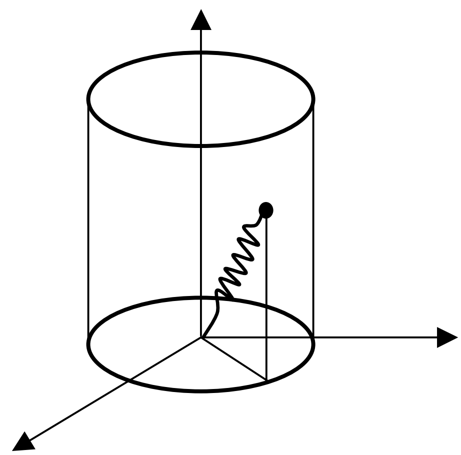
Assignment 2

1. Consider a particle confined to move on the surface of cylinder of radius R, and bound to the origin by a spring of constant k and l_0=0 (see Fig).

a) Prove that the Lagrangian is $L = \frac{m}{2}\left(R^2\dot{\theta}^2 + \dot{z}^2\right) - \frac{k}{2}\left(R^2 + z^2\right)$

b) Next find the conjugate momenta, the Hamiltonian and Hamilton's equations of motion. Based on these equations, what type of motion do you expect for the particle? Will there be oscillatory motion? How about linear motion?



2. For a particle with speed v=(v_x, v_y, v_z), the relativistic Lagrangian is:

$$L_{rel} = -mc^2\sqrt{1-\frac{\vec{\mathrm{v}}^2}{c^2}} - q\phi(\vec{r},t) + q\vec{\mathrm{v}}\cdot\vec{\mathrm{A}}(\vec{r},t)$$

a) Show that this equation reduces to the usual non-relativistic form is |v|/c<<1.

b) Find the canonical momenta p_x, p_y, p_z, the Hamiltonian H and Hamilton's equations of motion. Hint: is it easier to try to find d/dt(p-qA), rather than dp/dt.

3. Formulate the double-pendulum problem (from previous assignment) in terms of Hamiltonian and Hamilton's equations of motion. Try to find H both directly, and using Eq. 8.27 from Goldstein.

4. A Hamiltonian with one degree of freedom has the form:

$$H = \frac{p^2}{2\alpha} - bqpe^{-\alpha t} + \frac{b\alpha}{2} q^2 e^{-\alpha t}\left(\alpha + be^{-\alpha t}\right) + \frac{kq^2}{2}$$

Where b, α and k are constants.

a) Find a Lagrangian corresponding to this Hamiltonian.

b) Find an equivalent Lagrangian that is not explicitly time dependent.

5. A pendulum is constructed by attaching a mass m to a string of fixed length l. One end of the string is fixed at A to the top of a circle of radius $R<2l/\pi$ (see fig). Assuming motion confined to the xy plane, show that the Hamiltonian is

$$H = \frac{p^2}{2m(R\phi - l)^2} - mg\left[R(1-\cos\phi) + (l - R\phi)\sin\phi\right]$$

where p is the momentum associated with ϕ. Show that the angular frequency of small oscillations about stable equilibrium is

$$\omega = \sqrt{g \Big/ \left(l - \frac{R\pi}{2}\right)}$$