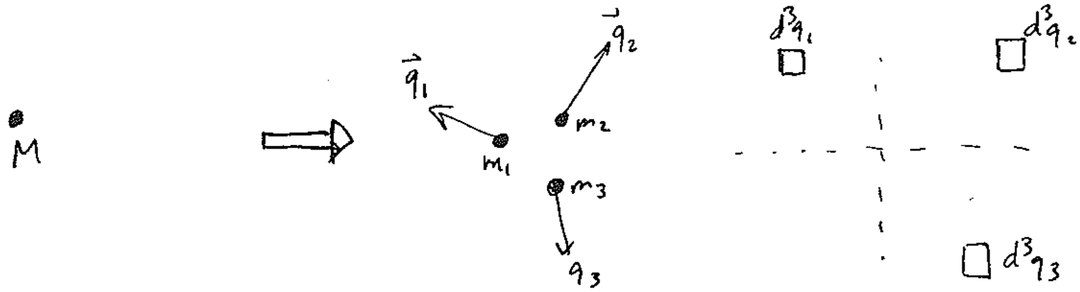
LAST TIME: $d\Gamma$: prob/unit time of decaying to specified final state e.g. volume d^3q_i around q_i



$$d\Gamma = |M_{fi}|^2 \cdot \frac{1}{2M} \frac{1}{2\omega_{q_1}} \cdots \frac{1}{2\omega_{q_n}} (2\pi)^4 \delta^4(p_f - p_i) \frac{d^3q_1}{(2\pi)^3} \cdots \frac{d^3q_n}{(2\pi)^3}$$

Q: particle of mass M decaying to 2 distinguishable particles of mass $m < \frac{M}{2}$.

$\Phi\phi\phi$ interaction $\longrightarrow$ $M_{fi} = \lambda$

A:

$\omega_{q_1} = \sqrt{m^2 + q_1^2}$

$\omega_{q_2} = \sqrt{m^2 + q_2^2}$

$$d\Gamma = \left\{ \lambda^2 \cdot \frac{1}{2M} \frac{1}{2\sqrt{m^2+q_1^2}} \frac{1}{2\sqrt{m^2+q_2^2}} (2\pi)^4 \delta^3(\vec{q}_1 + \vec{q}_2)\, \delta\left(M - \sqrt{m^2+q_1^2} - \sqrt{m^2+q_2^2}\right) \frac{d^3q_1}{(2\pi)^3} \frac{d^3q_2}{(2\pi)^3} \right.$$

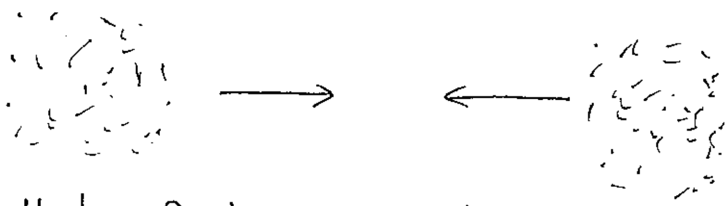
Γ : integrate over q_1, q_2 $\quad \delta^3(\vec{q}_1 + \vec{q}_2) \Rightarrow q_2 = -q_1$

$$\Gamma = \int \frac{d^3q_1}{(2\pi)^3} \frac{\lambda^2}{2M} \frac{1}{4(m^2+q_1^2)} (2\pi)\, \delta\left(M - 2\sqrt{m^2+q_1^2}\right)$$

$= $ etc...

$\nwarrow$ $\delta(f(q_1)) = \dfrac{\delta(q_1 - q_1^0)}{|f'(q_1^0)|}$ where $f(q_1^0) = 0$

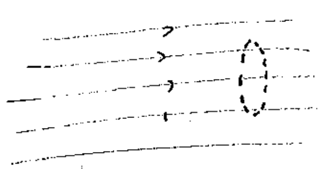
$\boxed{\text{2 initial particles}}$ e.g. collider experiments



collide 2 beams, observe # events/unit time

Differential scattering cross section:

$$d\sigma = \frac{\text{\# events/time to specified set of final particles for single target particle at rest}}{\text{luminosity of incoming beam}}$$

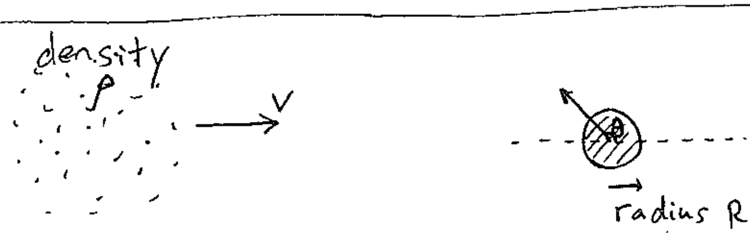


Luminosity $\mathcal{L}$ = # particles/area/time

$d\sigma \rightarrow$ units of area $\sim$ area of incoming beam that undergoes a given scattering

$\rightarrow$ indep of details of experiment.

