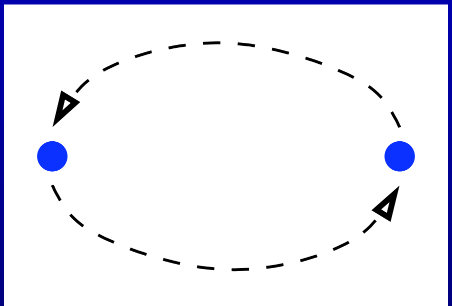
$$\Psi(\mathbf{r}_1, \mathbf{r}_2) = \pm\Psi(\mathbf{r}_2, \mathbf{r}_1).$$



Particle statistics

In 3 space dimensions indistinguishable particles can be bosons or fermions,

$$\Psi(\mathbf{r}_1, \mathbf{r}_2) = \pm\Psi(\mathbf{r}_2, \mathbf{r}_1).$$



In 2 space dimensions we can have exotic particles called “anyons”, so that

$$\Psi(\mathbf{r}_1, \mathbf{r}_2) = e^{i\theta}\Psi(\mathbf{r}_2, \mathbf{r}_1), \quad \theta \neq 0, \pi.$$

Anyons and quantum computation

ACADEMIC PRESS

SCIENCE DIRECT®

Annals of Physics 303 (2003) 2–30

ANNALS of PHYSICS

www.elsevier.com/locate/aop

Fault-tolerant quantum computation by anyons

A.Yu. Kitaev*

L.D. Landau Institute for Theoretical Physics, 117940, Kosygina St. 2, Germany

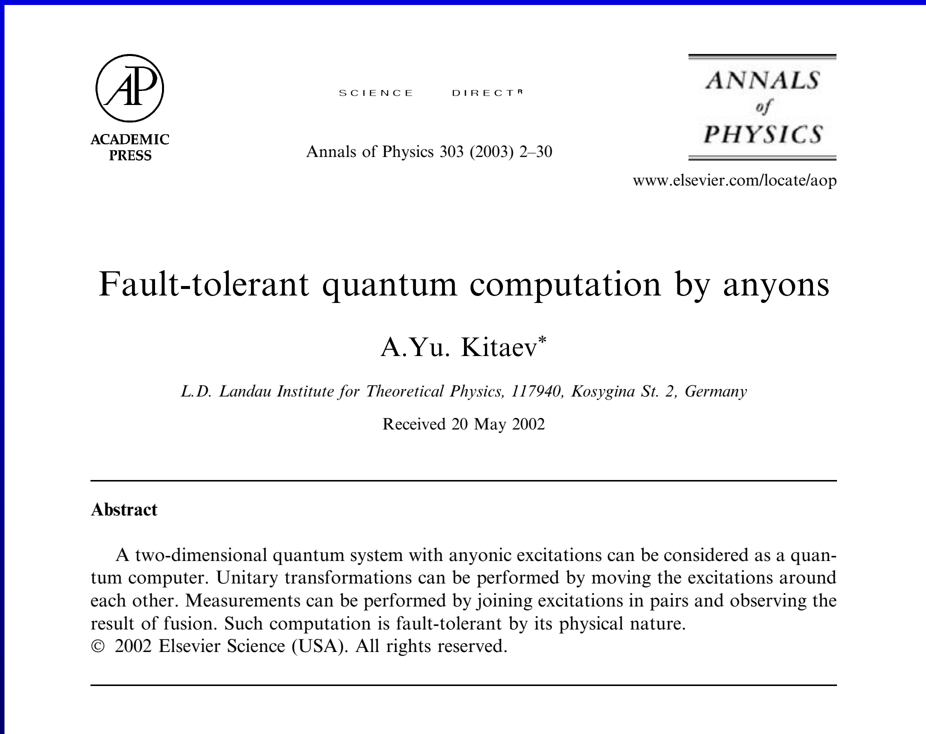
Received 20 May 2002

Abstract

A two-dimensional quantum system with anyonic excitations can be considered as a quantum computer. Unitary transformations can be performed by moving the excitations around each other. Measurements can be performed by joining excitations in pairs and observing the result of fusion. Such computation is fault-tolerant by its physical nature.

Quantum computation can be performed in a fault-tolerant way by braiding non-abelian anyons.

Anyons and quantum computation


ACADEMIC PRESS

SCIENCE DIRECT®

Annals of Physics 303 (2003) 2–30

ANNALS of PHYSICS

www.elsevier.com/locate/aop

Fault-tolerant quantum computation by anyons

A.Yu. Kitaev*

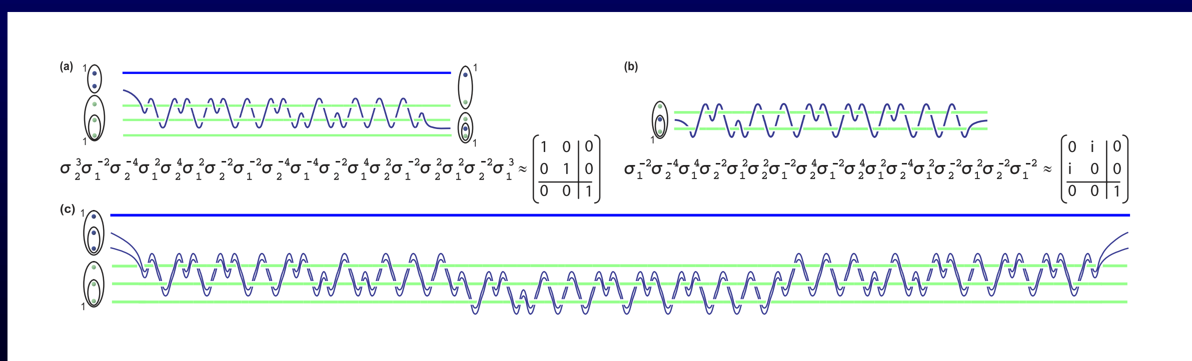
L.D. Landau Institute for Theoretical Physics, 117940, Kosygina St. 2, Germany

Received 20 May 2002

Abstract

A two-dimensional quantum system with anyonic excitations can be considered as a quantum computer. Unitary transformations can be performed by moving the excitations around each other. Measurements can be performed by joining excitations in pairs and observing the result of fusion. Such computation is fault-tolerant by its physical nature.

Quantum computation can be performed in a fault-tolerant way by braiding non-abelian anyons.



Bonesteel et al., PRL **95**, 140503 (2005).

Anyons in FQH fluids

Abelian anyons are known to occur as excitations of the fractional quantum Hall fluids described by Laughlin wavefunctions

$$\Psi(\{z_i\}) = \prod_{i<j}(z_i - z_j)^m e^{-\sum_i |z_i|^2/4},$$

with m odd integer. These have exchange phase

$$\theta = \frac{\pi}{m}$$

and charge

$$q = \pm\frac{e}{m}.$$

Anyons in FQH fluids

Abelian anyons are known to occur as excitations of the fractional quantum Hall fluids described by Laughlin wavefunctions

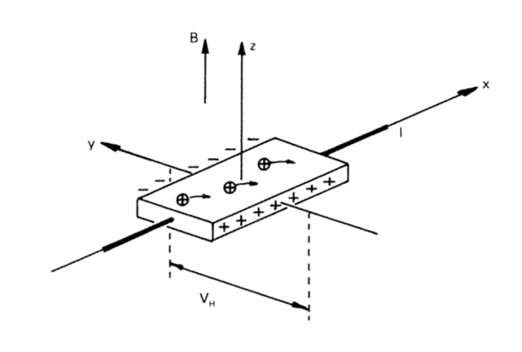
$$\Psi(\{z_i\}) = \prod_{i<j}(z_i - z_j)^m e^{-\sum_i |z_i|^2/4},$$

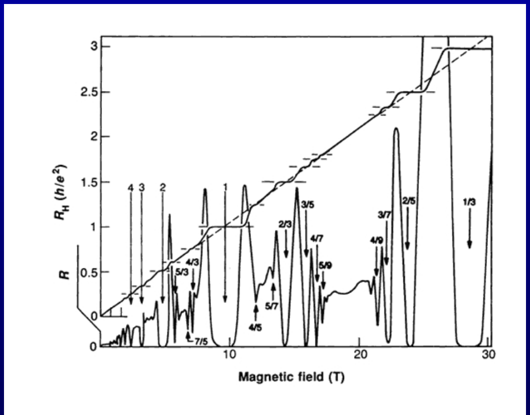
with m odd integer. These have exchange phase

$$\theta = \frac{\pi}{m}$$

and charge

$$q = \pm\frac{e}{m}.$$



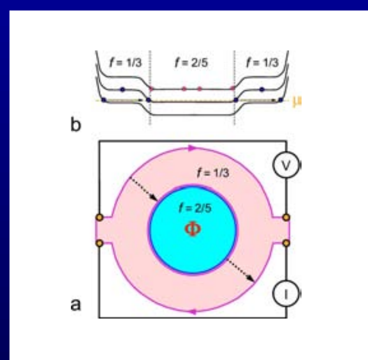


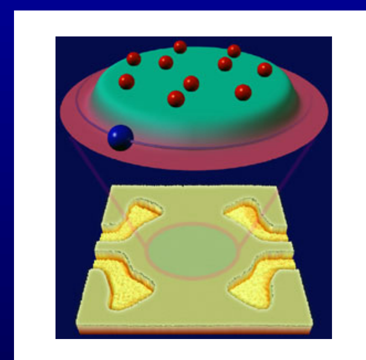
Robert Laughlin (left) receiving the Nobel Prize for physics from Swedish King Carl XVI Gustaf at the ceremonies in Stockholm, Sweden, on December 10, 1998. (AP Photo/Jonas Ekstromer)

- Fractional charges $\frac{e}{3}$ have been observed by resonant tunneling experiments in the $\nu = \frac{1}{3}$ FQH state [Goldman and Su, 1995] .

- Fractional charges $\frac{e}{3}$ have been observed by resonant tunneling experiments in the $\nu = \frac{1}{3}$ FQH state [Goldman and Su, 1995] .

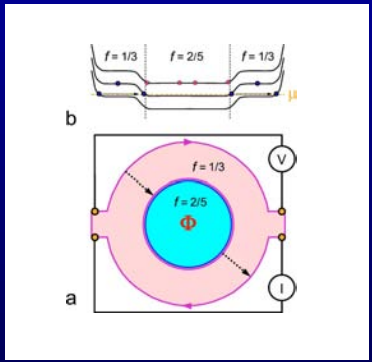
- Fractional statistics have been observed only recently in an Aharonov-Bohm type quasiparticle interferometer [Camino, Zhou and Goldman, 2005].

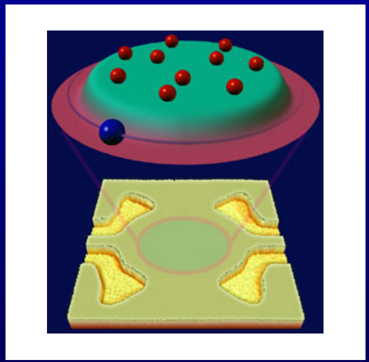




- Fractional charges $\frac{e}{3}$ have been observed by resonant tunneling experiments in the $\nu = \frac{1}{3}$ FQH state [Goldman and Su, 1995] .

- Fractional statistics have been observed only recently in an Aharonov-Bohm type quasiparticle interferometer [Camino, Zhou and Goldman, 2005].





- Non-abelian anyons occur in the so called Moore-Read “Pfaffian” state which may be realized in the $\nu = \frac{5}{2}$ FQH state. Experimentally as yet unconfirmed.

Anyons, fractionalization and strong correlations

Anyons and charge fractionalization typically occur in strongly correlated electron systems.

Anyons, fractionalization and strong correlations

Anyons and charge fractionalization typically occur in strongly correlated electron systems.

Strongly correlated = { many-body wavefunction Ψ *cannot* be written as a single Slater determinant of the constituent electron single-particle states.

Anyons, fractionalization and strong correlations

Anyons and charge fractionalization typically occur in strongly correlated electron systems.

Strongly correlated = { many-body wavefunction Ψ *cannot* be written as a single Slater determinant of the constituent electron single-particle states.

Question:

Are fractional statistics and fractional charges inextricably linked to strong correlations?

Anyons, fractionalization and strong correlations

Anyons and charge fractionalization typically occur in strongly correlated electron systems.

