Physics 200 Problem Set 2

Problem 1

Using the transformation rules

$$x' = \gamma(x - vt)$$

and

$$x = \gamma(x' + vt')$$

that we derived in class, derive the rule that determines t' in terms of x and t. That is, show that

$$t' = \gamma(t - \frac{v}{c^2}x)$$

Problem 2

Approximately how much shorter than its actual length would a stationary observer (on the Earth) measure a large passenger jet to be during a typical flight (order of magnitude is fine).

Problem 3

Marta, 7 years old, is tired of her younger sister Gretel, age 5, getting all the attention. She decides to take a trip to Proxima Centauri (the nearest star), 4 light years away. She travels to the star and back, always at a constant speed, such that when she returns, she is two years younger than Gretel. How old are she and Gretel when she returns (assume that they are exactly two years apart initially).

Problem 4

One day, Kermit the Frog sits down for lunch and opens a can of Flies & Grubs to eat. Exactly 3μs later (in Kermit's frame), Miss Piggy opens her own can of Flies & Grubs at her house 5km away (she is about to feed her pet salamander). Around this time, Gonzo happens to be driving at constant velocity on the straight road connecting the two houses, and it turns out that in Gonzo's frame of reference, Kermit and Miss Piggy open their cans simultaneously. How fast, and towards whose house, is Gonzo driving?

Problem 5

Milt and Ethel pass each other in rocket ships travelling in opposite directions, each at speed v in a certain frame. They both set their clocks to zero when they pass. In this frame, there are markers in either direction from the point where Milt and Ethel pass, each located a distance D from this point.

a) How far apart does Milt observe the markers to be?
b) What does Milt's clock read when he passes the marker?
c) What is Ethel's trajectory $x_M(t_M)$ in Milt's frame (assume that both Milt and Ethel assign coordinates $(x = 0, t = 0)$ to the event where the two ships pass). Using this, determine what velocity Milt measures Ethel to have.
d) Relative to the two markers, where does Milt observe Ethel to be when he passes his marker.
e) What time does Milt observe on Ethel's clock when he passes his marker?
f) Milt and Ethel both stop their clocks when they pass their marker. An observer in the original frame argues (correctly) that the clocks must read the same time at this point, because the situation is completely symmetrical. He concludes that neither Milt nor Ethel can have observed any time dilation on the other's clock. Using your analysis in the previous parts, explain why his conclusion is not correct.

Physics 200 Problem Set 3

Problem 1

A moving ruler has clocks on either end that are synchronized in the frame of the ruler. An observer watching the moving ruler sees the clock on the left read 2:00 at $t = 0$ and at $x = 0$, but observes the clock on the right to read 2:00 at $t = 10ns$ when this clock is at a position $x = 5m$. How long is the ruler in it's own frame, and what is its velocity relative to the observer? (Hint: use the invariant interval for the first part)

Problem 2

A rocket ship is traveling towards a distance star at velocity $v = 0.9c$. The star emits a jet of plasma traveling at $0.3c$ in a direction perpendicular to the direction towards the oncoming ship. What components do observers on the ship measure for the velocity of the jet? Give separate answers for the component towards the ship, and the component perpendicular to the direction between the ship and the star. (Hint: use the Lorentz transformation to solve this, as we did in deriving the one-dimensional velocity transform. Think carefully about how to define your axes. Express your answers in terms of c.)

Problem 3

Albert Einstein is out for a walk one day when he notices a flying sausage pass by at three fifths of the speed of light. The sausage is oriented in it's direction of motion (to minimize air resistance), and Albert observes the sausage to be 40cm long. One second later, a hungry beagle who is chasing the sausage passes by, traveling at 4/5 times the speed of light in the same direction.

a) How long does the beagle observe the sausage to be?

b) How long is the sausage in it's own frame?

c) According to his dog watch, how long after he passes Albert does it take the beagle to catch up with the sausage?

Problem 4

A flying space bus carries a group of space tourists on a round trip journey from a space station to a nearby nebula. The bus has an initial (positive) velocity of $v = 0.2c$ and a constant negative acceleration *as measured in the original frame* of $a = -0.1c/year$. When they arrive back at the space station, how much time has elapsed on the space station clocks, and how much time has elapsed on the clocks carried by the tourists?

