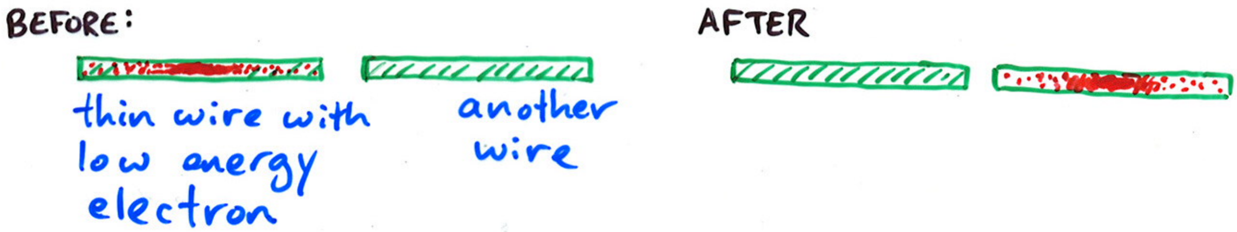
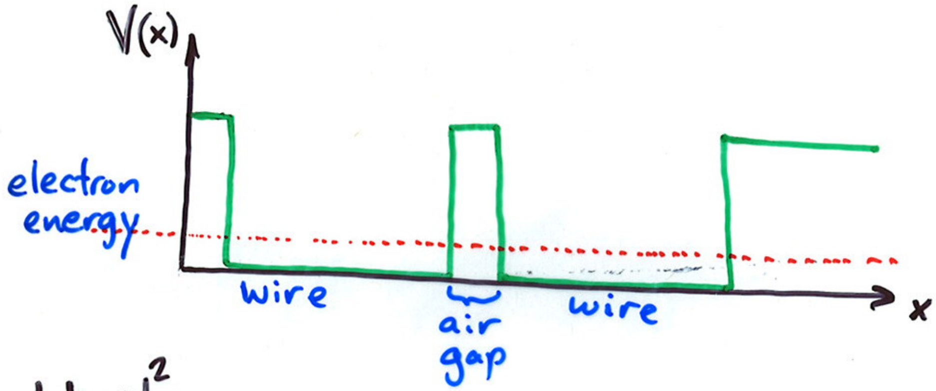
