STATISTICAL MECHANICS OF UTILITY AND EQUILIBRIUM: ANALOGIES BETWEEN ECONOMICS AND STATISTICAL PHYSICS

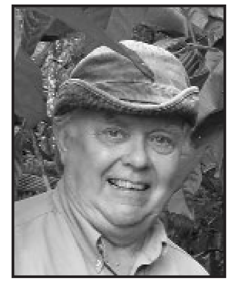
BY BIRGER BERGERSEN

Ideas from very disparate fields of knowledge often cross fertilize, and the relationship between physics and economics is no exception. Unfortunately, on some occasions the analogy doesn't quite work out. Historically, neoclassical economics has borrowed many ideas from equilibrium thermodynamics, and by concentrating on the key concepts of value, equilibrium, irreversibility and dimensionality (the possibility of scarce resource substitution) I will argue that most of the resulting analogies are misleading. Better analogies can be found in modern non-equilibrium statistical mechanics.

In the next section, I caution against the reliance in conventional economic theory on aggregate quantities such as GDP and indexes, and contrast this approach with the multidimensional approach of ecological and feminist economists, and other areas of science where it is generally accepted that quantities of different physical dimension should not be added [1,2].

The section on 'Utility and Money' argues that conventional economic theory fails to appreciate the subtlety of the first law of thermodynamics in attempts to find an analogy with thermodynamics and equilibrium theories of economics. Consequences for theories of utility and money are explored.

The last section, 'Equilibrium', is concerned with the concept of equilibrium in physics and economics. I argue that if the economics of production is incorporated into the theory, equilibrium theory is not applicable. The consequences are many and serious.



SUMMARY

Much has been made of the extent to which mainstream economics is founded on analogies with equilibrium concepts in physics. I argue that more fruitful analogies can be found with current ideas in non-equilibrium statistical mechanics. I begin by discussing the extent to which aggregate quantities are meaningful and the assumption of neoclassical economics that assets can be *substituted* and related to a single one dimensional measure. Next I move on to the concept of *utility* and *money* in the context of state *vs*. process variables of thermodynamics. The rest of the paper is concerned with exploring differences between *equilibrium* and *non-equilibrium* statistical mechanics. The absence of extremum properties in the latter case gives rise to serious stability concerns, unlike the situation in equilibrium theory where extremum properties guarantee stability. I also explore the issue of "illusion of control" which arises in multiagent systems where agents have minds of their own, but has no analogy in simulations of physical systems of atoms and molecules. Finally I discuss the concepts of *irreversibility* which can be ignored in equilibrium and *dissipative structures* which appear far from equilibrium.

AGGREGATES, DIMENSIONALITY AND ABILITY TO SUBSTITUTE

Economists like to describe an economy in terms of aggregate quantities such as gross domestic product (GDP). Similarly a thermodynamic description of matter makes do with just a few variables such as temperature, pressure and mass. Environmental and feminists economists object to this analogy. The economist Marilyn Waring [1] argues that the concept of GDP is seriously flawed because of the failure to include important economic contributions from subsistence farming, unpaid house work, care giving and volunteering. Nor are depreciations of natural resources and environmental values included in a meaningful way, although the associated values are vital for the maintenance of the fabric of society. In the introduction to the second edition of her book [1] Waring reports on attempts to improve the situation by assigning a monetary value to these assets. But, as she points out, it makes little sense to add the money made by the arms manufacturer and drug pusher to the value of the work of the daughter who takes care of her Alzheimer parent. Similarly, a species which becomes extinct remains extinct, while business losses can be replaced. Numerous attempts have been tried to produce indexes that correct for the bias inherent in the GDP. An example is the human development index promoted by the United Nations. Unfortunately, it seems impossible to

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find an objective single index. Indeed, competing indexes that claim to take into account sustainability issues produce divergent results [2]. The problem is that converting assets to a single variable such as money or an index implicitly assumes that assets can be substituted. Ecology teaches us that this is not the case, just as nutritionists stress the importance of consuming food from the different food groups, and that we need to worry about trace elements and vitamins. Recently ecological economists have been thinking in terms of multidimensional capital. Common divisions are natural, social, human, physical and financial capital.

