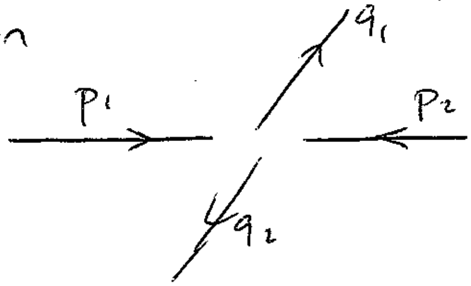
LAST TIME:

$$d\sigma = \frac{1}{|\vec{v}_1 - \vec{v}_2|} \frac{1}{4E_{p_1} E_{p_2}} |M_{fi}|^2 (2\pi)^4 \delta^4(p_f - p_i) \prod \frac{d^3 q_i}{2E_{q_i} (2\pi)^3}$$

EXAMPLE: scattering of scalar particles with $V = \frac{\lambda}{4!}\phi^4$ interaction



STEP 1: Calculate M_{fi}

leading order

$$\langle 0 | a_{q_1} \sqrt{2E_{q_1}}\, a_{q_2} \sqrt{2E_{q_2}} \left\{ -i \frac{\lambda}{4!} \int d^4x\, \phi^4(x) \right\} a^\dagger_{p_1} \sqrt{2E_{p_1}}\, a^\dagger_{p_2} \sqrt{2E_{p_2}} | 0 \rangle$$

$$= -24 i \frac{\lambda}{4!} \int d^3x \langle 0 | a_{q_1} a_{q_2}\, \phi(x) \phi(x) \phi(x) \phi(x)\, a^\dagger_{p_1} a^\dagger_{p_2} | 0 \rangle \sqrt{2E_{q_1}} \sqrt{2E_{q_2}} \sqrt{2E_{p_1}} \sqrt{2E_{p_2}}$$

$$\phi(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left(a_{\vec{p}}\, e^{-ip\cdot x} + a^\dagger_p e^{ip\cdot x} \right)$$

$$\sqrt{2E_q}\, \underbrace{a_q\, \phi}(x) = e^{iq\cdot x}$$

$$\sqrt{2E_p}\, \underbrace{\phi(x)\, a^\dagger_p} = e^{-ip\cdot x}$$

$$= -i\lambda (2\pi)^4 \delta^4(p_f - p_i) \equiv i (2\pi)^4 M_{fi}\, \delta^4(p_f - p_i)$$

$$\therefore M_{fi} = -\lambda$$

CENTER of MASS FRAME: $\vec{p}_2 = -\vec{p}_1 \quad E_{p_2} = E_{p_1} = E$

$$d\sigma = \frac{1}{2\,|v|}\,\frac{1}{4E^2}\,\lambda^2 (2\pi)^4\,\delta(E_{q_1}+E_{q_2}-2E)\,\delta^3(\vec{q}_1+\vec{q}_2)\,\frac{d^3\vec{q}_1}{(2\pi)^3(2E_{q_1})}\,\frac{d^3\vec{q}_2}{(2\pi)^3(2E_{q_2})}$$

must have $\vec{q}_2 = -\vec{q}_1 \quad E_{q_1} = E_{q_2} = E$

then $\delta^3(\vec{q}_1+\vec{q}_2)\,d^3q_2 = 1$

$$\delta(E_{q_1}+E_{q_2}-2E)\,d^3q_1$$

$$= \delta(2E_q - 2E)\,q^2\,dq\,d\Omega$$

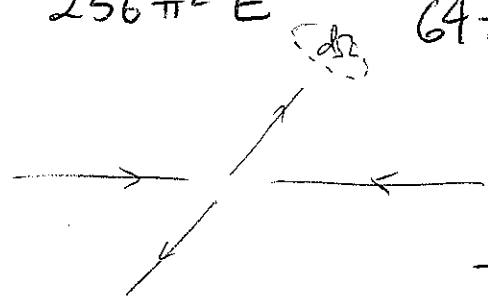
$$= \delta\left(2\sqrt{q^2+m^2} - 2E\right) q^2\,dq\,d\Omega$$

$$= \frac{q\sqrt{q^2+m^2}}{2}\,d\Omega$$

use $v = \dfrac{q}{E} = \dfrac{p}{E}$

$$\therefore\ d\sigma = \frac{\lambda^2\,d\Omega}{256\pi^2 E^2} = \frac{\lambda^2\,d\Omega}{64\pi^2 E_{cm}^2}$$

$E_{cm} = 2E$ = total c.o.m. energy



→ uniform distribution of scattered particles

→ cross section decreases as $\frac{1}{E^2}$ as $E\uparrow$

Total cross section:

→ integrate over HALF SPHERE since $(\vec{q}_1, -\vec{q}_1) \approx (\vec{q}_1, \vec{q}_1)$ for identical particles

$$\sigma = \frac{\lambda^2}{32\pi E_{cm}^2}$$