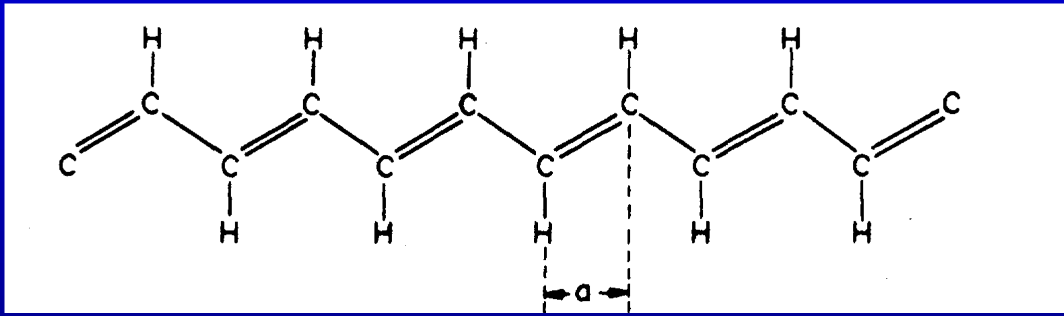
Strongly correlated $=$ $\left\{\right.$ many-body wavefunction Ψ *cannot* be written as a single Slater determinant of the constituent electron single-particle states.

Question:
Are fractional statistics and fractional charges inextricably linked to strong correlations?

Answer:
Not necessarily.

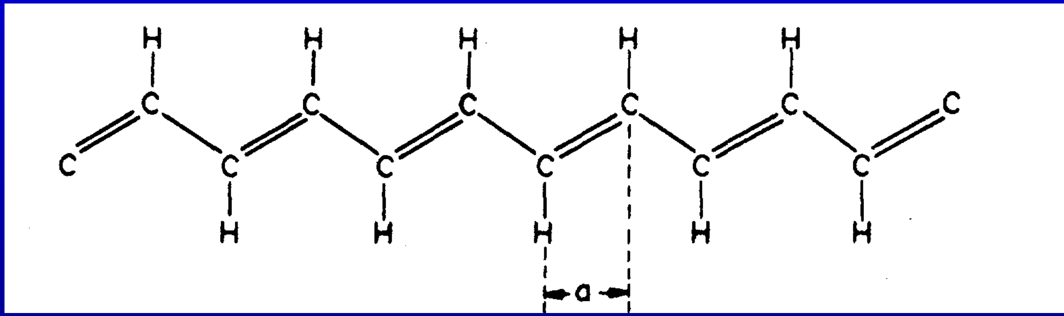
Fractional charges in polyacetylene

[Su, Schrieffer and Heeger, 1979]



Fractional charges in polyacetylene

[Su, Schrieffer and Heeger, 1979]



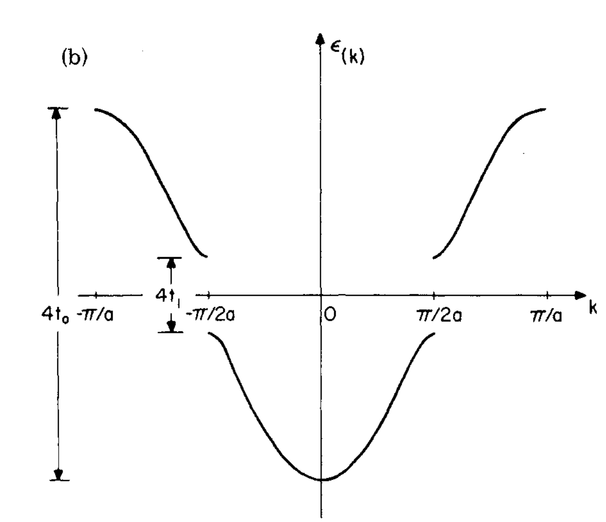
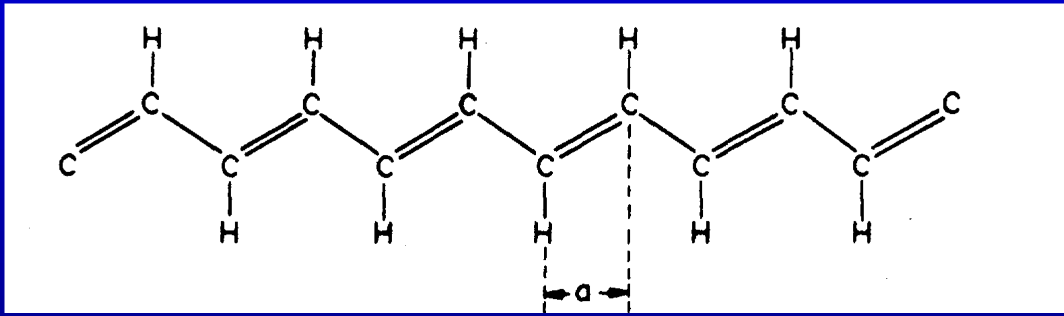


FIG. 1. (a) *Trans* configuration of $(CH)_x$. $a = 1.2$ Å. (b) π-band structure of perfectly dimerized $(CH)_x$.

band structure

Fractional charges in polyacetylene

[Su, Schrieffer and Heeger, 1979]



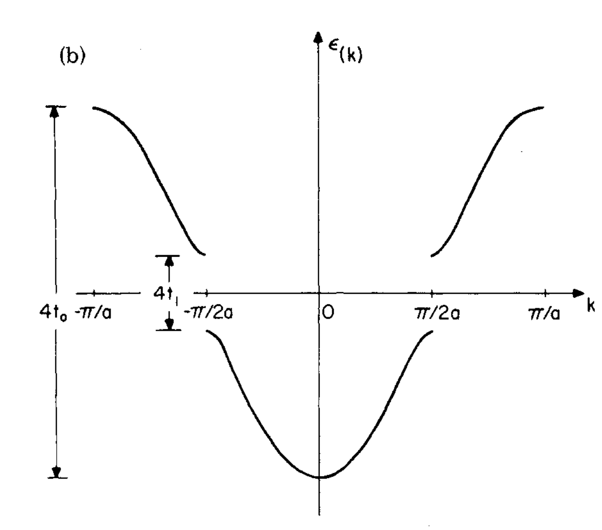


FIG. 1. (a) *Trans* configuration of $(CH)_x$. $a = 1.2$ Å. (b) π-band structure of perfectly dimerized $(CH)_x$.

band structure

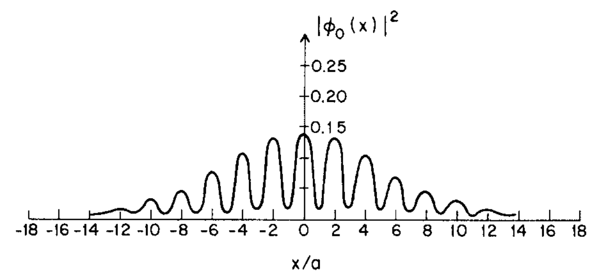


FIG. 3. Probability distribution of the localized electronic state at the center of the gap.

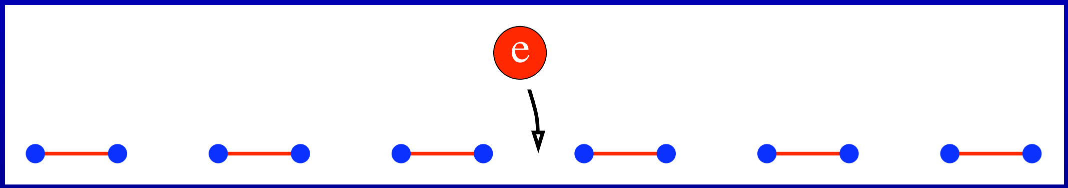
bound state at domain wall

For spinless electrons the charge associated with a domain wall between two degenerate ground states is

$$\delta Q = \pm \frac{e}{2}$$

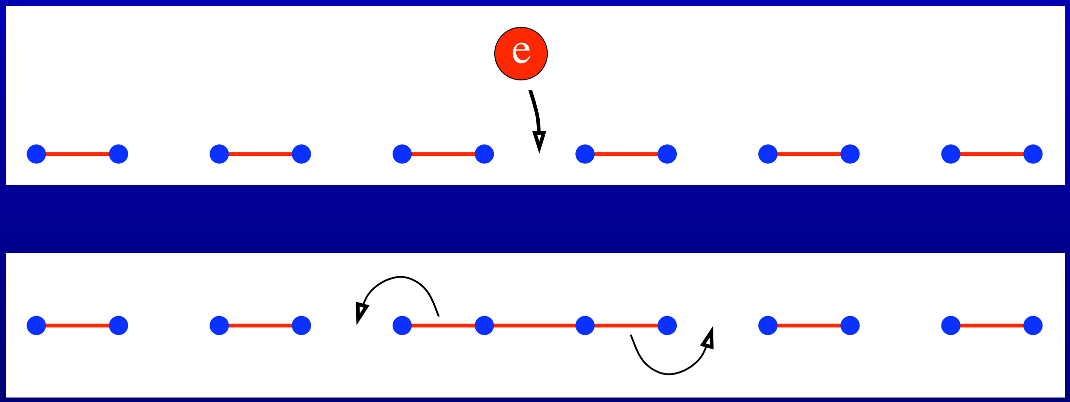
For spinless electrons the charge associated with a domain wall between two degenerate ground states is

$$\delta Q = \pm \frac{e}{2}$$



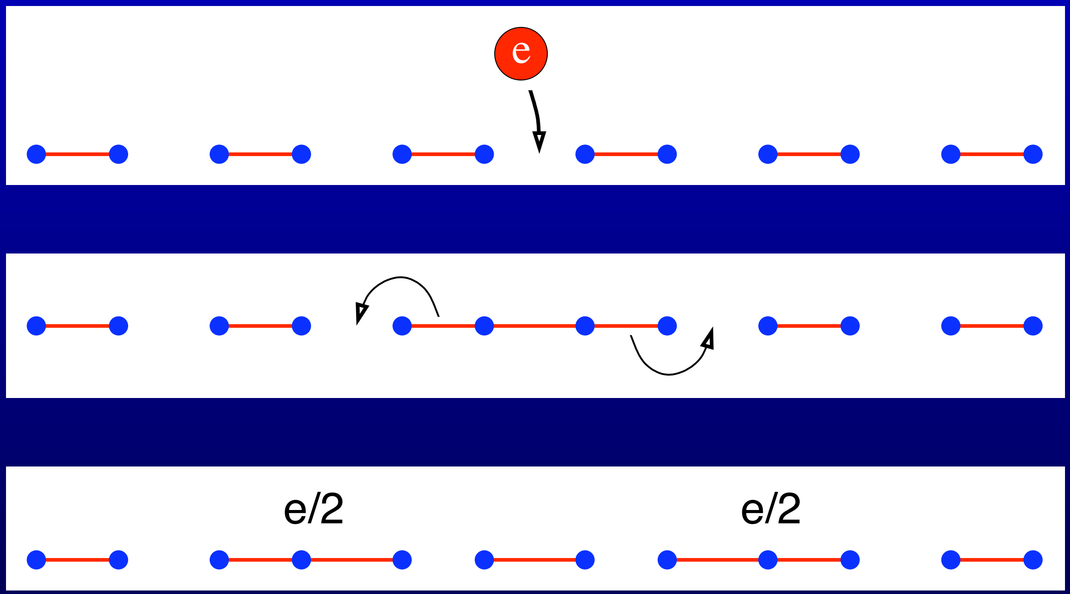
For spinless electrons the charge associated with a domain wall between two degenerate ground states is

$$\delta Q = \pm \frac{e}{2}$$



For spinless electrons the charge associated with a domain wall between two degenerate ground states is

$$\delta Q = \pm \frac{e}{2}$$



In 3 space dimensions Jackiw and Rebbi argued that $\frac{1}{2}$ of a fermion can be bound to the monopole in the Yang-Mills gauge field:

PHYSICAL REVIEW D VOLUME 13, NUMBER 12 15 JUNE 1976

Solitons with fermion number 1/2*

R. Jackiw and C. Rebbi
Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 23 December 1975)

We study the structure of soliton-monopole systems when Fermi fields are present. We show that the existence of a nondegenerate, isolated, zero-energy, *c*-number solution of the Dirac equation implies that the soliton is a degenerate doublet with Fermi number ± 1/2. We find such solutions in the theory of Yang-Mills monopoles and dyons.

In 3 space dimensions Jackiw and Rebbi argued that $\frac{1}{2}$ of a fermion can be bound to the monopole in the Yang-Mills gauge field:

PHYSICAL REVIEW D VOLUME 13, NUMBER 12 15 JUNE 1976

Solitons with fermion number 1/2*

R. Jackiw and C. Rebbi
Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 23 December 1975)

We study the structure of soliton-monopole systems when Fermi fields are present. We show that the existence of a nondegenerate, isolated, zero-energy, *c*-number solution of the Dirac equation implies that the soliton is a degenerate doublet with Fermi number ± 1/2. We find such solutions in the theory of Yang-Mills monopoles and dyons.

