Problem Set 8

Problem 1

Hand in the other part of the homework on Monday.

Problem 2

Consider the action

$$S = \int \frac{1}{2}\partial_\mu \bar{\psi} \partial^\mu \psi - \frac{1}{2} m^2 \bar{\psi}\psi$$

where ψ is considered to be a complex field. Show that the action can be written as a sum of independent quadratic actions for eight real fields (*hint: a good start is to write out the action explicitly in terms of the real and imaginary parts of each component of* ψ), and explain why such an action is physically unacceptable.

Problem 3

a) Show that if $u(0)$ satisfies $(m\gamma^0 - m)u(0) = 0$ that $u(\vec{p}) = M(\Lambda_{\vec{p}})u(0)$ satisfies

$$(p_\mu \gamma^\mu - m)u(\vec{p}) = 0\ ,$$

where $\Lambda_{\vec{p}}$ is the boost up to momentum p.

Note: the fact that $\bar{\psi}\psi$ *transforms like a scalar implies that*

$$M^\dagger(\Lambda)\gamma^0 = \gamma^0 M^{-1}(\Lambda)$$

This, and the fact that $\bar{\psi}\gamma^\mu\psi$ *transforms like a vector implies that*

$$M^{-1}(\Lambda)\gamma^\mu M(\Lambda) = \Lambda^\mu{}_\nu \gamma^\nu$$

b) Let ξ_r, $r = 1, 2$ be orthonormal two component vectors, and let

$$u_r(\vec{p}) = M(\Lambda_{\vec{p}})\sqrt{m} \begin{pmatrix} \xi_r \\ \xi_r \end{pmatrix}$$

Show that

$$\sum_s u_s(\vec{p})\bar{u}_s(\vec{p}) = \gamma^\mu p_\mu + m$$

Note, you do not need to write out $M(\Lambda_{\vec{p}})$ explicitly.