Physics 200 Problem Set 7

Problem 1

a) X-rays with wavelength 0.1nm scatter off a thin metal foil. What is the observed wavelength for X-rays that scatter directly backwards from electrons in the metal, assuming the electrons in the metal are at rest?

b) In an actual metal, the electrons are not all at rest inside the metal. Instead, we can think of the free electrons as bouncing around inside the metal, with characteristic speeds of around $10^6 m/s$. As a result, we actually observe a range of wavelengths for the backwards-scattered X-rays. To estimate this range, determine the shift in wavelength of an X-ray that scatters directly backward from an electron which is initially moving towards the X-ray photon with velocity v (which could be positive or negative). *Hint: a simple way to do this is to go to the frame of the electron, use the Compton scattering formula there, and then go back to the original frame. Remember that there is a Doppler shift for the light when going between frames.* Taking v to be between $10^6 m/s$ and $-10^6 m/s$, determine the range of possible wavelengths for the scattered X-rays.

Problem 2

In a diffraction experiment, the intensity profile for the diffraction pattern on a screen is measured to be proportional to the function

$$I(x) = \frac{1}{1+x^2}$$

a) If we do the experiment with individual photons, what is the probability that a given photon will hit the screen in the interval $2 < x < 3$? *Hint: the function above does not integrate to 1.*

b) What is the probability that the first four photons will all hit the screen in the region $x > 0$?

Problem 3

Suppose we set up a sequence of three polarizers with orientations 45 degrees, 75 degrees, and 45 degrees, where the angles refer to how much each polarizer is rotated relative to the vertical direction.

a) If we send three vertically (i.e. 0 degrees) polarized photons in, either 0,1,2, or 3 photons might pass through all the polarizers. What is the probability for each of these possibilities?

b) Suppose we we send 10000 vertically polarized photons in. On average, how many photons do we expect to make it through all these polarizers?

c) If we want to maximize the probability that a photon will pass through all three polarizers, what should we choose for the initial polarization state?

Problem 4: Electron microscopy

You are probably familiar with the idea that the resolving power of light microscopes is limited by the wavelength of the light being used, and that we can get better resolution by using electron microscopes. In this problem, we'll use our knowledge of the double slit experiment to understand this better. The basic idea of a transmission microscope is that we send light (or electrons) through a sample, reconstruct an image based on what we observe on the other side. The resolution is defined as the smallest distance between features on the sample such that we can still distinguish them as two independent features (rather than just a blur).

Now, let's imagine that our sample is a thin piece of foil with two very narrow slits, separated by a distance d (assume that the slits are much narrower than d). In this case, we can send light/electrons through the sample and look at the interference pattern that emerges. In this case, we can ask how close together the slits can be before we can no longer tell that there are two slits rather than just one. This is the resolution of our microscope. In the double slit experiment, the important feature that tells us that there are two slits is the series of bright and dark regions that arise due to constructive/destructive interference from the two slits. As we make the slits closer together, this pattern spreads out, and eventually even the first dark region will be at an angle greater than $\pi/2$. For separations much less than this, we will have only constructive interference between the slits for all observable angles, and the image will be the same as if we just had a single slit that was twice as wide.

In summary, if the slits are closer than some distance R (where the first dark region in the pattern is at an angle $\pi/2$), we will not be able to distinguish from the interference pattern (our "image") whether there is one slit or two, so we define this to be the resolution.

a) Explain (using a precise equation) why the double slit interference pattern spreads out when we make the slits closer together.

b) Calculate the resolution in terms of the wavelength of the light/electrons used. What is the minimum resolution if we use optical light?

c) For electrons, the wavelength that determines the interference pattern is the de Broglie wavelength, $\lambda = h/p$. In a standard Transmission Electron Microscope, the kinetic energy of the electrons used is about 100 keV. What is the resolution in this case?

d) If we want to resolve atomic scale features, what how fast do our electrons need to travel (relative to the speed of light)?

e) By accelerating electrons to high enough energies at particle accelerators, physicists have been able to "see" that individual protons actually have internal structure and are composed of quarks (these experiments are known as "deep inelastic scattering" experiments). This required a resolution of around $10^{-16}m$. What energy (in eV) did the electrons need to have for this, and what γ does this correspond to?