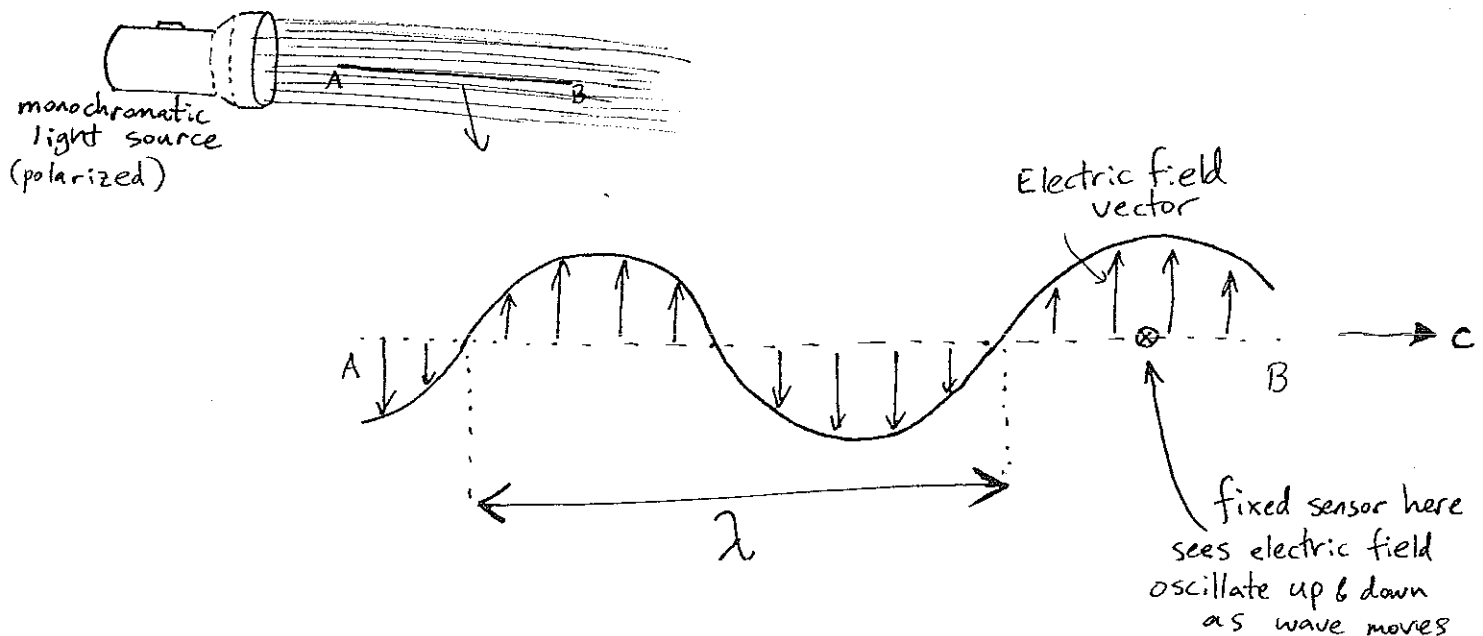# Physics 200 Tutorial 6: Light!

In this tutorial, we will review some of the basic properties of light and think about how light appears to observers in different frames of reference. So far, the only thing we've discussed about light is that its velocity is c regardless of which frame of reference the velocity is being measured in. But of course, light has other properties than its velocity.

Question 1

What are some characteristics that distinguish light from one source vs light from some other source? Answer in terms of properties that we can actually perceive (rather than in mathematical terms).

Question 2

As we discussed briefly in the first week, light has a mathematical description as a wave solution to the equations that describe the evolution of electric and magnetic fields. To visualize this solution, we can ask what the electric field looks like at some instant in time for the points along some line inside a light beam:

monochromatic
light source
(polarized)

Electric field
vector

λ

A

B

c

fixed sensor here
sees electric field
oscillate up & down
as wave moves

In the picture, the curve represents a plot of the electric field at each point along the dotted line (the path from A to B inside the light beam) at some instant in time. There are some places where the electric field points up and some places where the electric field points down (at any given time). The distance between two successive "peaks" (places where the electric field is maximum) in the picture is called the WAVELENGTH, and is usually denoted by λ.

a) How are the characteristics that you listed in question 1 related to the mathematical properties of the wave (i.e. features of the diagram above)?

b) The whole wave is moving at velocity c, so the pattern in the picture will shift to the right as time passes. At any fixed point in space, the electric field in the light wave will therefore appear to oscillate up and down. In terms of c and λ, how long does one of these oscillations take?

Your answer to this question is the PERIOD of the light wave, which is the inverse of the FREQUENCY. Let's call the period T and the frequency f. Note that the period, the frequency, and the wavelength are all determined in terms of one another.