Problem 5

For Enid's 80th birthday, her husband Abraham decides to build a new 4 meter long garage for Enid's 4 meter long 1958 Edsel convertible. Unfortunately, the same day, Enid decides to give herself a birthday present and trades in her 4 meter long 1958 Edsel in for a 5 meter long 2007 Hummer. As she arrives back home, she sees the new garage, and not wanting to hurt Abraham's feelings, accelerates to $0.6c$. Since $\gamma = 5/4$ for $v = 0.6c$, she knows that Abraham will observe observe her vehicle to be only 4 meters long and therefore should see her Hummer (barely) fit in the garage he has made. Everything works out as planned, and Abraham closes the door of the garage just when he observes the front of the Hummer reach the far inside wall of the garage.

Now, you might ask, how can it be that the 5 meter long Hummer can fit in the 4 meter long garage? To resolve this apparent paradox, let's use a spacetime diagram.

a) On the attached spacetime diagram, assume that each tick represents 4 meters, and that the (x, t) coordinates correspond to the frame of the garage. Take the door of the garage to be at $x = 0$ and the far inside wall to be at $x = 4m$. Also, assume that the back of the Hummer is at $x' = 0$ and that this coincides with the door of the garage at $t = t' = 0$.

Draw (carefully!) the trajectories of the front and back of the garage and the Hummer on the spacetime diagram. Indicate with an star the event where the front end of the car reaches the far inside wall of the garage. Indicate with an 'A' the event where Abraham closes the door of the garage. Indicate with an 'E' the back end of the Hummer in Ethel's frame when she observes the front end of the Hummer reaches the wall of the garage. Finally, indicate with a 'Z' where Ethel observes the front of her car to be at the same time that she observes Abraham to close the door.

b) Based on your diagram, explain the resolution of the paradox.

Optional bonus problem 6

Hand in separately; worth an extra 1% on your assignments grade.

A rocket is initially stationary in some frame. If the rocket then turns on its thrusters such that the acceleration in its own frame is always A, determine the trajectory of the rocket in the original frame.

