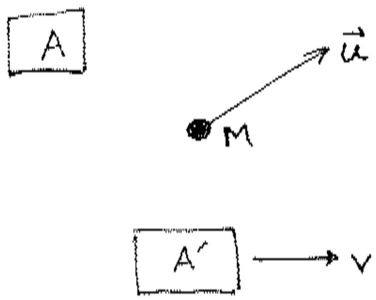
A note about Lorentz transforms for energy & momentum.

When using the formulae

$$E' = \gamma (E - v p_x)$$
$$p_x' = \gamma \left(p_x - \frac{v}{c^2} E\right)$$
$$p_y' = p_y$$
$$p_z' = p_z$$

it is very important to understand what v and γ refer to. We use these formulae to determine the energy and momentum of an object in some new frame moving at velocity v in the $\hat{x}$ direction of some original frame where the same object has energy E and momentum $\vec{p}$.



For example, in the picture above, we might want to know the energy and momentum of the object of mass M as observed by the observer A'. To do this, we should use the formulae at the top, plugging in E and $\vec{p}$ as observed for the observer A in the original frame, and using v and $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ as the velocity and γ of the observer A'. Note that v and γ here <u>have nothing to do with the object of mass M</u>. They are always just defined based on the velocity of the new frame in the original frame. This is also true for the ordinary Lorentz transform for coordinates and for velocities (where the velocity of the object in question is usually denoted by $\vec{u}$ in the original frame and $\vec{u}'$ in the new frame).

In the example above, we need to calculate E and $\vec{p}$ in the original frame in order to find E' and $\vec{p}'$. As usual, we have:

$$E = \gamma_{\vec{u}} m c^2$$

$$\vec{p} = \gamma_{\vec{u}} m \vec{u}$$

Here, we have another γ appearing, but since we're now calculating the properties of a particular object in a particular frame, γ is calculated using the velocity of that object. So here, we have

$$\gamma_{\vec{u}} = \frac{1}{\sqrt{1 - \frac{\vec{u}^2}{c^2}}}$$

which has nothing to do with v.

SUMMARY:
- When calculating energies & momenta of objects in a given frame, γ refers to the velocity of the object
- When doing Lorentz transformations, the γ & v in the Lorentz transformation formulae refer to the velocity of the new frame relative to the old frame