In the above cases charge fractionalization occurs as a result of fermions coupling to a soliton configuration of a background (scalar or gauge) field.

Interactions play no significant role, systems can be regarded as **weakly correlated**.

In 3 space dimensions Jackiw and Rebbi argued that $\frac{1}{2}$ of a fermion can be bound to the monopole in the Yang-Mills gauge field:

PHYSICAL REVIEW D VOLUME 13, NUMBER 12 15 JUNE 1976

Solitons with fermion number 1/2*

R. Jackiw and C. Rebbi

Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

(Received 23 December 1975)

We study the structure of soliton-monopole systems when Fermi fields are present. We show that the existence of a nondegenerate, isolated, zero-energy, *c*-number solution of the Dirac equation implies that the soliton is a degenerate doublet with Fermi number ± 1/2. We find such solutions in the theory of Yang-Mills monopoles and dyons.

In the above cases charge fractionalization occurs as a result of fermions coupling to a soliton configuration of a background (scalar or gauge) field.

Interactions play no significant role, systems can be regarded as **weakly correlated**.

In $d = 1, 3$ particle statistics is trivial (fermions or bosons):

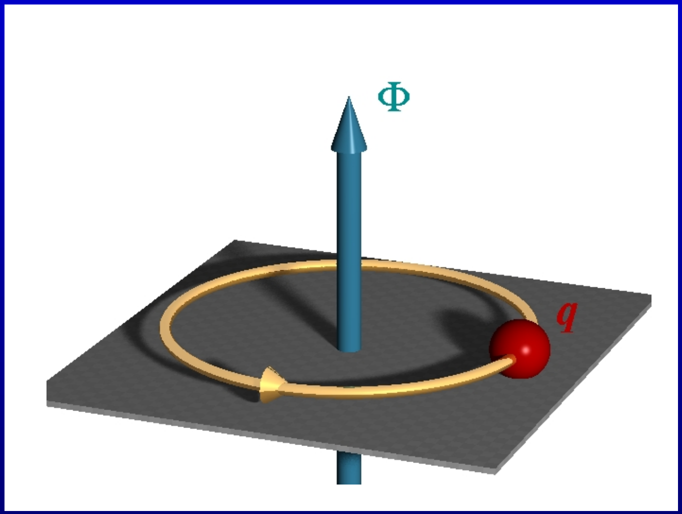
Need a two-dimensional example!

Anyon as a charge-flux composite

A simple toy model based on the Aharonov-Bohm effect.

Anyon as a charge-flux composite

A simple toy model based on the Aharonov-Bohm effect.

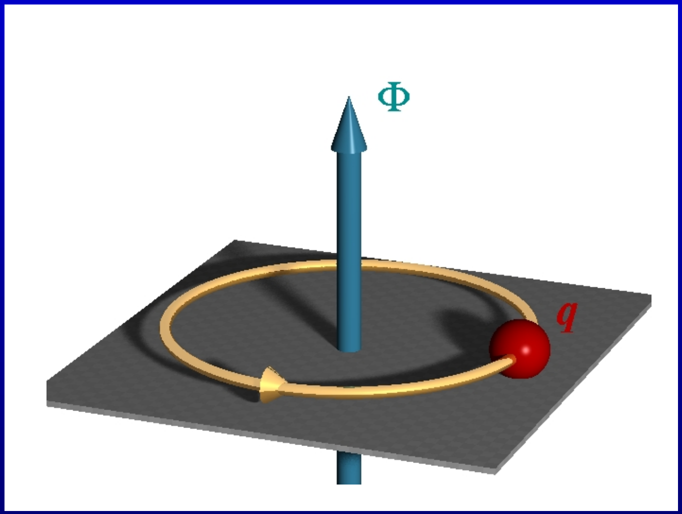


A charge q encircling flux Φ
acquires Aharonov-Bohm phase

$$2\pi\left(\frac{q}{e}\right)\left(\frac{\Phi}{\Phi_0}\right)$$

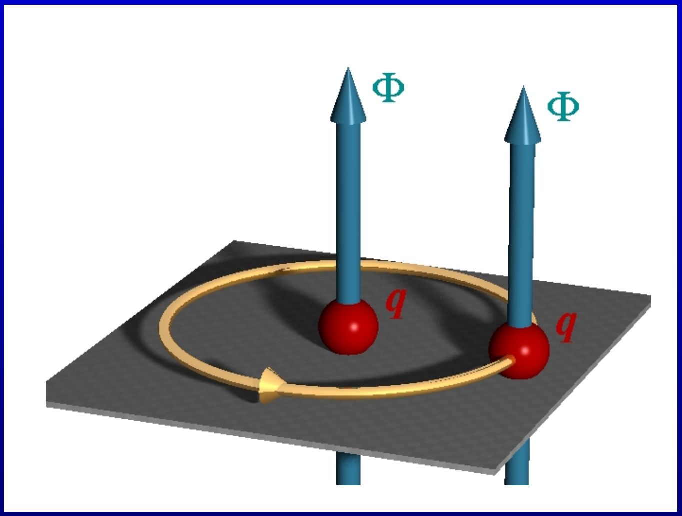
Anyon as a charge-flux composite

A simple toy model based on the Aharonov-Bohm effect.



A charge q encircling flux Φ acquires Aharonov-Bohm phase

$$2\pi \left(\frac{q}{e}\right)\left(\frac{\Phi}{\Phi_0}\right)$$



A charge-flux composite (q, Φ) encircling another (q, Φ) acquires

$$4\pi \left(\frac{q}{e}\right)\left(\frac{\Phi}{\Phi_0}\right)$$

Since the encircling operation can be thought of as two exchanges, the (q, Φ) composite particles have exchange phase

$$\theta = 2\pi \left(\frac{q}{e}\right) \left(\frac{\Phi}{\Phi_0}\right).$$

Since the encircling operation can be thought of as two exchanges, the (q, Φ) composite particles have exchange phase

$$\theta = 2\pi \left(\frac{q}{e}\right) \left(\frac{\Phi}{\Phi_0}\right).$$

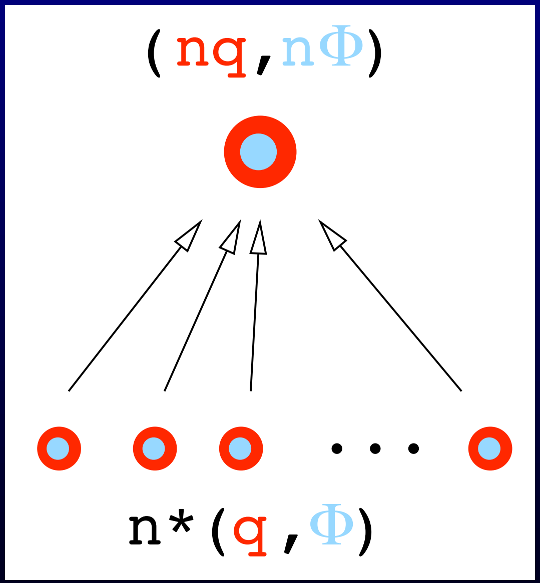
For fractional charge q and flux Φ these composite particles could be **anyons**.

Since the encircling operation can be thought of as two exchanges, the (q, Φ) composite particles have exchange phase

$$\theta = 2\pi \left(\frac{q}{e}\right) \left(\frac{\Phi}{\Phi_0}\right).$$

For fractional charge q and flux Φ these composite particles could be **anyons**.

Anyon fusion:

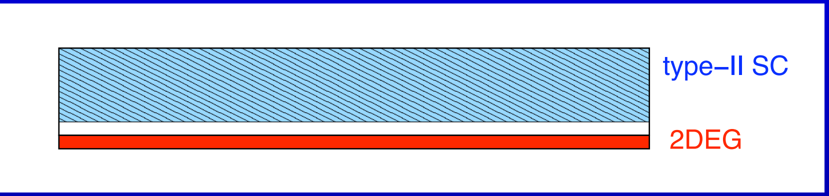


Fusing together n identical particles with statistical phase θ_0 results in a particle with statistical phase

$$\theta = n^2 \theta_0.$$

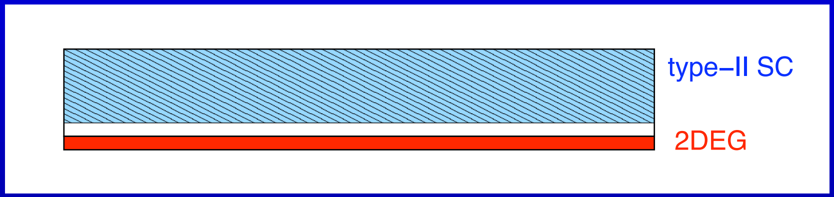
A new device:

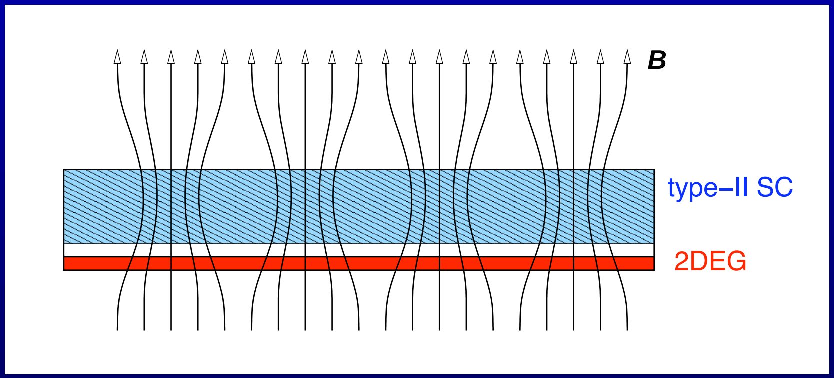
Superconductor-semiconductor heterostructure.



A new device:

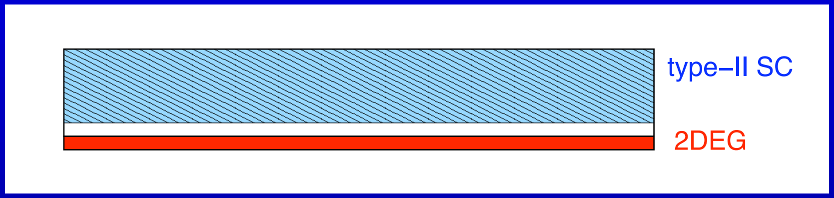
Superconductor-semiconductor heterostructure.

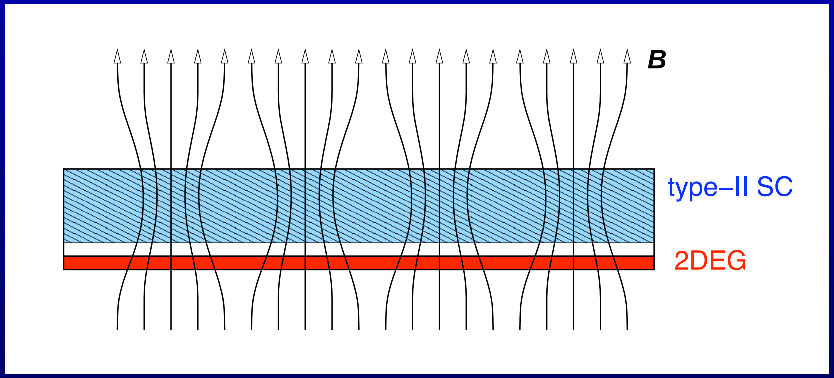




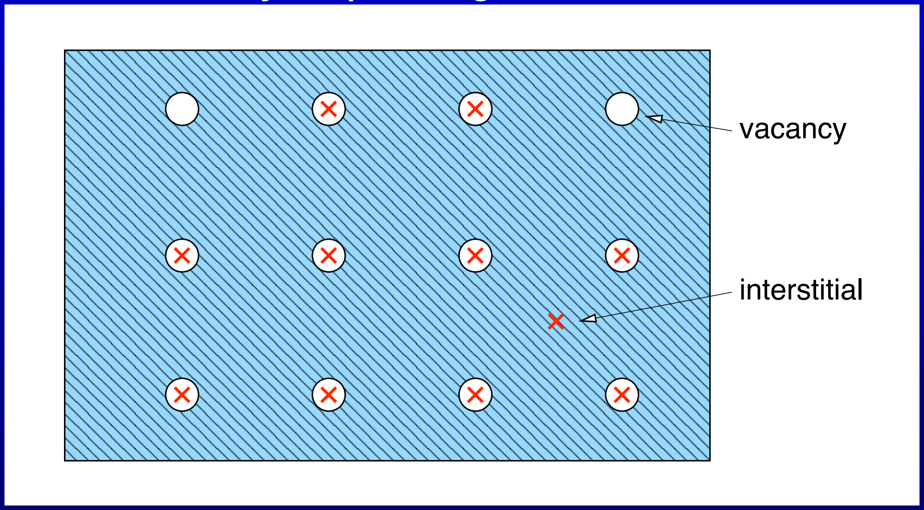
A new device:

Superconductor-semiconductor heterostructure.