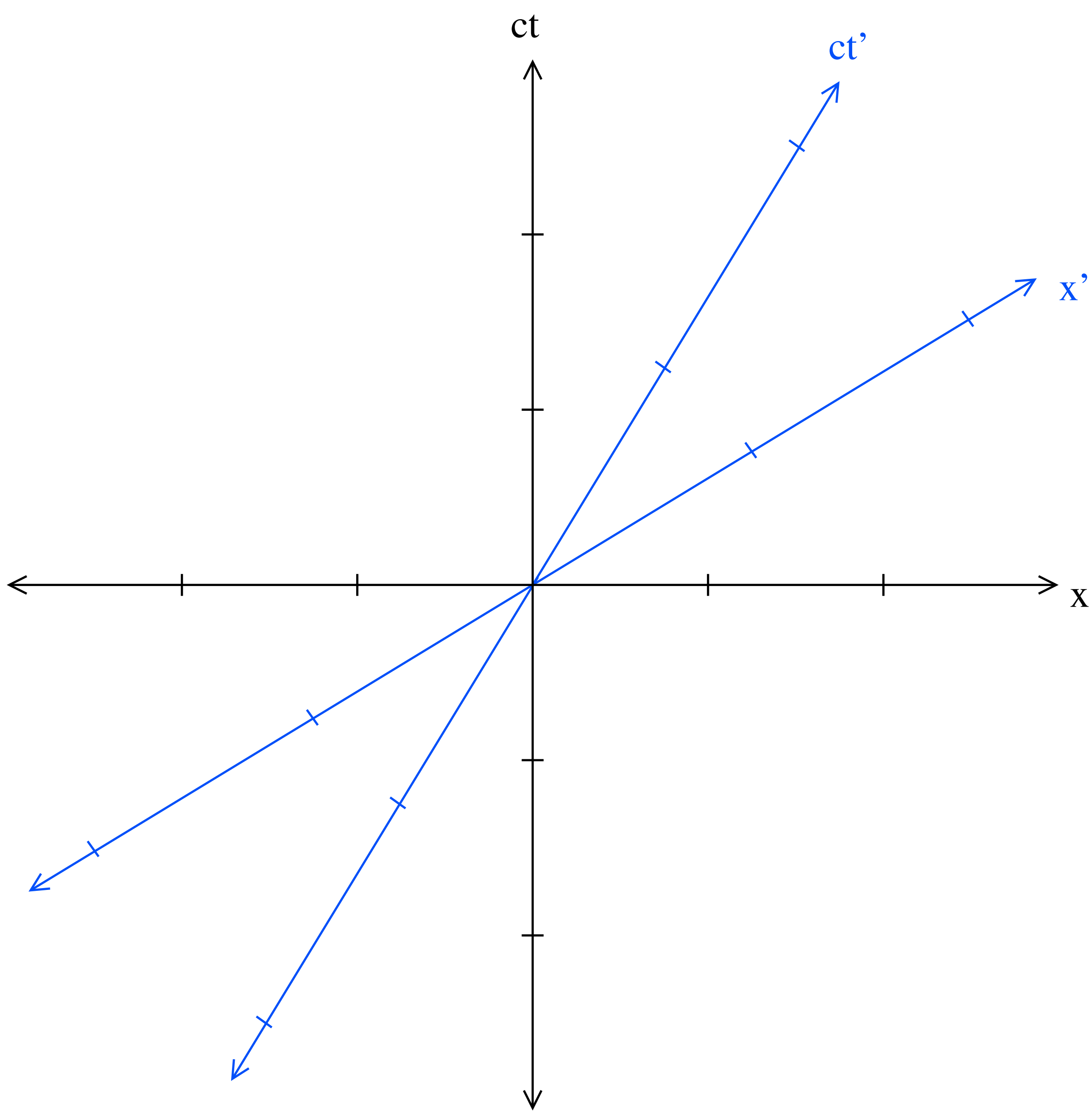

Figure 1: Lorentz transformation for $v = 0.6c$. Use this for problem 5.

Physics 200 Problem Set 2

Hint: Draw pictures!

Problem 1

One day, Kermit the Frog sits down for lunch and opens a can of Flies & Grubs to eat. Exactly 3 μs later (in Kermit's frame), Miss Piggy opens her own can of Flies & Grubs at her house 5km away (she is about to feed her pet salamander). Around this time, Gonzo happens to be driving at constant velocity on the straight road connecting the two houses, and it turns out that in Gonzos frame of reference, Kermit and Miss Piggy open their cans simultaneously. How fast, and towards whose house, is Gonzo driving?

Problem 2

A spaceship leaves the Earth traveling at speed $4/5c$ towards planet Elvis, 4 light years away.

a) In the Earth's frame of reference, planet Elvis suddenly turns blue when the ship is halfway there. In the ship's frame of reference, how far away is the ship from planet Elvis when the planet turns blue? Assume that planet Elvis is stationary in the frame of the Earth.

b) How much do the people on board the ship age between the time planet Elvis turns blue (in their frame) and the time they reach the planet?

Problem 3

At $t = 0$ in the Earth's frame, astronomers spot a small asteroid 1 light year away (in the Earth's frame of reference) traveling directly towards the Earth at $4/5c$. To prevent certain doom for all humanity, they immediately launch a missile, which travels at 3/5 c towards the rocket. Their plan is to have the missile explode just as it passes the asteroid. To achieve this, the missile is equipped with a timer, such that the missile will explode when the time runs out.

a) What should the timer on the missile be set to (you may wish to solve parts b and c first, though this is not necessary)?

b) If we call the direction towards the asteroid the $\hat{x}$ direction, and define the Earth to be at $x = 0$, what is the trajectory $x(t)$ of the asteroid in the frame of the Earth?

c) What is the trajectory $x'(t')$ of the asteroid in the frame of the missile? At what t' does the asteroid reach $x' = 0$? Does this agree with your answer for a (if you have already done it)?

d) What is the velocity of the missile in the asteroid's frame of reference?

Problem 4

Shaquille O'Neal's new car (the Shaqmobile) is equipped with a powerful laser that sends pulses of light directly upwards in the frame of reference of his car (he can use it to project an image of himself on the clouds above). One evening, Shaq is driving his car at three fifths the speed of light, when he sees Kobe Bryant standing by the side of the road. Just as he passes Kobe, he sends up a pulse of light. The clouds are 1 km high that night, and the pulse hits the clouds directly above Steve Nash (also standing beside the same road).

b) How far apart are Kobe and Steve Nash from each other?

c) What are the horizontal and vertical component's of the light's velocity in Kobe's frame of reference?