Creation and annihilation operators for fermions

Consider a quantum mechanical system of non-interacting fermions. Suppose that the single-particle energy eigenstates of the system are described by wavefunctions $\psi_p(x)$ (if there is no potential, p might label the momentum).

Q: If we have a system of two particles, with one particle in state p and the other particle in state q, what is the wavefunction $\psi(x_1, x_2)$ for the state $|(p,q)\rangle$ of the two particles in terms of the single-particle wavefunctions? *Hint: how would your answer be different if we were talking about bosons?*

Q: Based on your answer to the previous question, how is the state $|(q,p)\rangle$ (obtained by swapping the two particles) related to the state $|(p,q)\rangle$?

Q: Now suppose we have a quantum field theory system that can describe arbitrarily many of these non-interacting particles. In this theory, there will be an operator $a_p^\dagger$ that creates a particle in the state p. In terms of these operators, how do we write the state $|(p,q)\rangle$?

Q: Based on your previous two answers, how is $a_p^\dagger a_q^\dagger$ related to $a_q^\dagger a_p^\dagger$?

Q: If we take $p = q$, what does the previous relation imply about the state $a_p^\dagger a_p^\dagger |0\rangle$?

Q: On the subspace of states with basis $|0\rangle$ and $|p\rangle = a_p^\dagger |0\rangle$, how is the operator $a_p^\dagger$ represented (as a two by two matrix)?

Q: In terms of this matrix, calculate the matrix $a_p a_p^\dagger + a_p^\dagger a_p$ and the matrix $a_p a_p^\dagger - a_p^\dagger a_p$. Which one is proportional to the identity matrix?