Some economists think in terms of multidimensional capital and argue against the reliance on GDP and other indices. Quantities of different dimension should not be added.

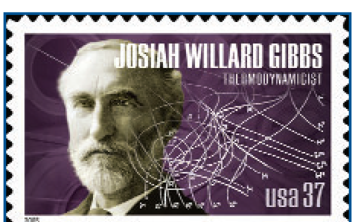
In their book "*Priceless*" Frank Ackerman and Lisa Heinzerling [3] document efforts to put a price on the environmental efforts to save threatened or endangered species through surveys of what it would be worth to each household (an approach referred to as contingent valuation in the economics literature). Apparently it is worth $216 to the average American household to save bald eagles and $67 to save gray wolves. Adding it up this would mean that the eagles are worth $23 billion, but the wolves only $7 billion. This addition makes no sense, apart from the fact that such surveys are easy to manipulate. There are societal values associated with belonging to a civilized society, which go beyond the sum of individual values, and which are priceless, as suggested by the book title.

UTILITY AND MONEY

The concept of utility is important in economics as a measure of the desirability of goods and services. Maximization of utility is commonly seen as analogous to minimizing a free energy in thermodynamics. A fundamental objection to this approach was made by J. Willard Gibbs, the founder of modern thermodynamics, over a century ago. Mirowski [4], in his book "*More heat than light*", describes the relationship between Gibbs and Irving Fisher, who many people regard as one of the founders of 20th century economics. Gibbs was co-supervisor of Fisher's Ph.D. thesis and chaired his final thesis defense. Fisher based his economic theory on a close analogy with Gibbs' theory of equilibrium thermodynamics; *e.g.*, the equilibrium price of a commodity is given by its marginal utility (derivative of the utility with respect to quantity). The thermodynamic analogy is that in equilibrium, at a given temperature, the pressure of a gas is the derivative of the Helmholtz free energy with respect to the volume. Gibbs objected that this identification reflected a poor understanding of the first law of thermodynamics

$$\mathrm{d}E = \text{đ}Q - \text{đ}W \qquad (1)$$

where E is the *energy*, Q is the *heat* given to the system and W the *work* done by the system. In elementary texts the first law is simply presented as the law of conservation of energy, but it is more subtle than that, as reflected in the different notation for the differential on the right and left hand side of (1). The energy is a *state variable, i.e.* it is determined by the state of the system and has an *exact differential*. Q and W are *process variables* and have *inexact differentials*. Knowledge of the state of the system does not allow us to determine its "heat" or "work". Gibbs found it unreasonable to expect the utility to have exact differentials. A similar point was made already in 1883 by Bertrand [5], who dismisses the marginalism of Walras by pointing out that, if you want to make money buying and selling commodities and equities, timing is everything. Furthermore, the timing of the big players in turn affects the prices. However, marginalist theories assign a fundamental value to the change in utility when a product is acquired, and requires that at equilibrium it equals the price. What if the concept of *fundamental value* is just an illusion?



J. Willard Gibbs was co-supervisor of the thesis of Irving Fisher, who became an extremely influential US economist. Fisher's theories were based on a close analogy with equilibrium thermodynamics.

A similar controversy in physics was settled in the mid 19th century. Prior to Joule's demonstration of the mechanical equivalence of heat, many scientists adhered to the Lavoisier *caloric theory* according to which heat was considered to be a fluid that flowed from hot to cold, but could neither be created or destroyed. Gibbs objection then is that marginalist economics is a kind of caloric theory.

Utility theory assigns values to outcomes, but if the goals are obtaining status, influence and respect, process may be more important than outcome.

