Physics 200 Problem Set 9

Problem 1

Batman and Robin are trying to develop a new electron gun to help neutralize positively charged criminals. They have already built a small device that can produce a beam of electrons with speed $1000 m/s$ all in the same direction. They decide that the beam is too wide, so they attach a metal plate with a very small hole in it, hoping that the electrons passing through the hole will make a beam only as wide as the hole. Instead, they find that the beam actually spreads out more the smaller they make the hole. More precisely, they find that the electrons coming out of the hole are no longer all traveling the same direction, but are found in some cone whose opening angle increases as the size of the hole is decreased.

a) Explain this mysterious result using the uncertainty principle. Note that for three dimensional situations, we have a separate uncertainty relation for each direction,

$$\Delta x \Delta p_x \geq \frac{\hbar}{2} \qquad \Delta y \Delta p_y \geq \frac{\hbar}{2} \qquad \Delta z \Delta p_z \geq \frac{\hbar}{2}$$

b) If the hole has diameter $1 \mu m$, estimate the opening angle of the beam coming through the hole.

Problem 2

After a long career of racing past each other at relativistic speeds, Milt and Ethel decide to slow down and start a physics consulting firm called "All Things Quantum." One day, a scientist from the molecule production company We-Make-Gas Inc. shows up with a sealed tube of gas which she believes is composed of a new kind of molecule with only a single electron. Having recently taken physics 200, she knows that electrons bound in atoms or molecules are always found at certain discrete energies, and asks Milt and Ethel to help her find these energy levels for her molecule.

Milt and Ethel are glad to help (for a small fee), so they carry out a couple of experiments. First, they heat up the gas and find that it has a very simple emission spectrum, with light at wavelengths

$$\lambda = 250 nm, \lambda = 300 nm, \lambda = 1250 nm$$

Next, they illuminate the gas with photons and find that for wavelengths smaller than 200nm, electrons are liberated from some of the atoms while for wavelengths of 250nm or 1250nm, there is a significant amount of absorption of the light (the gas is not heated in this experiment, so you can assume that the electrons around all of the molecules start in their lowest energy state).

If E = 0eV is the energy of an electron far away from the molecule, what are the allowed energy levels for electrons in the new molecule (in eV)?

Hint: for this problem, the basic idea is that electrons can jump from one energy level to another by either emitting or absorbing a photon whose energy is equal to the difference in energies between the two states.

Problem 3

We have seen in tutorial 12 that the energy for an electron in a wire of length L can only take on certain special values,

$$E_n = \frac{n^2 h^2}{8 m L^2} \qquad n = 1, 2, 3, \ldots.$$

We found that the wavefunctions corresponding to these energies are

$$\psi_n(x,t) = \sqrt{\frac{2}{L}} \sin(\frac{n \pi x}{L}) e^{-2\pi i (E/h) t}$$

These states with a definite frequency are the ENERGY EIGENSTATES for the system, since they have a definite frequency (i.e. the real and imaginary parts just oscillate up and down with time), and energy is determined in terms of frequency by $E = hf$. Just as for position eigenstates and momentum eigenstates, we can always write the wavefunction for a general state as a superposition of the energy eigenstates. Suppose we have a wire of length $L = 5nm$ and an electron in the wire is in a superposition

$$\psi(x,t) = \frac{1}{\sqrt{2}}\psi_1(x,t) + \frac{1}{\sqrt{2}}\psi_2(x,t)$$

a) If we make a measurement of the energy of this state at time $t = 0$, what are the possible values we might find and what are the probabilities for each?

b) If we instead wait and do the measurement of the energy at time $t = 10^{-13}s$, what are the possible values we might find and what are the probabilities for each? Is your answer the same or different than your answer in a?

For the energy eigenstate wavefunction $\psi_n(x,t)$, calculate the probability density. Does it change with time?

d) Sketch $\psi_1(x,t)$ (solid) and $\psi_2(x,t)$ (dotted) on the same graph for $0 \le x \le 5nm$ at $t = 0$. Sketch $\psi_1(x,t)$ (solid) and $\psi_2(x,t)$ (dotted) on the same graph for $0 \le x \le 5nm$ at $t = 4mL^2/h$ (*Note: you should find that both wavefunctions are real at this time*).

e) Based on your sketches in part c, is the probability density for the superposition of energy eigenstates $(\psi_1 + \psi_2)/\sqrt{2}$ the same or different at these two times?

Problem 4

An electron in the lowest energy state of a hydrogen atom has a spherically symmetric three-dimensional wavefunction

$$\psi(x,y,z) = \frac{1}{\sqrt{\pi}a^{\frac{3}{2}}}e^{-r/a}$$

where $r = \sqrt{x^2 + y^2 + z^2}$ is the distance to the proton and a is the Bohr radius, $a \approx 0.0529nm$.

a) Sketch the probability density as a function of the radius r. At what distance from the proton is the probability density for finding the electron greatest?

b) Suppose we measure the electron's position to determine how far it is from the proton. Argue that the probability for finding the electron between radius r and radius $r + dr$ is given by

$$P(r)dr$$

where

$$P(r) = 4\pi r^2 |\psi(r)|^2 \ .$$

Assume that Δr is small enough that we can treat $\psi(r)$ as a constant over this interval of radius. (Hint: how would you calculate the amount of mass in a star between radius r and radius $r + \Delta r$ if I told you the mass density as a function of radius was $\rho(r)$?)

c) The function $P(r)$ gives the relative probability for finding the electron at radius r if we do a measurement. Sketch this function and determine (relative to the Bohr radius a) where it is maximum. Explain why your result is different from your result for part a.

d) Suppose we measure the distance between the electron and the proton for one thousand identical hydrogen atoms (all in their lowest energy state). Using the result from part b, calculate the how many times we would expect to find an electron closer than the Bohr radius of the proton.