Q:

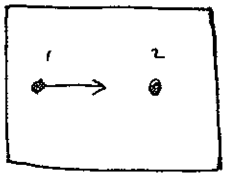


classical

What is luminosity? A: $\rho \cdot v$

What is σ to scatter in backwards direction $90° \leq \theta \leq 180°$?

A: $\frac{\pi}{2} R^2$

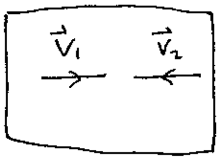


REST FRAME

$$d\sigma = \frac{\text{Rate in frame of target}}{\text{Luminosity in rest frame}}$$

$$= \frac{\text{Rate (rest)}}{\underbrace{v_1^{rest}}_{\text{velocity}}\ \underbrace{\rho_1^{rest}}_{\text{density}}}$$

Can express in terms of other frames



LAB FRAME

Time dilation: $\text{Rate(lab)} = \frac{1}{\gamma_2}\,\text{Rate (rest)}$

$$(\overset{\text{density}}{\rho}, \overset{\text{current}}{\rho\vec{v}}) \quad \text{transforms as 4-vector}$$

$$\begin{pmatrix} \rho_{rest} \\ \rho_{rest} \cdot v_{rest} \end{pmatrix} = \begin{pmatrix} \gamma_2 & -\gamma_2 v_2 \\ -\gamma_2 v_2 & \gamma_2 \end{pmatrix} \begin{pmatrix} \rho_{lab} \\ \rho_{lab} \cdot v_{lab}^1 \end{pmatrix}$$

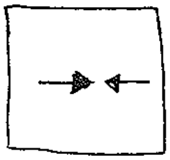
$$\rho_{rest}\, v_{rest} = \gamma_2\, \rho_{lab} \left(v_{lab}^1 - v_{lab}^2\right)$$

$$d\sigma = \frac{\text{Rate (lab)}}{\rho_{lab}\left(v_{lab}^1 - v_{lab}^2\right)}$$

Work at finite volume, single incoming + target particles.

$$\rho_{lab} = \frac{1}{V}$$

$$\text{Rate} = \frac{\text{Prob of transition}}{\text{Time.}} \quad \rightsquigarrow \text{calculate like last time}$$



- initial state $a^\dagger_{p_1} a^\dagger_{p_2}|0\rangle$
  properly normalized for box
- act w. $\mathcal{U}(\frac{T}{2}, -\frac{T}{2})$
- find overlap w. final state $a^\dagger_{q_1}\cdots a^\dagger_{q_n}|0\rangle$

$$P = \left|\langle 0| a_{q_1}\cdots a_{q_n}\; \mathcal{U}\; a^\dagger_{p_1} a^\dagger_{p_2}|0\rangle\right|^2$$

find $$\frac{P}{T} = |M_{fi}|^2 \frac{1}{V^{n+2}} \frac{1}{2\omega_{p_1}} \frac{1}{2\omega_{p_2}} \frac{1}{2\omega_{q_1}} \cdots \frac{1}{2\omega_{q_n}} V\cdot(V\delta_{p_f,p_i}) \cdot (T\delta_{E_f,E_i})$$

to specific final state $\vec{q}_1 \cdots \vec{q}_n$

$$\frac{P}{T} = |M_{fi}|^2 \frac{1}{V^2} \frac{1}{2\omega_{p_1}} \cdots \frac{1}{2\omega_{q_n}} V\cdot(V\delta_{p_f\, p_i})(T\delta_{E_f,E_i}) \cdot \frac{d^3q_1}{(2\pi)^3} \cdots \frac{d^3q_n}{(2\pi)^3}$$

to any state with final momenta in box d^3q_1 around $\vec{q}_1$, d^3q_2 around $\vec{q}_2$ ...

$$d\sigma = \frac{\overbrace{\text{Rate (lab)}}^{P/T}}{\underbrace{\rho_{lab}}_{1/V} \cdot (v^1_{lab} - v^2_{lab})}$$

$$\boxed{d\sigma = \frac{1}{|\vec{v}^1 - \vec{v}^2|} \frac{1}{4E_{p_1}E_{p_2}} |M_{fi}|^2 (2\pi)^4 \delta^4(p_f - p_i) \prod_{i=1}^{n} \frac{d^3q_i}{2E_{q_i}(2\pi)^3}}$$

assume colinear