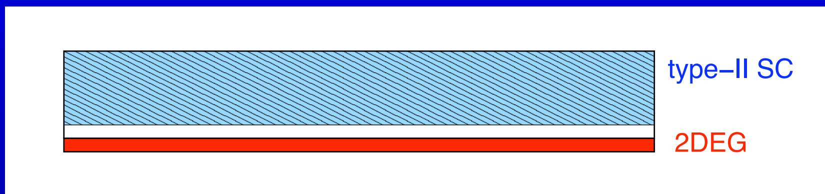
Periodic array of pinning sites for vortices

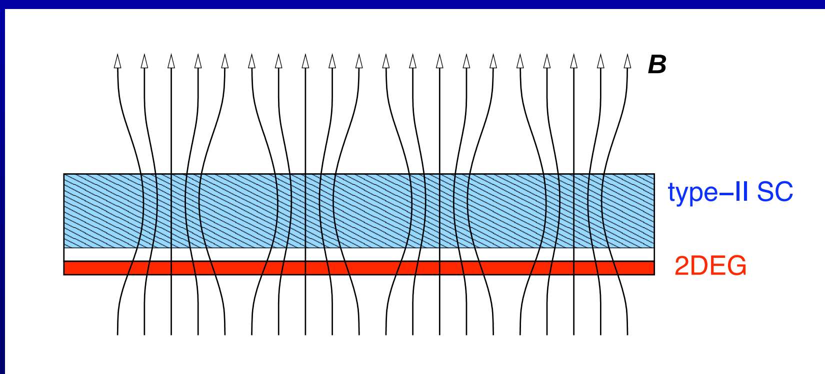


[top view]

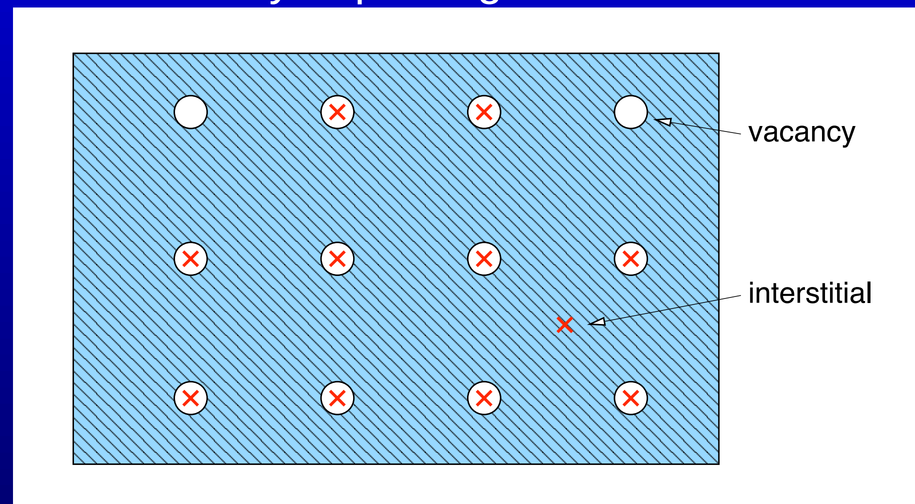
A new device:

Superconductor-semiconductor heterostructure.



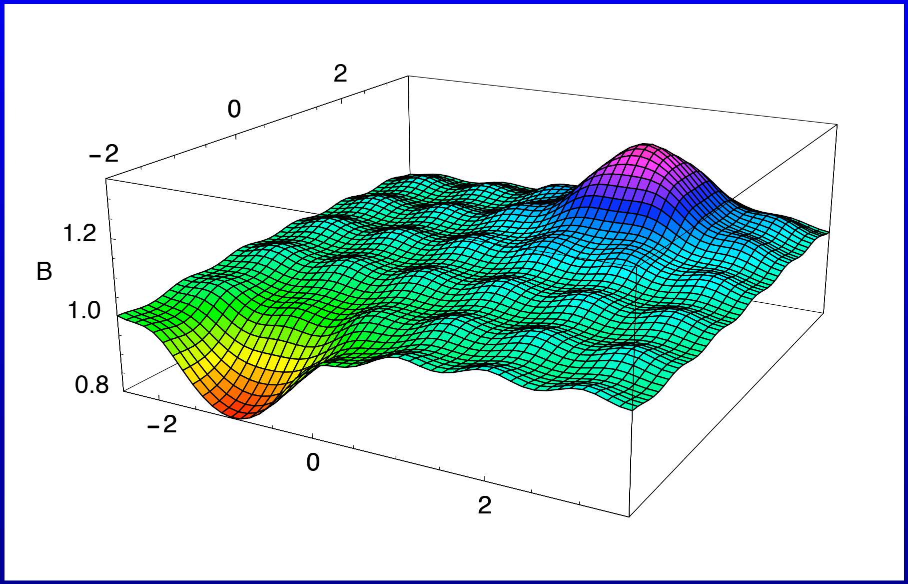


Periodic array of pinning sites for vortices



[top view]

- Superconductor quantizes magnetic flux in the units of $\Phi_0/2 = hc/2e$.
- Vortices preferentially occupy the pinning sites.
- Vacancies and interstitials in the vortex lattice produce localized flux surplus or deficit $\pm\Phi_0/2$.



Magnetic field profile with one vacancy and one interstitial, based on a simple London model with penetration depth $\lambda = a$ intervortex spacing.

Three principal claims

At 2DEG filling fraction $\nu = 1$ bound to each defect there is charge $+e/2$ for interstitial and $-e/2$ for vacancy.

Three principal claims

At 2DEG filling fraction $\nu = 1$ bound to each defect there is charge $+e/2$ for interstitial and $-e/2$ for vacancy.

These $(\pm e/2, \pm\Phi_0/2)$ charge-flux composites behave as anyons with exchange phase $\theta = \pi/4$.

Three principal claims

At 2DEG filling fraction $\nu = 1$ bound to each defect there is charge $+e/2$ for interstitial and $-e/2$ for vacancy.

These $(\pm e/2, \pm\Phi_0/2)$ charge-flux composites behave as anyons with exchange phase $\theta = \pi/4$.

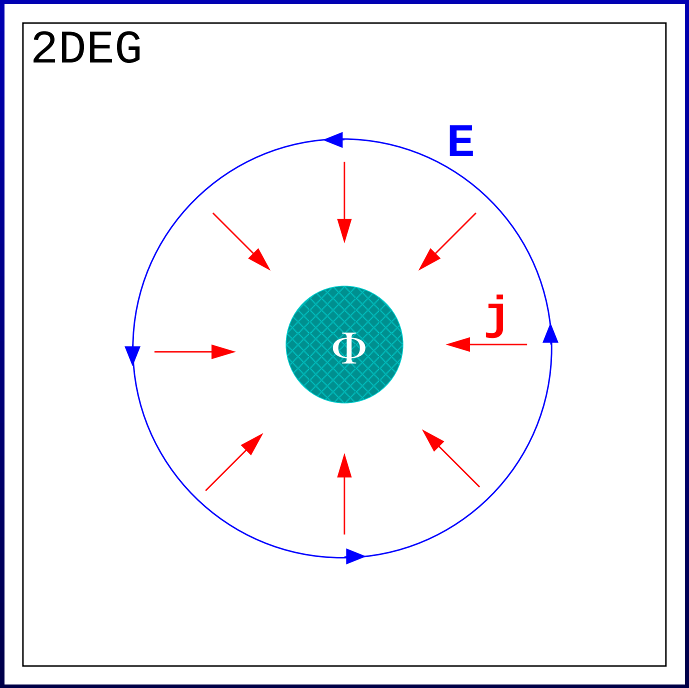
At 2DEG filling fraction $\nu = \frac{5}{2}$ bound to each defect should be a quasiparticle of the Moore-Read pfaffian state with non-Abelian statistics.

Fractional charges: Simple general argument

Consider the effect on 2DEG of *adiabatically* adding or removing vortex in a perfect flux lattice.

Fractional charges: Simple general argument

Consider the effect on 2DEG of *adiabatically* adding or removing vortex in a perfect flux lattice.



According to the Faraday's law

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t},$$

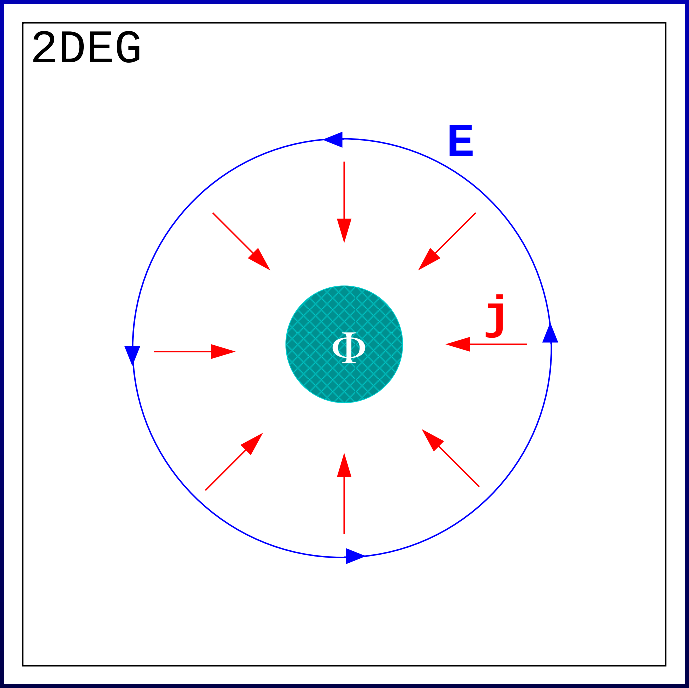
time-dependent flux produces electric field. The field, in turn, gives rise to Hall current

$$\mathbf{j} = \sigma_{xy}(\hat{z} \times \mathbf{E}),$$

with $\sigma_{xy} = e^2/h$ at $\nu = 1$.

Fractional charges: Simple general argument

Consider the effect on 2DEG of *adiabatically* adding or removing vortex in a perfect flux lattice.



According to the Faraday's law

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t},$$

time-dependent flux produces electric field. The field, in turn, gives rise to Hall current

$$\mathbf{j} = \sigma_{xy}(\hat{z} \times \mathbf{E}),$$

with $\sigma_{xy} = e^2/h$ at $\nu = 1$. Integrate:

$$\delta Q = \frac{e^2}{hc}\int_{t_1}^{t_2} dt \left(\frac{d\Phi}{dt}\right) = e\left(\frac{\delta\Phi}{\Phi_0}\right)$$

Statistical angle

Naive counting would assign the $(e/2, \Phi_0/2)$ object statistical phase $\theta_0 = 2\pi(\frac{1}{2} \times \frac{1}{2}) = \pi/2$.

Statistical angle

Naive counting would assign the $(e/2, \Phi_0/2)$ object statistical phase $\theta_0 = 2\pi(\frac{1}{2} \times \frac{1}{2}) = \pi/2$.

This is, however, incorrect because it ignores the intrinsic statistical phase of the electron.

Statistical angle

Naive counting would assign the $(e/2, \Phi_0/2)$ object statistical phase $\theta_0 = 2\pi(\frac{1}{2} \times \frac{1}{2}) = \pi/2$.

This is, however, incorrect because it ignores the intrinsic statistical phase of the electron.

Consider fusing two interstitials,

$$\underbrace{(e/2, \Phi_0/2)}_{\theta_0} + \underbrace{(e/2, \Phi_0/2)}_{\theta_0} \longrightarrow \underbrace{(e, \Phi_0)}_{\pi}.$$

Statistical angle

Naive counting would assign the $(e/2, \Phi_0/2)$ object statistical phase $\theta_0 = 2\pi(\frac{1}{2} \times \frac{1}{2}) = \pi/2$.

This is, however, incorrect because it ignores the intrinsic statistical phase of the electron.

Consider fusing two interstitials,

$$\underbrace{(e/2, \Phi_0/2)}_{\theta_0} + \underbrace{(e/2, \Phi_0/2)}_{\theta_0} \longrightarrow \underbrace{(e, \Phi_0)}_{\pi}.$$

According to the fusion rule $\theta = n^2\theta_0$ with $n = 2$ and $\theta = \pi$ we have

$$\theta_0 = \frac{\pi}{4}.$$

Non-Abelian physics at $\nu = \frac{5}{2}$

The Moore-Read “pfaffian” state can be thought of as superconducting condensate of composite fermion pairs with charge $2e$.

