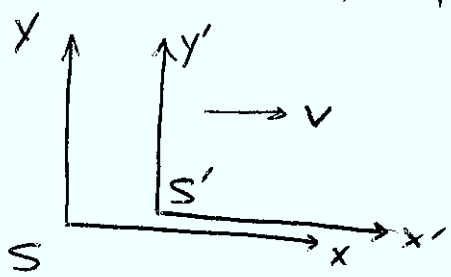
LAST TIME: Lorentz transform

$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$
$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$

v: velocity of frame S' relative to frame S



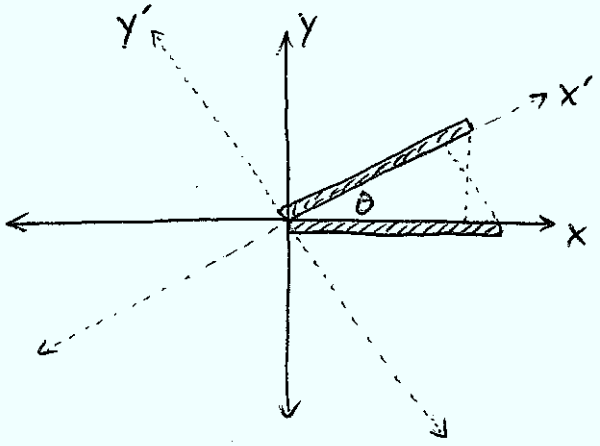
velocity tform:

$$u_x' = \frac{u_x - v}{1 - \frac{u_x v}{c^2}}$$

Observers don't agree on: time intervals, distances, simultaneity, before, after

Are there any "absolute" quantities that everyone can agree on?

DEMO: rotated coordinate systems



→ each observer sees other's meter stick to be shorter in their own x direction

→ silly to compare only x-distances: these depend on arbitrary choice of coordinate axes

Should measure Δx and Δy, will find

$$(\Delta x)^2 + (\Delta y)^2 = (\Delta x')^2 + (\Delta y')^2 = \text{LENGTH}^2$$

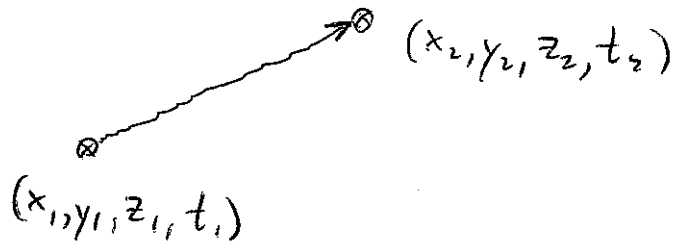
Length is INVARIANT under rotations ∴ sensible thing for two observers with rotated coords. to compare

$$x' = \cos\theta x + \sin\theta y$$
$$y' = \cos\theta y - \sin\theta x$$

↖ L.T. similar but rotates x,t.

Try to find something invariant under Lorentz transformations.

Hint: observers agree on speed of light.



For any two events, define

$$I = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - c^2(\Delta t)^2$$

$I = 0$ if light goes from (x_1, y_1, z_1) at t_1 to (x_2, y_2, z_2) at t_2 → all observers agree $I=0$ for these 2 events

More generally: for ANY two events, I has the same value in all reference frames.

CHECK: $(\Delta x')^2 + (\Delta y')^2 + (\Delta z')^2 - c^2(\Delta t')^2$ plug in from L.T.

$$= (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - c^2(\Delta t)^2$$

I = "INVARIANT INTERVAL"

What does it mean?