According to Mirowski [4] the objections of Gibbs and others were never satisfactorily countered by neoclassical economists. Behavioral economics, based on the *prospect theory* of Kahneman and Tversky [6,7], does address this problem by assuming that preferences are affected by *framing*, providing reference points. With sequential events this reference point will change in time and choices will depend on the order in which the events take place. A typical case of evolving frames is when we try something cautiously to find out if we like it or not. Depending on the outcome we either plunge in or shy away. Clearly, when it comes to acquired tastes, value and utility will be process and not state variables.

Seen as a process variable the value of a human life is the life lived, not the value of the assets collected at its end. If one's goals are obtaining status, power, influence and respect, process may be more important than outcome. The 2005 economics prize in memory of Alfred Nobel was given to two game theorists Thomas Schelling and Robert Aumann. Aumann [8] applies game theory within the rigid framework of conventional theory, in which preferences are set permanently, and the possibility ignored that values and preferences are changing as a result of process in repeated games. Schelling [9], on the other hand is aware of the role of process. His acceptance speech stresses the role of taboos and that once a taboo is broken an irreversible process is set in motion. Indeed, if utility is a process rather than a state variable, the rational choice model of sociology (see, *e.g.*, [10]) does not make sense. The severity of the objections will naturally depend on the context. For decisions taken once, or over a short period of time, the problem may not be severe, after all the caloric theory of heat also worked occasionally.

Another quantity which could be considered more as a process than a state variable is *money*. In individual transactions money generally changes hand in well defined amounts while the aggregate quantity *money supply* is a rather murky concept [11]. As the authors of *Evolution, money and war* [12] points out, "money in the bank" may not mean much in times of turmoil.

In the recent economic crisis trillions of dollar worth of assets vanished in thin air. The "stimulus packages" governments employ to alleviate the damage restore some of these funds. But, if the resulting debt incurred by these measures is repaid by printing money, the debt is no more "real" than the value attributed to the original assets. On the other hand, depletion of natural capital through overuse of nonrenewable resources and environmental degradation represent real losses. Also, history teaches us that social, cultural and human capital destroyed by war and turmoil are much more difficult to replace than to destroy.

EQUILIBRIUM

A concept that often leads to false analogies between statistical mechanics, thermodynamics and economics is that of *equilibrium*. If the properties of a thermodynamic system are independent of time it is said to be in a *steady state*. Only if there are no macroscopic *currents* (*e.g.* flow of heat or particles) can the system be said to be in equilibrium. Equilibrium systems generally are closed systems, or systems in contact with a heat bath in which the pressure and/or temperature and chemical potential are *constant*. Non-equilibrium steady state systems typically involves driven system where external sources of heat, force or matter produce flow of heat or electrical and particle currents. Economics concerns itself with the transformation of natural resources, energy and labor into products and commerce. If there is no throughput there is no economy. From the standpoint of statistical mechanics the concept of economic equilibrium is a complete oxymoron – there can be no such thing. Nevertheless, modern economists often apply equilibrium concepts in which supply meets demand and markets are cleared, perhaps guided by a Walrasian auctioneer or Adam Smith's invisible hand. The chapter in *Wealth of Nations*, in which the invisible hand is introduced, can be found on the web [13]. It is noteworthy that the context is trade between complementary economies. I find it easy to accept that, when common interests are recognized, it is as if an invisible hand is guiding the participants toward a deal. Problems arise when interests conflict. The auctioneer on the other hand, if he existed, but not fed, would soon experience the fate of his physics cousin the Maxwell demon, and suffer a burnout. Once fed he becomes part of the system and subject to influence by vested interests. In the financial world independent auditors and credit rating agencies are supposed to fulfill a role somewhat analogous to the auctioneer. The failure of even highly reputable ones to do so, as came to light in the recent turmoil, illustrates the point.

It is crucial to maintain the distinction between equilibrium and steady state.