Non-Abelian physics at $\nu = \frac{5}{2}$

The Moore-Read “pfaffian” state can be thought of as superconducting condensate of composite fermion pairs with charge $2e$.

Thus, the natural size of flux quantum is $hc/2e$, just like in a superconductor.

Non-Abelian physics at $\nu = \frac{5}{2}$

The Moore-Read “pfaffian” state can be thought of as superconducting condensate of composite fermion pairs with charge $2e$.

Thus, the natural size of flux quantum is $hc/2e$, just like in a superconductor.

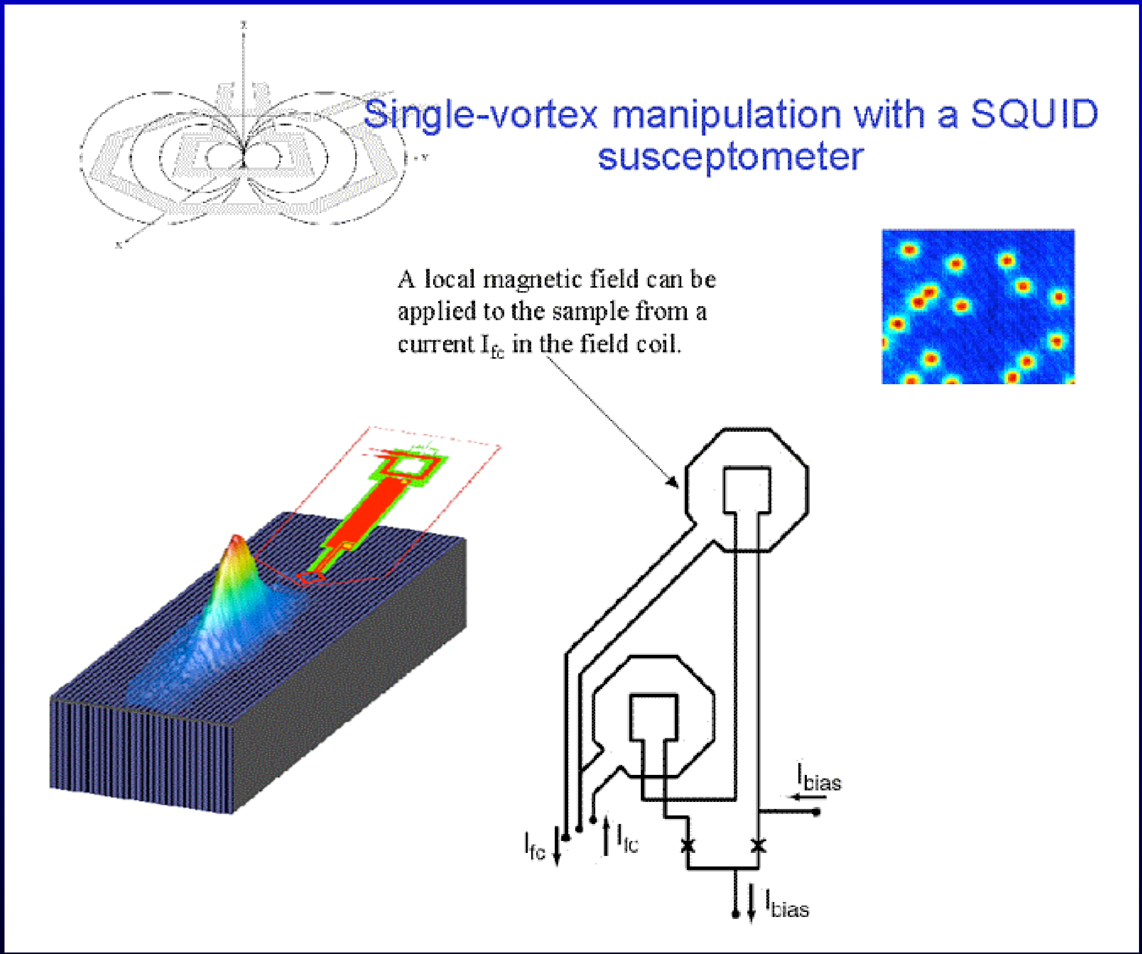
Vacancy or interstitial then should bind a quasiparticle of the pfaffian state. These are known to exhibit non-Abelian exchange statistics.

Practical issues

Vortices in superconductors can be created, imaged, and manipulated by a suite of techniques developed to study cuprates and other complex oxides.

Practical issues

Vortices in superconductors can be created, imaged, and manipulated by a suite of techniques developed to study cuprates and other complex oxides.



Moler Lab, Stanford

Simple continuum model

Consider 2-dimensional electron Hamiltonian,

$$\mathcal{H} = \frac{1}{2m_e}\left(\mathbf{p} - \frac{e}{c}\mathbf{A}\right)^2, \qquad \mathbf{A} = \left(\frac{1}{2}B_0 + \frac{\eta\Phi_0}{2\pi r^2}\right)(\mathbf{r} \times \hat{z}),$$

with m_e the electron mass, $\mathbf{p}$ the momentum operator in the x-y plane.

Simple continuum model

Consider 2-dimensional electron Hamiltonian,

$$\mathcal{H} = \frac{1}{2m_e}\left(\mathbf{p} - \frac{e}{c}\mathbf{A}\right)^2, \qquad \mathbf{A} = \left(\frac{1}{2}B_0 + \frac{\eta\Phi_0}{2\pi r^2}\right)(\mathbf{r} \times \hat{z}),$$

with m_e the electron mass, $\mathbf{p}$ the momentum operator in the x-y plane. The total field seen by an electron is

$$\mathbf{B}(\mathbf{r}) = \nabla \times \mathbf{A} = \hat{z}B_0 + \hat{z}\eta\Phi_0\delta(\mathbf{r}).$$

The δ-function serves as a crude representation of the half-flux removed by the vacancy.

Simple continuum model

Consider 2-dimensional electron Hamiltonian,

$$\mathcal{H} = \frac{1}{2m_e}\left(\mathbf{p} - \frac{e}{c}\mathbf{A}\right)^2, \qquad \mathbf{A} = \left(\frac{1}{2}B_0 + \frac{\eta\Phi_0}{2\pi r^2}\right)(\mathbf{r} \times \hat{z}),$$

with m_e the electron mass, $\mathbf{p}$ the momentum operator in the x-y plane. The total field seen by an electron is

$$\mathbf{B}(\mathbf{r}) = \nabla \times \mathbf{A} = \hat{z}B_0 + \hat{z}\eta\Phi_0\delta(\mathbf{r}).$$

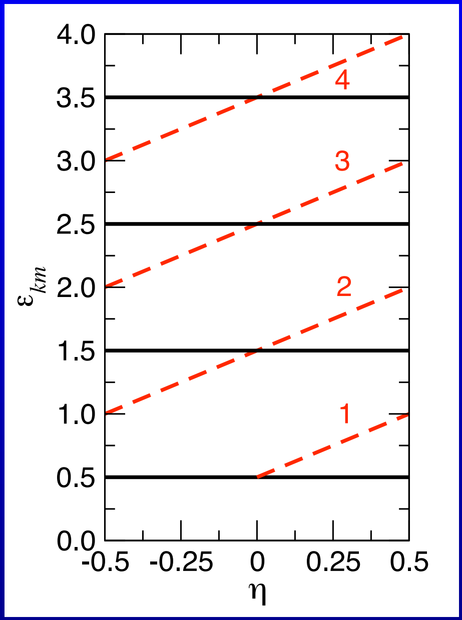
The δ-function serves as a crude representation of the half-flux removed by the vacancy.

This simple model contains the essence of the physics and is exactly soluble.

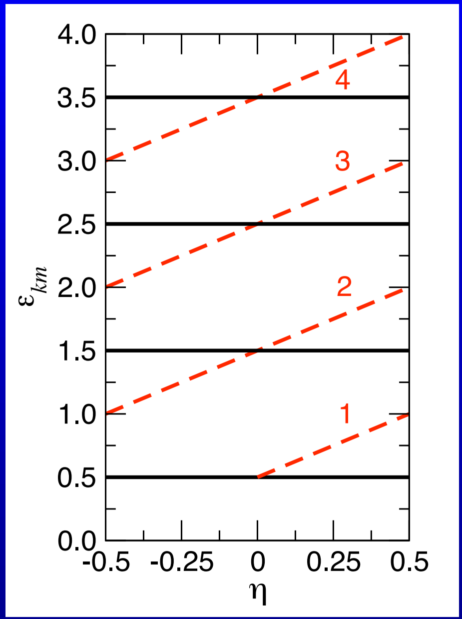
The single particle eigenstates $\psi_{km}(\mathbf{r})$ are labeled by the principal quantum number $k = 0, 1, 2, \ldots$ and an integer angular momentum m. The spectrum reads

$$\epsilon_{km} = \frac{1}{2}\hbar\omega_c \left[2k + 1 + |m + \eta| - (m + \eta)\right],$$

where $\omega_c = eB_0/m_e c$ is the cyclotron frequency.



The single particle eigenstates $\psi_{km}(\mathbf{r})$ are labeled by the principal quantum number $k = 0, 1, 2, \ldots$ and an integer angular momentum m. The spectrum reads



$$\epsilon_{km} = \frac{1}{2}\hbar\omega_c \left[2k + 1 + |m + \eta| - (m + \eta)\right],$$

where $\omega_c = eB_0/m_e c$ is the cyclotron frequency.

The eigenstates ψ_{0m} in the lowest Landau level are

$$\psi_{0m}(z) = A_m |z|^{-\eta} z^m e^{-|z|^2/4},$$

where $z = (x + iy)/\ell_B$, and $\ell_B = \sqrt{\hbar c/eB}$ is the magnetic length.

If we fill the lowest Landau level by spin-polarized electrons then the many-body wavefunction can be constructed as a Slater determinant of $\psi_{0m}(z_i)$, where z_i is a complex coordinate of the i-th electron,

$$\Psi(\{z_i\}) = \mathcal{N}_\eta \prod_i |z_i|^{-\eta} \prod_{i<j} (z_i - z_j) e^{-\sum_i |z_i|^2/4}.$$

If we fill the lowest Landau level by spin-polarized electrons then the many-body wavefunction can be constructed as a Slater determinant of $\psi_{0m}(z_i)$, where z_i is a complex coordinate of the i-th electron,

$$\Psi(\{z_i\}) = \mathcal{N}_\eta \prod_i |z_i|^{-\eta} \prod_{i<j}(z_i - z_j) e^{-\sum_i |z_i|^2/4}.$$

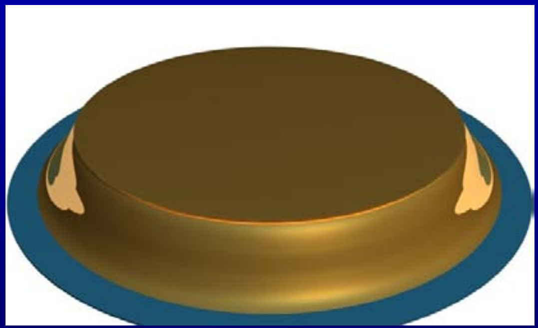
The charge density is given as $\rho(\mathbf{r}) = \langle\Psi_0|\hat{\rho}|\Psi_0\rangle$. For a droplet composed of N electrons occupying N lowest angular momentum states we obtain

$$\rho(\mathbf{r}) = e \sum_{m=0}^{N-1} |\psi_{0m}(\mathbf{r})|^2.$$

If we fill the lowest Landau level by spin-polarized electrons then the many-body wavefunction can be constructed as a Slater determinant of $\psi_{0m}(z_i)$, where z_i is a complex coordinate of the i-th electron,

$$\Psi(\{z_i\}) = \mathcal{N}_\eta \prod_i |z_i|^{-\eta} \prod_{i<j} (z_i - z_j) e^{-\sum_i |z_i|^2/4}.$$

The charge density is given as $\rho(\mathbf{r}) = \langle \Psi_0 | \hat{\rho} | \Psi_0 \rangle$. For a droplet composed of N electrons occupying N lowest angular momentum states we obtain



$N = 100$, $\eta = 0$

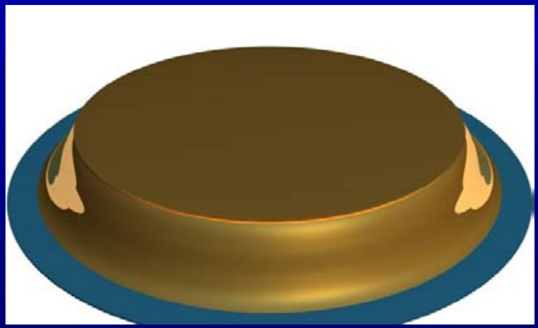
$$\rho(\mathbf{r}) = e \sum_{m=0}^{N-1} |\psi_{0m}(\mathbf{r})|^2.$$