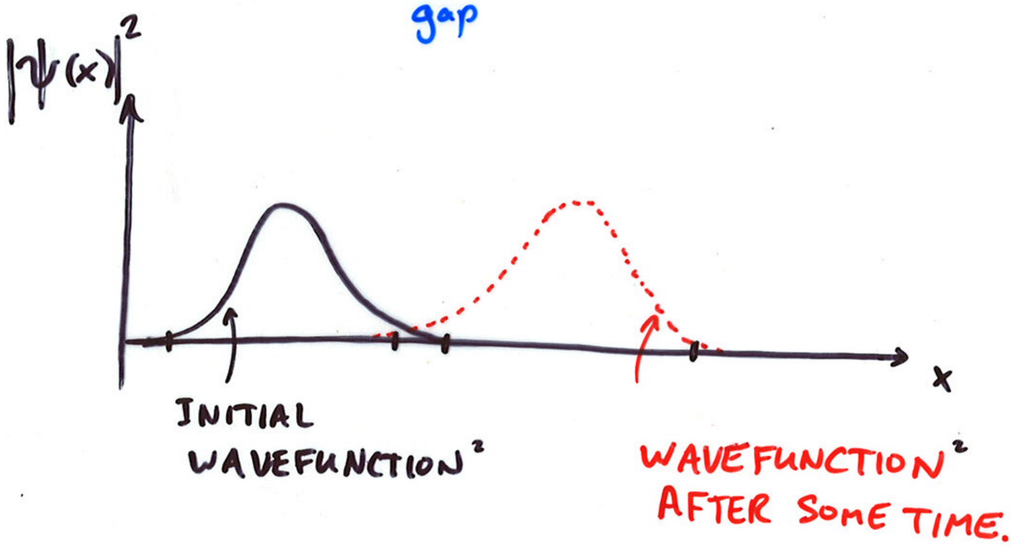
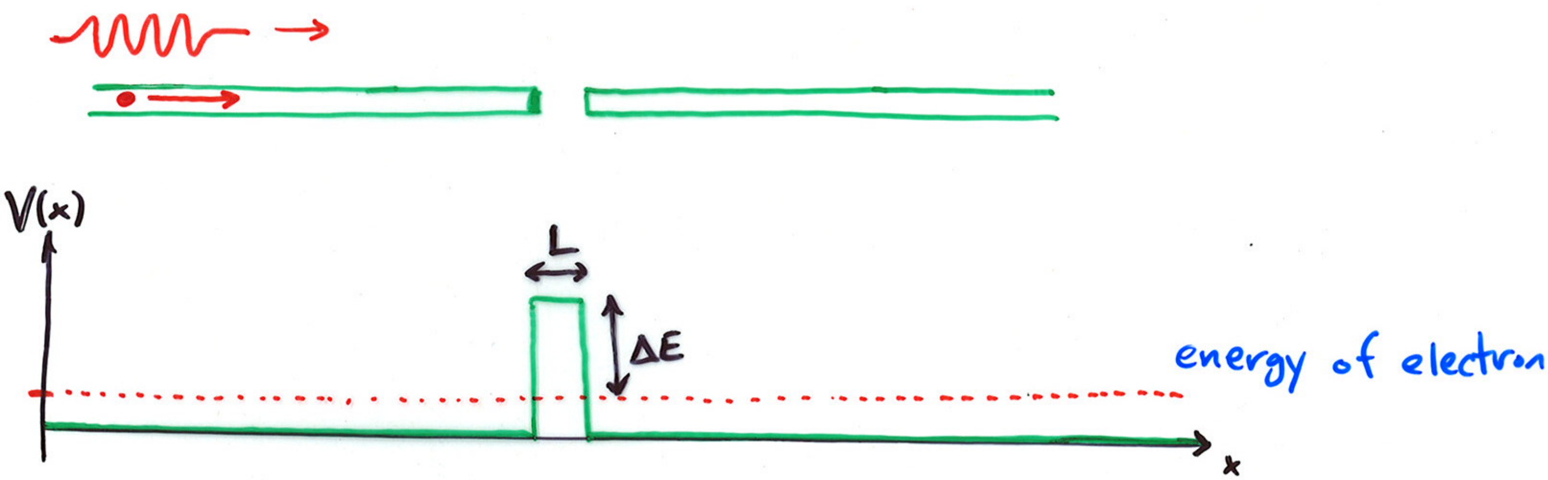
LAST TIME: Tunneling