Question 3

We know that all observers will agree on the velocity of a light pulse, but what about the wavelength? Let's try to answer this question based on what we know. Suppose we have a light source (e.g. a star) emitting light with period T in its own frame. If the light source is moving with velocity v relative to an observer, with components $v\cos(\theta)$ and $v\sin(\theta)$ towards and perpendicular to the observer as shown below, what is the period (or frequency or wavelength) of the light that the observer sees?
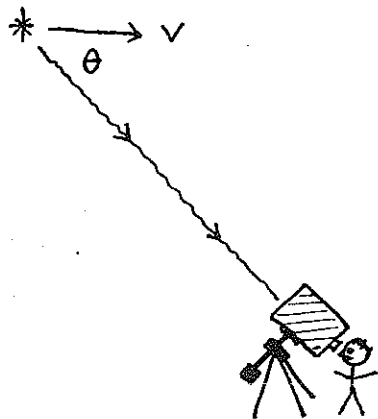
Let's try to work out a formula for this using basic relativity. Imagine that a source of light emits a very short pulse: only a single wavelength. In the frame of the source, the time between when the beginning of the pulse is emitted and when the end of the pulse is emitted is T, the period of the wave.

a) What is the time between when the beginning of the pulse is emitted and when the end of the pulse is emitted in the frame of the observer?

$t_1 = 0$ L_1 t_2 v

θ

L_2

D

t_4

t_3
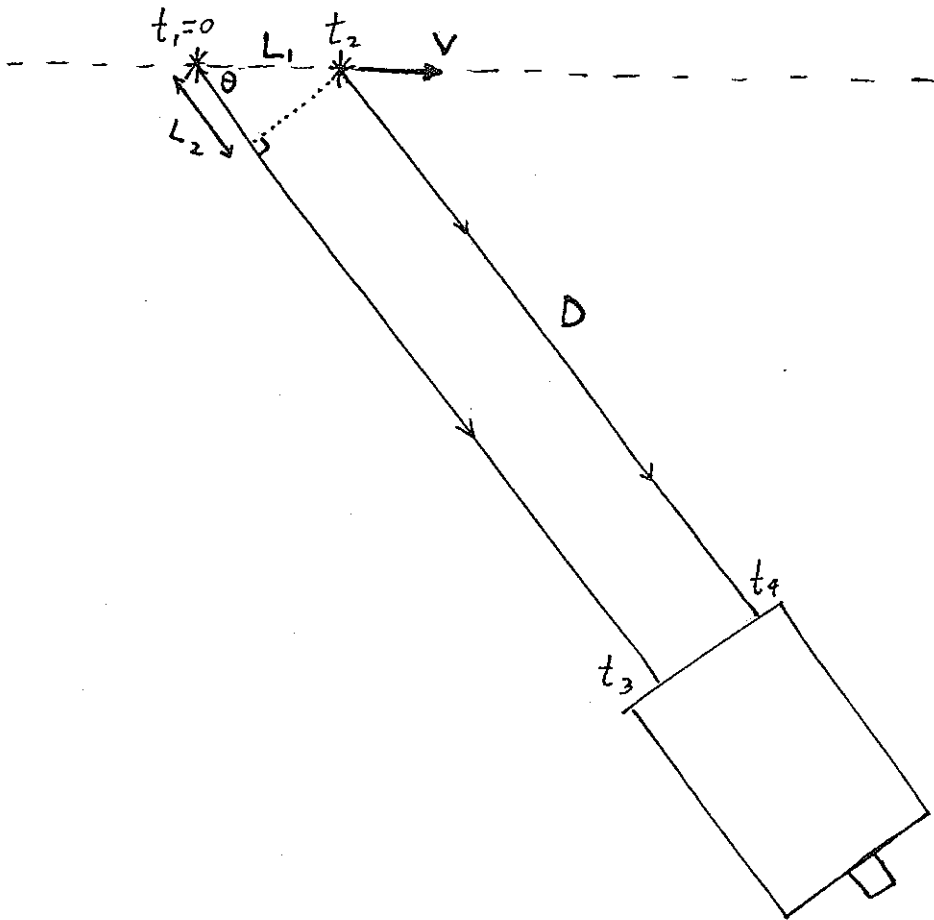
$t_1 =$

$t_2 =$

$L_1 =$

$L_2 =$

$t_3 =$

$t_4 =$

b) The picture above shows the location of the source at the time $t_1=0$ when the beginning of the pulse is emitted and at the time t_2 (= your answer from part a) when the end of the pulse is emitted. To find the *observed* period of the light we want to know the time difference between the time t_3 when the beginning of the pulse reaches the telescope and the time t_4 when the end of the pulse reaches the telescope. Fill in the table above (your answers should be in terms of T, v, γ, θ, D, and c). Now determine the observed period by using $T_{obs} = t_4 - t_3$ (this should be in terms of T, v, γ, θ, and c).