If we fill the lowest Landau level by spin-polarized electrons then the many-body wavefunction can be constructed as a Slater determinant of $\psi_{0m}(z_i)$, where z_i is a complex coordinate of the i-th electron,

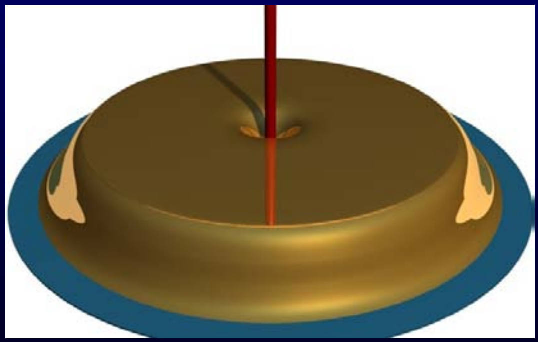
$$\Psi(\{z_i\}) = \mathcal{N}_\eta \prod_i |z_i|^{-\eta} \prod_{i<j}(z_i - z_j) e^{-\sum_i |z_i|^2/4}.$$

The charge density is given as $\rho(\mathbf{r}) = \langle\Psi_0|\hat{\rho}|\Psi_0\rangle$. For a droplet composed of N electrons occupying N lowest angular momentum states we obtain

$$\rho(\mathbf{r}) = e\sum_{m=0}^{N-1} |\psi_{0m}(\mathbf{r})|^2.$$

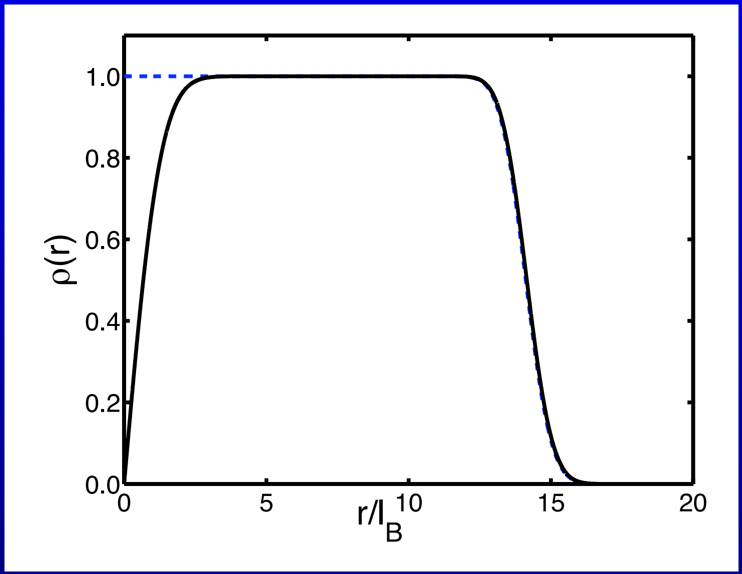


$N = 100$, $\eta = 0$

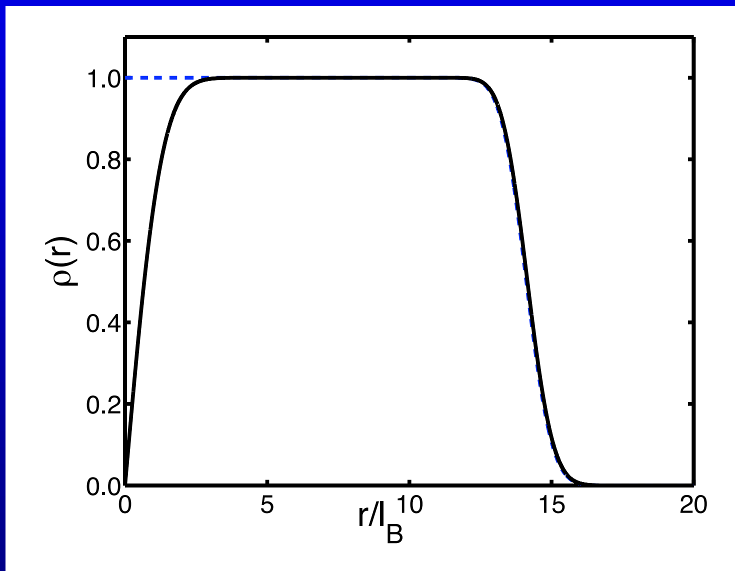


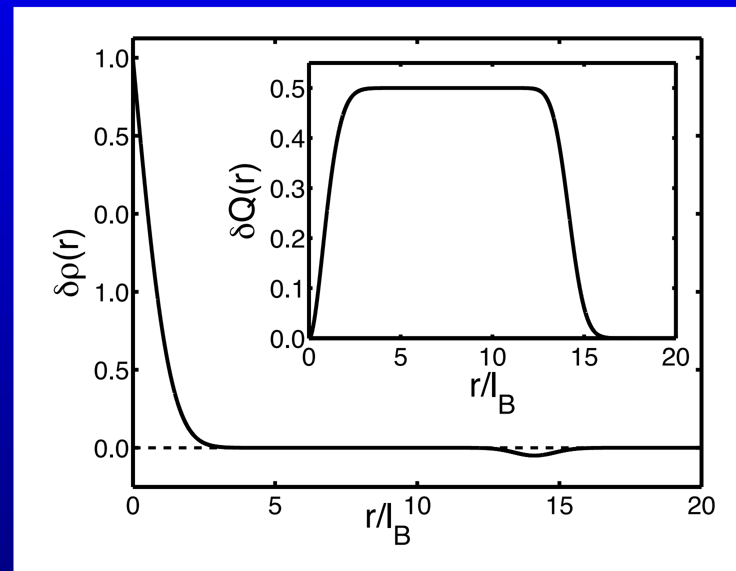
$N = 100$, $\eta = -1/2$

Look at the charge distribution more closely:



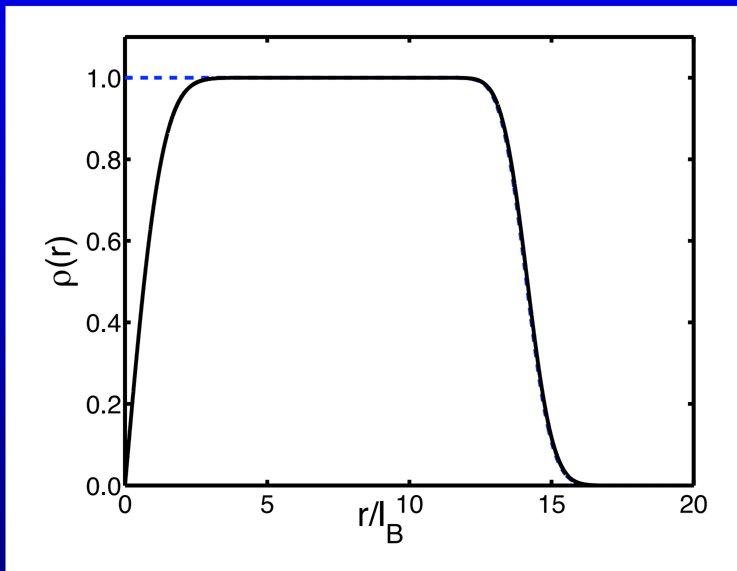
Look at the charge distribution more closely:

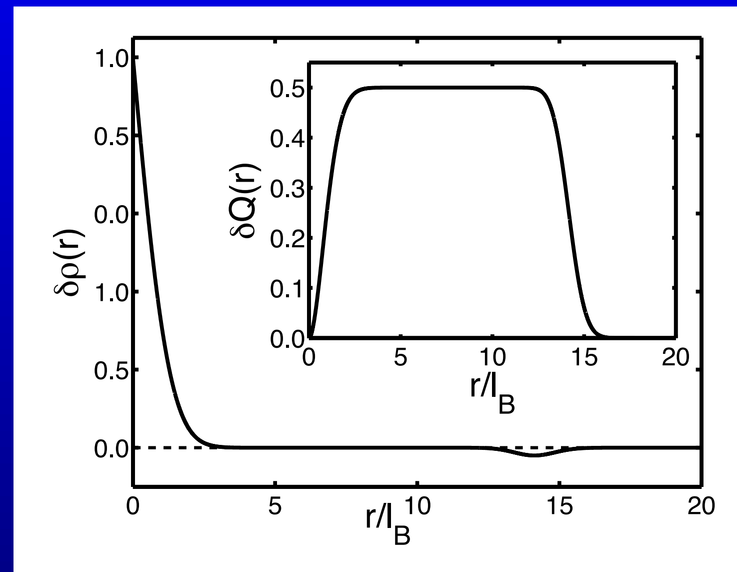




Inset shows the accumulated charge deficit $\delta Q(r) = 2\pi \int_0^r r' dr' \delta\rho(r')$ in units of e.

Look at the charge distribution more closely:





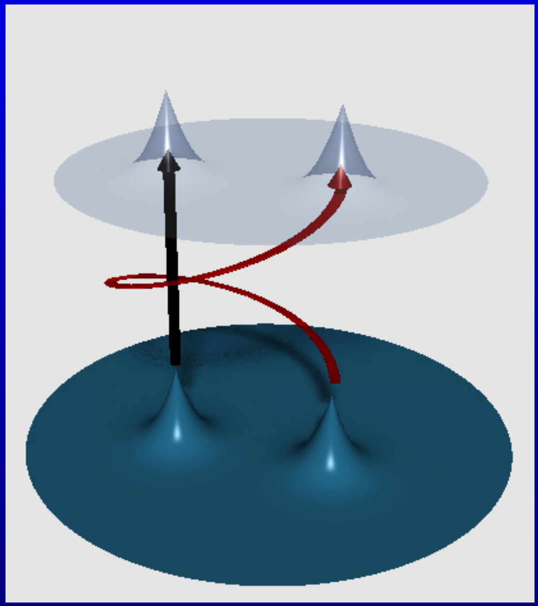
Inset shows the accumulated charge deficit $\delta Q(r) = 2\pi \int_0^r r' dr' \delta\rho(r')$ in units of e.

This confirms our first claim that vacancy binds fractional charge $-e/2$.

Particle statistics

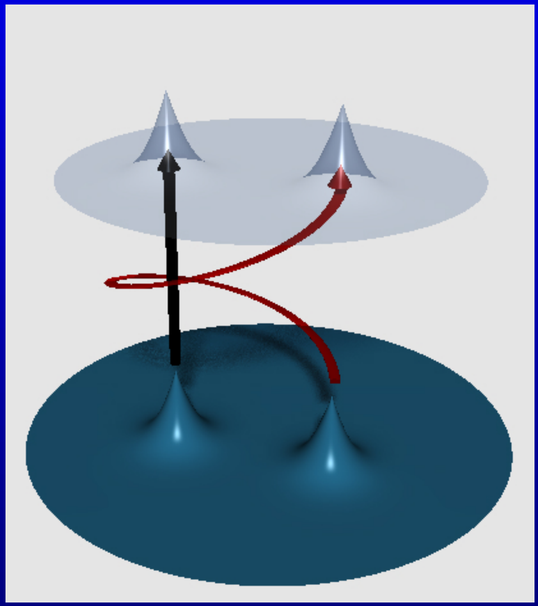
The statistical angle θ can be computed from the present model by evaluating the Berry phase when we adiabatically carry one vacancy around another [Arovas, Schrieffer and Wilczek, 1984]:

$$\gamma(\mathcal{C}) = i \oint_{\mathcal{C}} dw \langle \Psi_w | \frac{\partial}{\partial w} \Psi_w \rangle .$$



Particle statistics

The statistical angle θ can be computed from the present model by evaluating the Berry phase when we adiabatically carry one vacancy around another [Arovas, Schrieffer and Wilczek, 1984]:



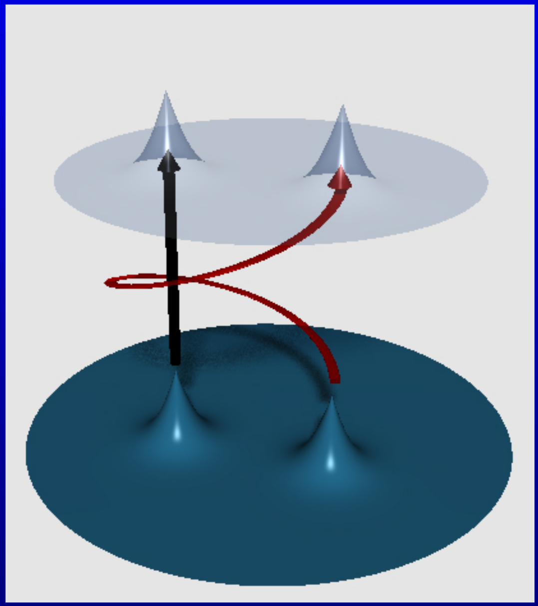
$$\gamma(\mathcal{C}) = i \oint_{\mathcal{C}} dw \langle \Psi_w | \frac{\partial}{\partial w} \Psi_w \rangle .$$

The first vacancy is at w while the second remains at the origin.