One difference between equilibrium and nonequilibrium steady states in physics is that the zeroth law of thermodynamics applies to the former but not to the latter. This law states for thermodynamic "forces" to be balanced the pressure, temperature and chemical potential must be constant at equilibrium. In contrast, in non-equilibrium steady states temperature, pressure and chemical potential gradients may occur. Unbalanced thermodynamic "forces" also occur when the system is driven by seemingly unrelated gradients. This gives rise to a number of important effects such as the thermoelectric effect in which an electromotive force is produced by a temperature difference, the thermomolecular effect in which a temperature difference produces a pressure gradient in a rarefied gas, and the Soret effect in which a temperature gradient tend to segregate components of liquid mixtures. Recently, Smith and Foley [14] attempted a modern justification of Irving Fisher's analogy between Gibbs thermodynamics and economics. In such theories Boltzmann factors appear in which average wealth plays the role of termperature [15]. A clear consequence is that if agents are free to choose, prices for the same commodity should not vary within the system. This is contrary to everyday experience, every serious shopper knows that the same groceries tend to have different prices in stores that cater mainly to wealthy or poor customers, somewhat in analogy with the thermomolecular effect. On the macro scale when comparisons are made of wealth in different economies these are usually carried out at purchasing power parity (ppp) not at the actual exchange rates and the two rates are often quite different. It is not too difficult to think of analogies to the thermoelectric effect (migration?) or the Soret effect (ghetto formation?).

Another, perhaps more important difference between equilibrium states and non-equilibrium steady states is how the system responds when disturbed. Equilibrium systems satisfy variational principles, typically the entropy is maximized, subject to constraints, or the appropriate free energy is minimized. If such a system is weakly disturbed the system will move towards equilibrium, and stability is generally assured by the variational property. Similarly, economic calculations in general equilibrium theory are treated as constrained optimization problems in which the question of stability need not be addressed. These types of variational principles are generally not available in non-equilibrium steady states, while in economics the absence of an unambiguous choice as to what is optimized allows the analysts to bias the result in favor of their master's interests.

In thermodynamic equilibrium the appropriate free energy in minimized. There is no natural such quantity in an economic steady state. With no such principle, stability issues become much more serious.

In non-equilibrium theories stability issues are a major concern. Interestingly, when dynamics is introduced into neoclassical models instabilities and inconsistencies [13] pop up yielding untold riches to the connoisseurs of market foibles [16]. Typically, what happens in driven physical systems is that, when forcing is increased, *dissipative structures* appear [1]; *e.g.*, when sunlight reaches the earth more heat is absorbed at the surface than in the atmosphere. The air near the surface then becomes warmer than the air above. Since warm air is lighter than heavy air convective instabilities develop, which when combined with Coriolis forces gives rise to vortices, which may develop into storms. At certain latitudes these storms sometimes dissipate into devastating hurricanes. A much studied system is flow of water in pipes. At low flow velocities in small diameter pipes the flow is laminar, while turbulence suddenly appears at higher flow rates. The critical flow rate depends on the *Reynolds number*

$$Re = \frac{\rho u d}{\eta} \tag{2}$$

where ρ is the fluid density, u the flow velocity, d the diameter of the pipe and η the viscosity of the fluid. The critical number is sensitive to the roughness of the pipe and external noise. Typically the critical number is around 1000, but with extreme care clever experimenters can bring it up to 100 000. Ghashghaie *et. al.* [17] showed that there is a close statistical similarity between turbulent Kolmogorov cascades and exchange rate fluctuations, although Arnéodo *et. al.* [18] warned that the two phenomena differed in fundamental aspects. Is it unreasonable to expect that in an economy a too strong driving force in the form of changing technology and trading pattern, rapid population growth, economic inequality, climate change and unsustainable resource use, can set up dissipative structures in the form of wars, sectarian conflicts and market bubbles and crashes. In conventional economics such phenomena are dismissed as exogenous shocks. If one admits that the shocks are endogenous, once an instability occurs (or as Schelling puts it a taboo is broken) what happens next may still be impossible to predict. However, it may be possible to predict that an unstable situation is approaching. Whether events are endogenous or exogenous also matter when assigning blame when something bad happens. Market bubbles [19] and crashes are often blamed on "noisy traders" and uninformed herd mentality, although rational predators who target the noisy traders also muddy the water [20]. Presumably if all market players were trained economists everything would go smoothly. If, on the other hand, the system is intrinsically unstable, it is useless to blame the victims. The psychologist Ellen Langer coined the phrase *illusion of control* [21] to describe situations in which people attribute personal success to skill when they were just lucky; *e.g.* most people think of themselves as better than average drivers, and often attribute chance success to "skill". Satinover and Sornette [22] used the concept to describe observed behavior in the *minority* and *Parrendo* games while De Bondt and Thaler [23] showed that financial analysts are prone to attributing skill to lucky events and bad luck to mistakes.