ANSWER: $T_{obs} =$

check your answer with me or the TA.

c) Congratulations! You have just derived the formula for the DOPPLER SHIFT of light. Using your answers to part 3b and 2a, rewrite the Doppler shift formula as a formula relating the observed wavelength of light from the star to the wavelength emitted in the star's frame (this should be in terms of λ, v, γ, θ, and c):

ANSWER: $\lambda_{obs} =$

Problem 4

↑ 0.6 c

*

a) If we have a star emitting light with an average wavelength 500nm moving away from us at v=0.6c, what is the wavelength of the light we observe?

Earth

b) Can you explain why the effect in part b is called REDSHIFT and why it is extremely useful for astronomers?

* → 0.6c

c) What is the observed wavelength if the same star is travelling perpendicular to our line of sight?

Earth

Question 5: The expanding universe

By observing the redshift of distant galaxies, scientists have concluded that our universe is expanding. In this question, we'll understand better what this means and how redshift observations helped us come to this conclusion.

A good way to picture an expanding universe is to imagine a big rubber sheet with tacks stuck into it at various places (the galaxies). Now, as time passes, the whole sheet gets stretched out uniformly:
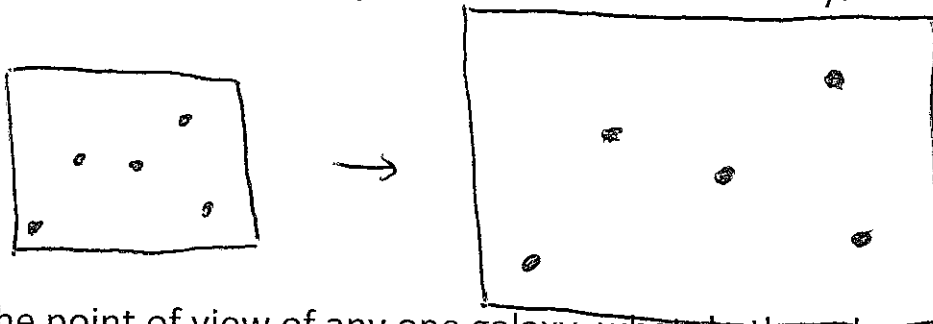
a) From the point of view of any one galaxy, what do the other galaxies appear to be doing?

b) If the sheet is expanding at a rate such that it doubles in a time T, and the distance between two galaxies is D at time 0, what is the distance between the galaxies at a later time t? (*Hint: what is the distance at time T, 2T, 3T, etc...*)

c) Now that you have the separation as a function of time, you can figure out the relative velocity of the two galaxies. For the two galaxies in part b), what is the velocity of one relative to the other at time t=0? How does this velocity depend on distance?

d) If a typical galaxy emits light at average wavelength λ in its rest frame, what would be the observed wavelength of light from a galaxy at a distance D if the universe is expanding in the manner above.

This relationship between redshift and distance is essentially what we actually observe when looking at distant galaxies.[1] It is this observation that allowed us to conclude the relationship between distance and velocity (your result from part c), which is known as Hubble's Law, and based on this relationship, we surmise the universe is expanding.

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[1] Our analysis here is a slight simplification, since we should really take into account the fact that the light from a distant galaxy takes time to reach us, and so the light we see now tells us about the galaxy's velocity at an earlier time.

Problem 6

a) When we start to discuss quantum mechanics, we will learn that light is made up of particles (photons) whose energy is given by $E = h f$, where f is the frequency of the light, and h is a constant (Planck's constant). What would be the energy of the particles making up light of wavelength λ?
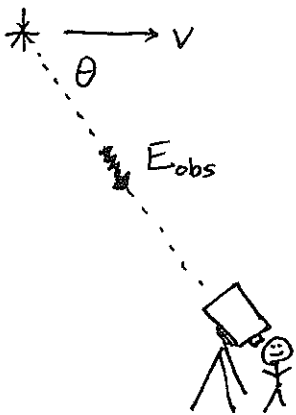
b) In the setup of problem 3, if the light from the star is made of particles with energy E_{obs}, what is energy E_{star} of these particles in the reference frame of the star (answer in terms of E_{obs}, v, c, γ, and θ)? The Lorentz transformations for energy and momentum are written on the board.

c) Using this result, and your result for part a, what is the relationship between the wavelength of light λ_{star} in the star's frame and the observed wavelength λ_{obs}. Does this agree with your formula in problem 3c?