Particle statistics

The statistical angle θ can be computed from the present model by evaluating the Berry phase when we adiabatically carry one vacancy around another [Arovas, Schrieffer and Wilczek, 1984]:

$$\gamma(\mathcal{C}) = i \oint_{\mathcal{C}} dw \langle \Psi_w | \frac{\partial}{\partial w} \Psi_w \rangle.$$



The first vacancy is at w while the second remains at the origin.

For two vacancies, located at w_a and w_b the many-body wavefunction reads

$$\Psi_{w_a w_b} = \mathcal{N}_{w_a w_b} \prod_i (z_i - w_a)^{1/2} (z_i - w_b)^{1/2} \prod_{i<j} (z_i - z_j) e^{-\sum_i |z_i|^2/4}. \qquad (1)$$

In the above we have performed a gauge trasformation into the “string gauge” in which all the phase information is explicit in the wavefunction.

We now take $w_a = w$ and $w_b = 0$ and compute the above Berry phase along a closed contour $\mathcal{C}$ that encloses the origin.

This gives

$$\gamma(\mathcal{C}) = -\pi \left(\frac{\Phi}{\Phi_0} - \frac{1}{2} \right) .$$

We now take $w_a = w$ and $w_b = 0$ and compute the above Berry phase along a closed contour $\mathcal{C}$ that encloses the origin.

This gives

$$\gamma(\mathcal{C}) = -\pi \left(\frac{\Phi}{\Phi_0} - \frac{1}{2} \right) .$$

- The first term reflects the Aharonov-Bohm phase that a charge $-e/2$ particle acquires on encircling flux Φ due to background magnetic field B_0.

We now take $w_a = w$ and $w_b = 0$ and compute the above Berry phase along a closed contour $\mathcal{C}$ that encloses the origin.

This gives

$$\gamma(\mathcal{C}) = -\pi\left(\frac{\Phi}{\Phi_0} - \frac{1}{2}\right).$$

- The first term reflects the Aharonov-Bohm phase that a charge $-e/2$ particle acquires on encircling flux Φ due to background magnetic field B_0.
- The second represents twice the statistical phase of the vacancy, $\theta = \pi/4$, confirming our earlier heuristic result.

Lattice model

When the effective Zeeman coupling in 2DEG is sufficiently strong, then, in addition to the periodic vector potential, the electrons also feel periodic scalar potential. This effect becomes important in diluted magnetic semiconductors, such as $Ga_{1-x}Mn_xAs$, where the effective gyro-magnetic ration can be of order $\sim$100.

Lattice model

When the effective Zeeman coupling in 2DEG is sufficiently strong, then, in addition to the periodic vector potential, the electrons also feel periodic scalar potential. This effect becomes important in diluted magnetic semiconductors, such as $Ga_{1-x}Mn_xAs$, where the effective gyro-magnetic ration can be of order ∼100. As argued by Berciu *et al.* [Nature, 2005] this situation is described by an effective tight-binding model

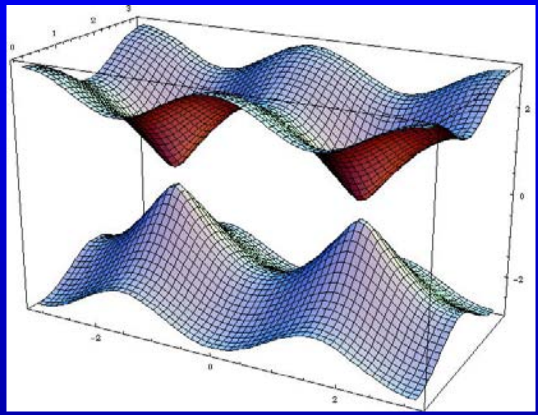
$$\mathcal{H} = \sum_{ij} (t_{ij} e^{i\theta_{ij}} c_j^\dagger c_i + \mathrm{h.c.}) + \sum_i \mu_i c_i^\dagger c_i,$$

with Peierls phase factors

$$\theta_{ij} = \frac{2\pi}{\Phi_0} \int_i^j \mathbf{A} \cdot d\mathbf{l}$$

corresponding to magnetic field of $\frac{1}{2}\Phi_0$ per plaquette.

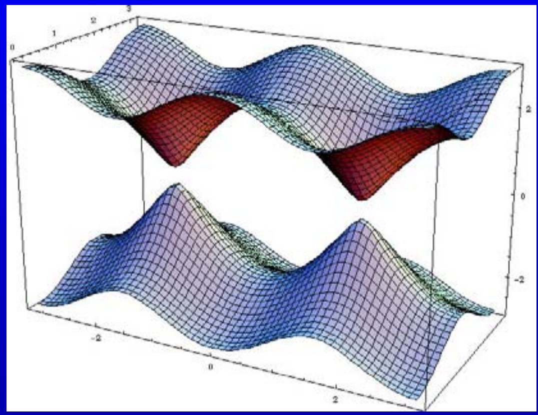
In *uniform field* and $\mu_i = \mu = \mathrm{const}$ $\mathcal{H}$ is easily diagonalized with the energy spectrum



$$E_{\mathbf{k}} = \mu \pm 2t\sqrt{\cos^2 k_x + \cos^2 k_y + 4\gamma^2 \sin^2 k_x \sin^2 k_y},$$

and $\gamma = t'/t$.

In *uniform field* and $\mu_i = \mu = \mathrm{const}$ $\mathcal{H}$ is easily diagonalized with the energy spectrum

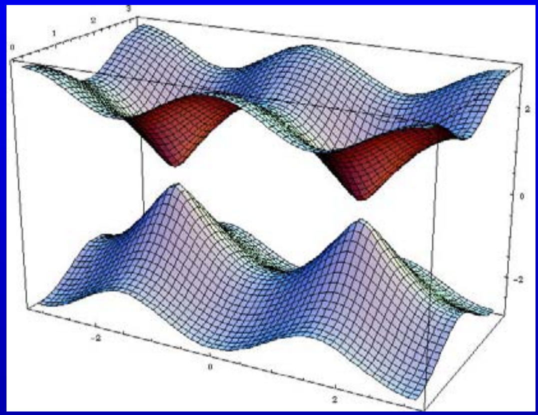


$$E_{\mathbf{k}} = \mu \pm 2t\sqrt{\cos^2 k_x + \cos^2 k_y + 4\gamma^2 \sin^2 k_x \sin^2 k_y},$$

and $\gamma = t'/t$.

Filling the lower band with spin polarized electrons corresponds to the previous case of the filled lowest Landau level.

In *uniform field* and $\mu_i = \mu = \text{const}$ $\mathcal{H}$ is easily diagonalized with the energy spectrum

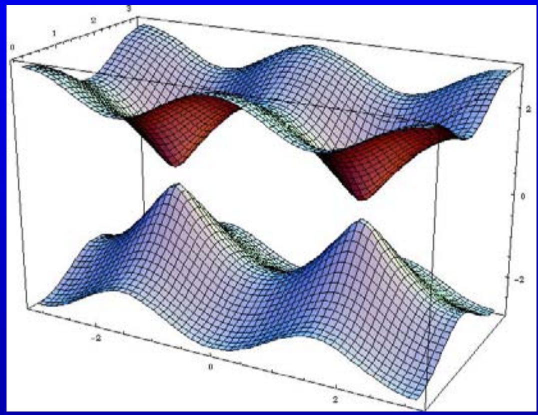


$$E_{\mathbf{k}} = \mu \pm 2t\sqrt{\cos^2 k_x + \cos^2 k_y + 4\gamma^2 \sin^2 k_x \sin^2 k_y},$$

and $\gamma = t'/t$.

Filling the lower band with spin polarized electrons corresponds to the previous case of the filled lowest Landau level.

We model vacancy/interstitial by removing/adding $\frac{1}{2}\Phi_0$ to a selected plaquette.



In *uniform field* and $\mu_i = \mu = \text{const}$ $\mathcal{H}$ is easily diagonalized with the energy spectrum

$$E_{\mathbf{k}} = \mu \pm 2t\sqrt{\cos^2 k_x + \cos^2 k_y + 4\gamma^2 \sin^2 k_x \sin^2 k_y},$$

and $\gamma = t'/t$.

Filling the lower band with spin polarized electrons corresponds to the previous case of the filled lowest Landau level.

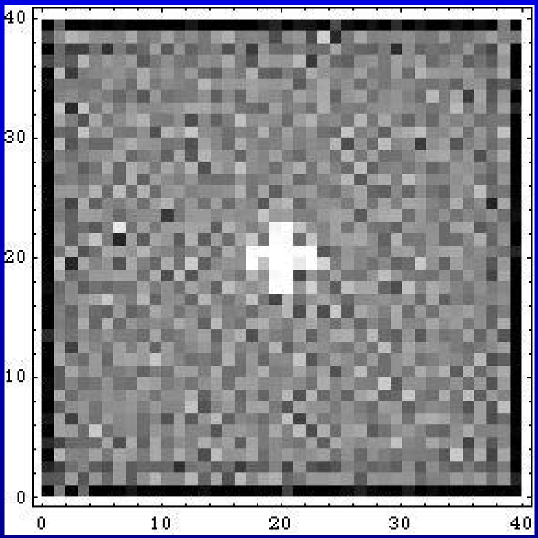
We model vacancy/interstitial by removing/adding $\frac{1}{2}\Phi_0$ to a selected plaquette.

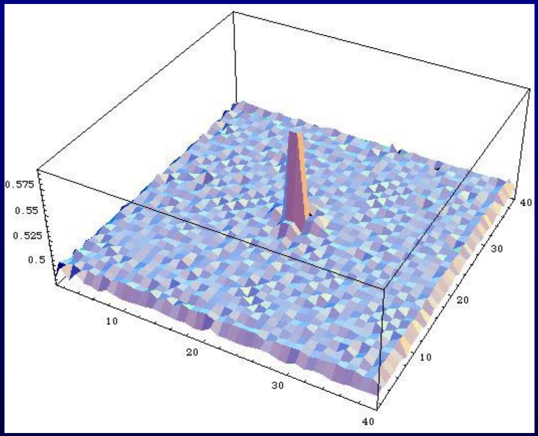
We have diagonalized the above Hamiltonian numericaly for system sizes up to 50×50 and various configurations of fluxes and μ_i's.

In all cases we find that vacancy/interstitial binds charge $\pm e/2$.

Sample result for a single interstitial and random μ_i with $\Delta\mu_i = 0.05t'$.

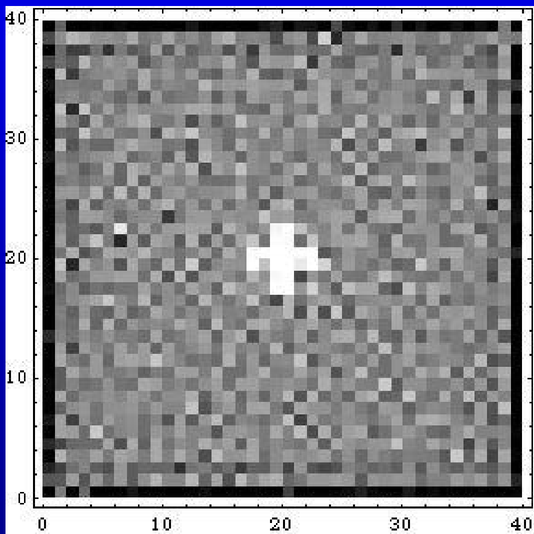
Sample result for a single interstitial and random μ_i with $\Delta\mu_i = 0.05t'$.