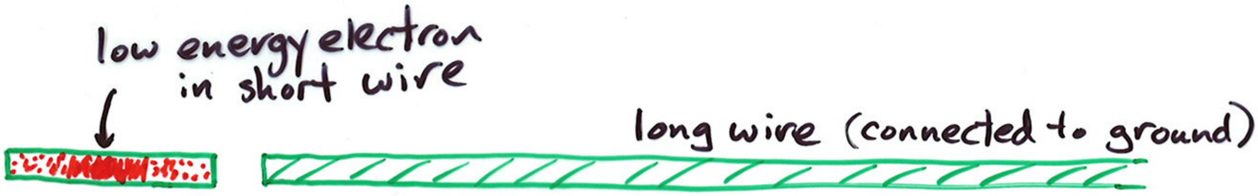
MORE TUNNELING (QUANTITATIVE)

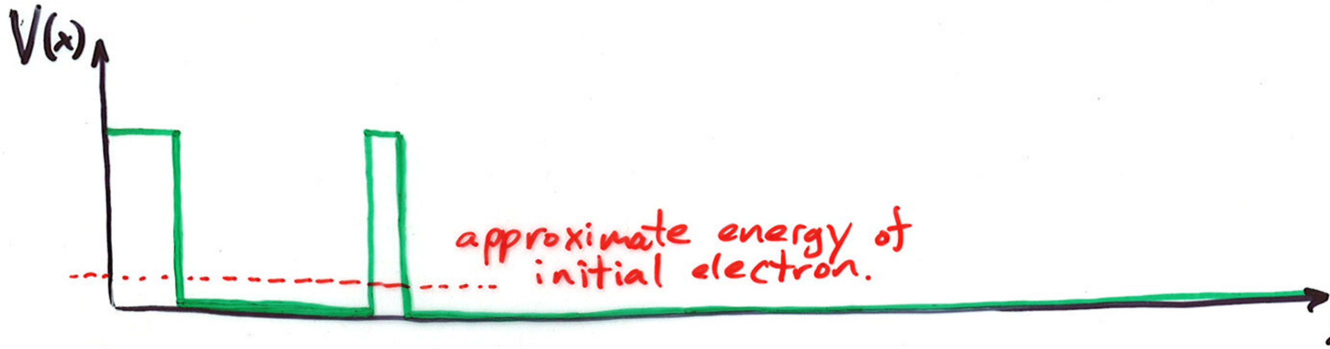


Probability of tunneling

$$P \approx e^{-2\sqrt{2}\,\frac{L\cdot\sqrt{m\Delta E}}{\hbar}}$$

e.g. $\Delta E \sim$ few eV $\Rightarrow L \sim$ fraction of 1nm for significant probability





What is the probability of finding the electron in the short wire as a function of time?

ANSWER:

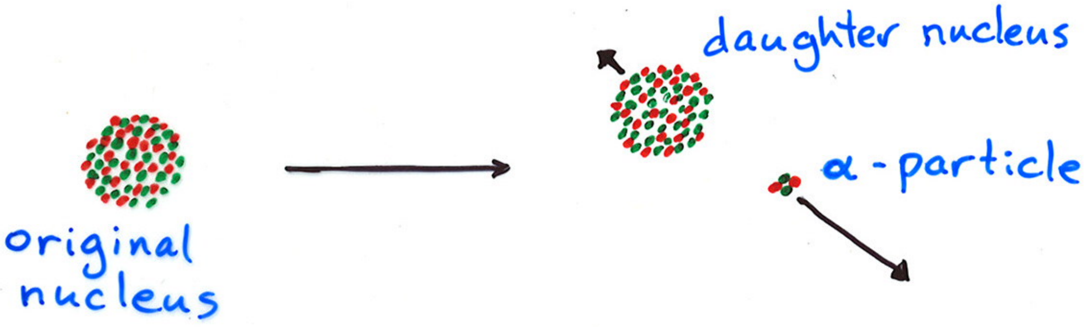
It decays exponentially $P \propto e^{-t/\tau}$