An important consequence of the assumption that the onset of turbulence depends on the Reynolds number (2) is that the critical flow velocity depends on system size - if the diameter of the pipe doubles the critical flow velocity is halved. Similarly an increase in mass density (inertia) and a decrease in viscosity (friction) reduces the stability region for laminar flow. Do we expect a system size dependence for market insta-

1. Ilya Prigogine obtained the Nobel Prize in chemistry in 1977 for his contributions to the theory of dissipative structures. This was one of the more controversial awards, but the importance of the concept cannot be denied.

bilities? If so, this should be a matter of concern in times of increasing globalization (some recent examples of work which raise this concern are [24,25]). Is there an analogy between viscosity and transaction cost, regulations and red tape? or between inertia and increasing dependence on intangibles in asset evaluation [2] [26]?

A more subtle, but equally important difference between equilibrium and non equilibrium systems is the role of fluctuations. In equilibrium they are (except for isolated *critical* points in parameter space) Gaussian, large deviations from equilibrium are very rare and fluctuations do not give rise to macroscopic currents and there is no *arrow of time*. If a film is made of a series of fluctuating events and the film is afterward run backwards, an observer will be unable to tell the difference. In contrast, fluctuations in non-equilibrium system frequently have a *rectifying* effect, providing a sense of direction. All living things have a *metabolism* that allow bacteria to swim and eukariotic cells to transport vital chemicals on filaments in the cell. Similarly, the mycelium of mycorrhizal funghi transport useful chemicals which the funghi trade with trees and plants in the forest. Physicists try to model these phenomena through Brownian ratchets [27]. Recently the concept of Brownian ratchets have been extended to situations of economic interest in *Parrondo games* [28] in which two losing strategies when combined result in a winning game. Of particular interest is the collective Parrondo game in which a large number of players vote to reach a common decision on one of two strategies although their interests diverge. No matter whether one votes according to ones own selfish interest [29] or for what is the best outcome for the collective [30], it is a losing game. But, if the decision is made randomly it is a winning game. Seen in an economic context the paradox is easy to resolve. If the two strategies are profit taking and reinvesting, choosing only one of them is a losing proposition over time, but if combined the situation may be sustainable. On the other hand, if only short term interest is the basis for strategy selection, the result is ruin.

CONCLUSION

As could be expected there has been criticism of equilibrium theory from many sources in the economics literature (see *e.g.* Ackerman [31] and references therein). A physicist reader perplexed by the economics literature will find the book by Keen [32] invaluable, as well his warning to the econophysics community [33]. Some of the points raised by me can also be found in the work of McCauley [34]. The analogies used by classical and neoclassical economists with physics are often pointed out. Sometimes the question is asked “why does something that works so well in physics not work in economics?”. The purpose of the present note has been to point out that this type of theory does not work so well in physics either. In many ways the lack of concern with dynamics makes the general equilibrium theory of economics like meteorology without the weather. I find it disconcerting that as late as 2006 in “The Economist” [35] a prominent place was given to a special report describing large scale computer modeling using Walrasian general equilibrium theory to attack important macroeconomic problems.

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2. It is generally accepted that the invention of the steam engine played a pivotal role in the industrial revolution. For the engine to work Watts needed the centrifugal governor. As the demand for more powerful engines grew, the mass of the flywheel was increased, and the friction of the shaft was reduced, with ensuing deleterious effects on the stability of the device. The need to solve this problem was important in the development of modern control engineering.

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