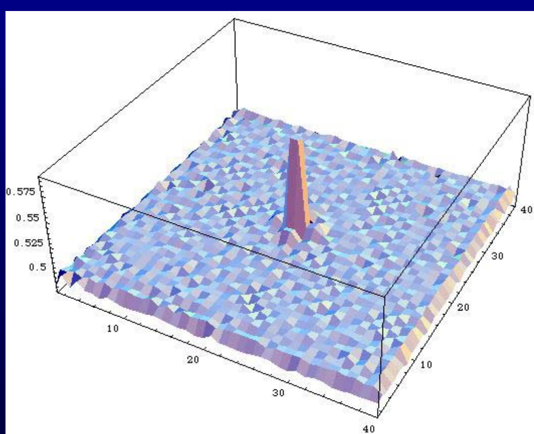




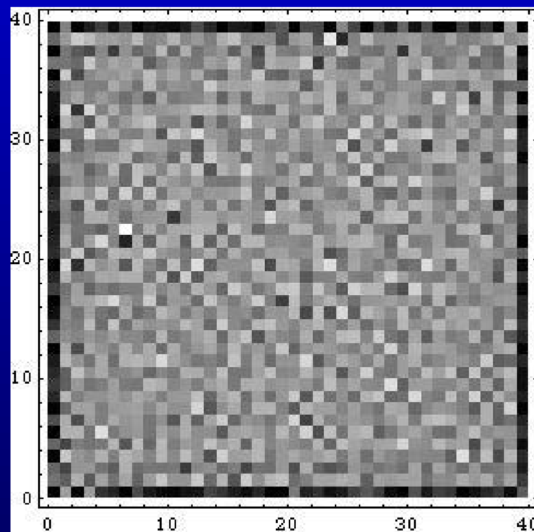
$\rho(\mathbf{r})$ with interstitial

Sample result for a single interstitial and random μ_i with $\Delta\mu_i = 0.05t'$.



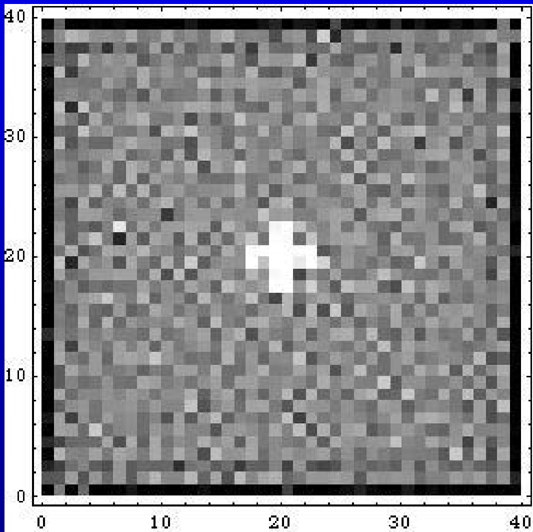


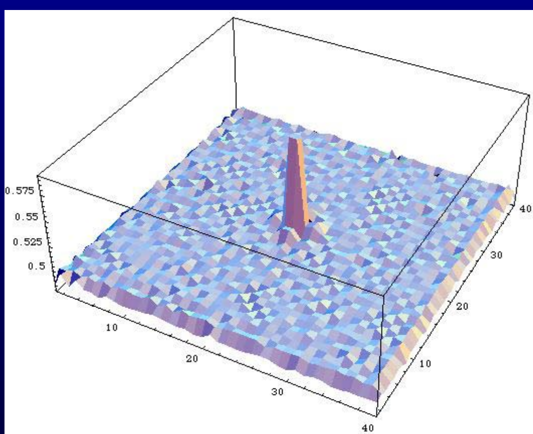
$\rho(\mathbf{r})$ with interstitial



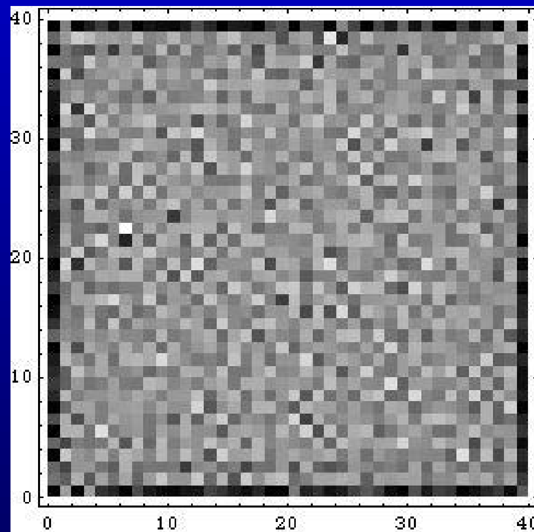
$\rho(\mathbf{r})$ without interstitial

Sample result for a single interstitial and random μ_i with $\Delta\mu_i = 0.05t'$.

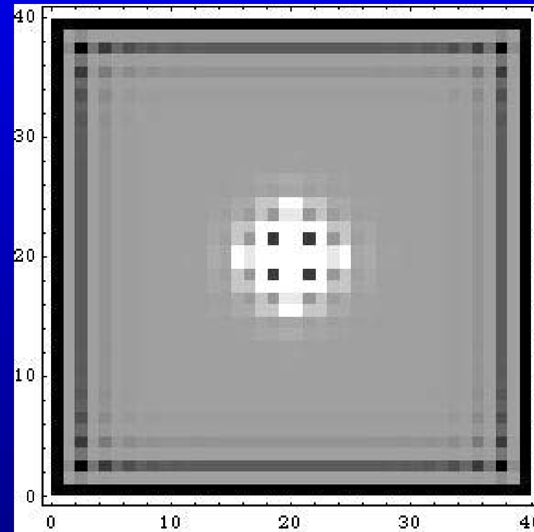


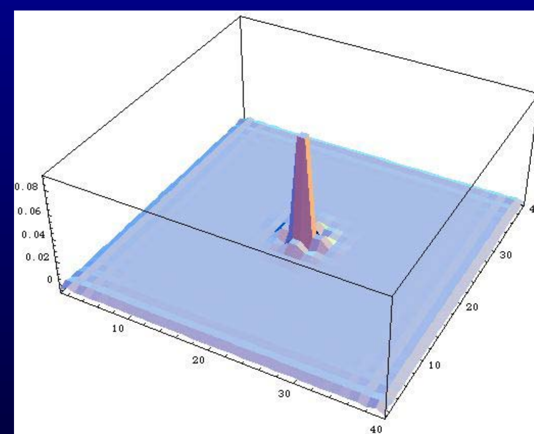


$\rho(\mathbf{r})$ with interstitial



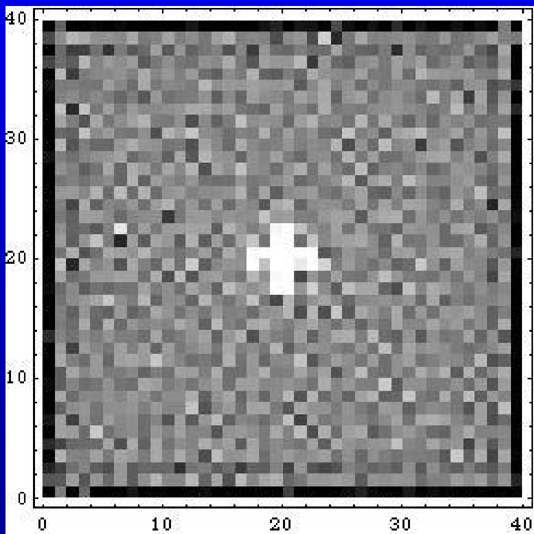
$\rho(\mathbf{r})$ without interstitial

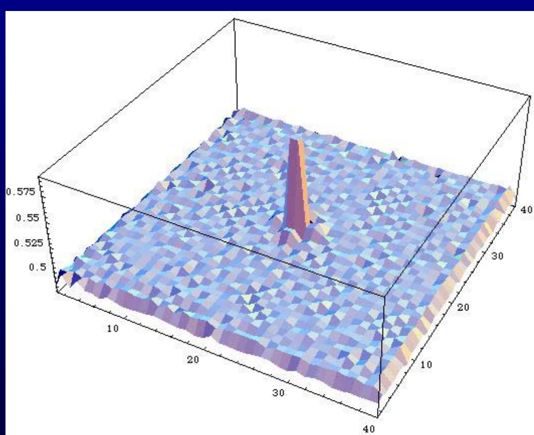




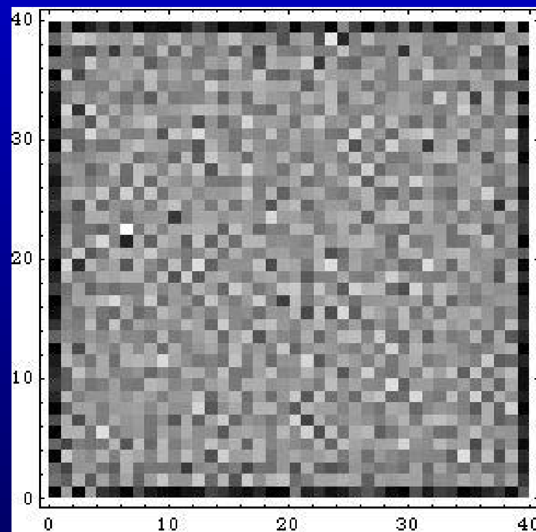
induced charge $\delta\rho(\mathbf{r})$

Sample result for a single interstitial and random μ_i with $\Delta\mu_i = 0.05t'$.

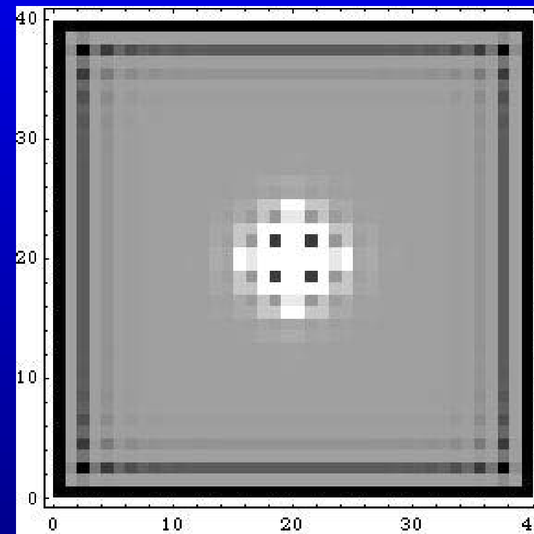


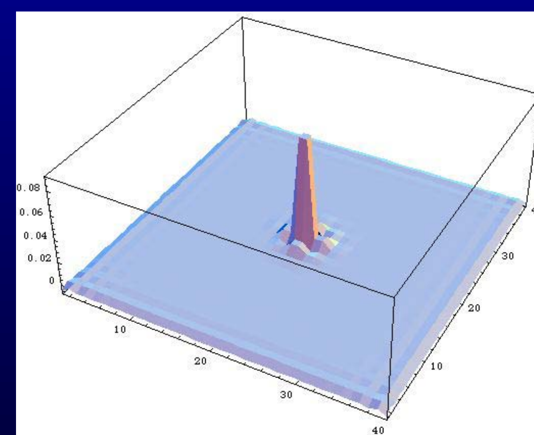


$\rho(\mathbf{r})$ with interstitial



$\rho(\mathbf{r})$ without interstitial





induced charge $\delta\rho(\mathbf{r})$

The induced charge integrates to $e/2$ to within machine accuracy.

Conclusions

- Unusual phenomena, such as charge and statistics fractionalization, can occur when two weakly interacting (and well understood) systems are brought to close proximity.

Conclusions

- Unusual phenomena, such as charge and statistics fractionalization, can occur when two weakly interacting (and well understood) systems are brought to close proximity.
- We showed, by general argument and detailed computation, that vacancy in a vortex lattice binds $-e/2$ charge in the adjacent 2DEG at unit filling.

Conclusions

- Unusual phenomena, such as charge and statistics fractionalization, can occur when two weakly interacting (and well understood) systems are brought to close proximity.
- We showed, by general argument and detailed computation, that vacancy in a vortex lattice binds $-e/2$ charge in the adjacent 2DEG at unit filling.
- Such a composite object behaves as abelian anyon with $\theta = \pi/4$ and can be manipulated by existing experimental techniques.

Conclusions

- Unusual phenomena, such as charge and statistics fractionalization, can occur when two weakly interacting (and well understood) systems are brought to close proximity.
- We showed, by general argument and detailed computation, that vacancy in a vortex lattice binds $-e/2$ charge in the adjacent 2DEG at unit filling.
- Such a composite object behaves as abelian anyon with $\theta = \pi/4$ and can be manipulated by existing experimental techniques.
- These superconductor-semiconductor heterostructures can be (and have been) built in a lab and could allow for controlled creation and manipulation of the fractional particles.

Conclusions

- Unusual phenomena, such as charge and statistics fractionalization, can occur when two weakly interacting (and well understood) systems are brought to close proximity.

- We showed, by general argument and detailed computation, that vacancy in a vortex lattice binds $-e/2$ charge in the adjacent 2DEG at unit filling.

- Such a composite object behaves as abelian anyon with $\theta = \pi/4$ and can be manipulated by existing experimental techniques.

- These superconductor-semiconductor heterostructures can be (and have been) built in a lab and could allow for controlled creation and manipulation of the fractional particles.

- At $\nu = \frac{5}{2}$ vacancy should bind a Moore-Read “non-Abelion” which may be used to implement topologically protected fault tolerant quantum gates.