τ = "lifetime" of electron in short wire

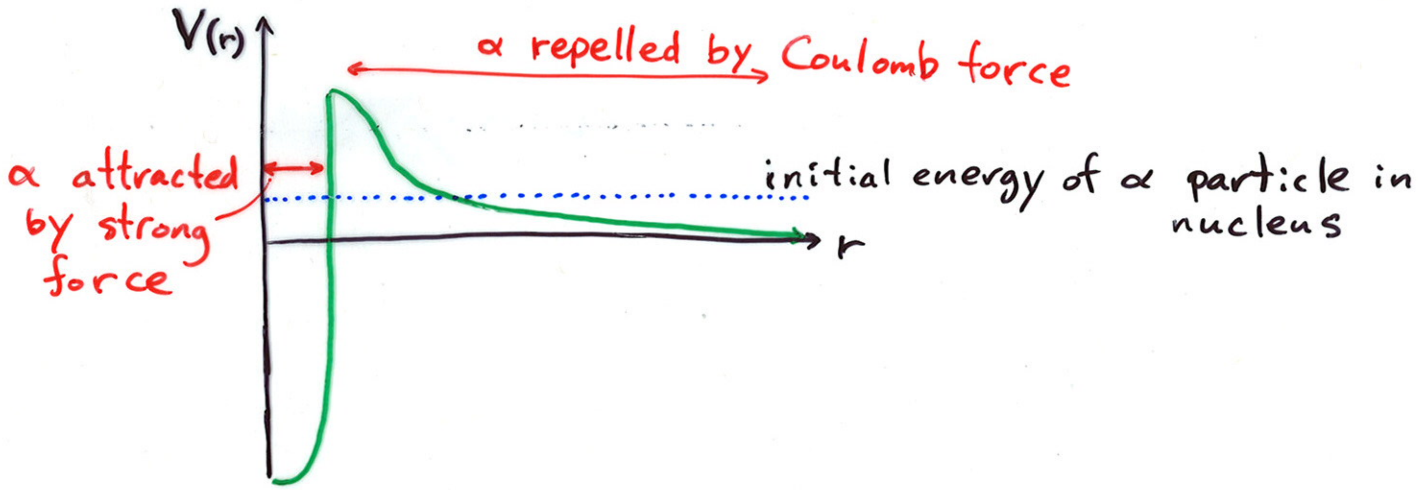
$\propto$ inverse tunneling prob. through barrier

$$\propto e^{2\sqrt{2}\,\frac{L\cdot\sqrt{m\,\Delta E}}{\hbar}}$$

RADIOACTIVE α-DECAY

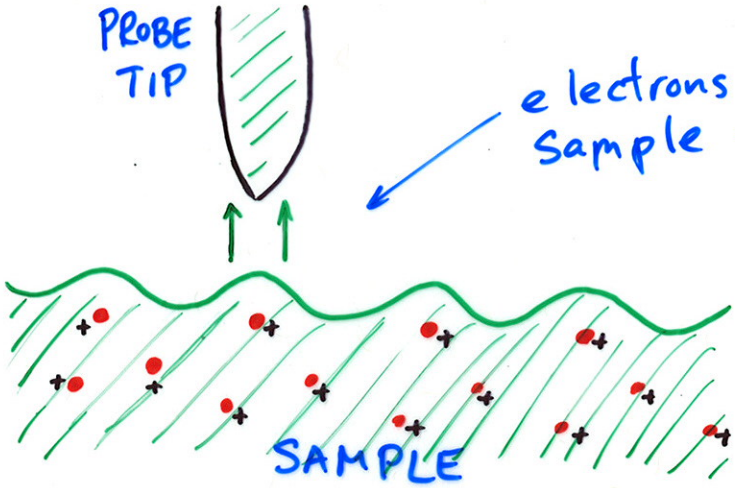


Potential for α particle:



Lifetime of nucleus = Average time for α-particle to tunnel through barrier

SCANNING-TUNNELING MICROSCOPES

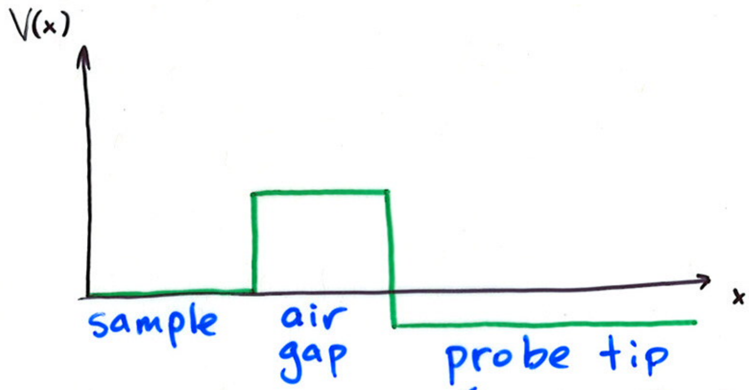


electrons can tunnel from sample to probe tip

current $\propto$ rate of tunneling

$\propto e^{-a\cdot(\text{SEPARATION})}$

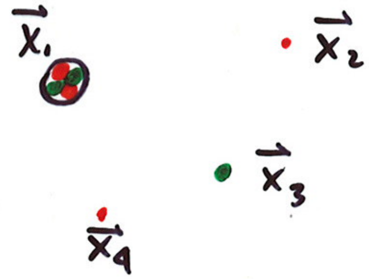
Move tip horizontally across surface + measure current to deduce height profile



(make V slightly <0 so net current of electrons into probe).

GENERAL QUANTUM SYSTEMS

Classical picture:



Quantum: superposition of classical configurations

wavefunction: $\psi(\vec{x}_1, \vec{x}_2, \dots, \vec{x}_N, t)$

General Schrödinger equation

$$
\begin{aligned}
i\hbar \frac{\partial \psi}{\partial t} = & -\frac{\hbar^2}{2m_1}\left(\frac{\partial^2\psi}{\partial x_1^2} + \frac{\partial^2\psi}{\partial y_1^2} + \frac{\partial^2\psi}{\partial z_1^2}\right) - \dots \\
& -\frac{\hbar^2}{2m_N}\left(\frac{\partial^2\psi}{\partial x_N^2} + \frac{\partial^2\psi}{\partial y_N^2} + \frac{\partial^2\psi}{\partial z_N^2}\right) \\
& + V(\vec{x}_1, \dots, \vec{x}_N)\,\psi
\end{aligned}
$$

energy of classical configuration

THE ENERGY SPECTRUM

Central problem: compute energies of bound states (always discrete)

ATOM

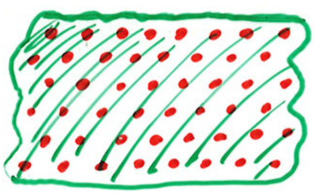
MOLECULE

→ determines emission & absorption spectra

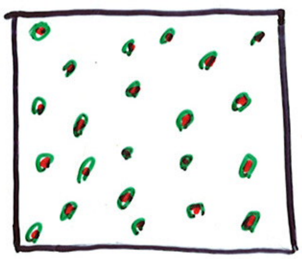
NUCLEI

→ determines nuclear masses & stability

SOLIDS



GASES



→ determines conductivity properties, heat capacity, equations of state, all of thermodynamics

} STATISTICAL MECHANICS & CONDENSED MATTER PHYSICS

TIME INDEPENDENT SCHRÖDINGER EQUATION

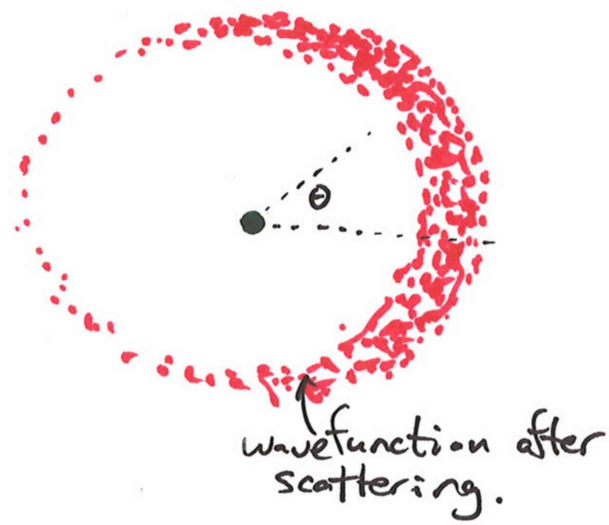
TIME DEPENDENT PROBLEMS

Evolution of wavefunction allows us to predict probabilities for various outcomes based on initial setup

e.g.

BEFORE:

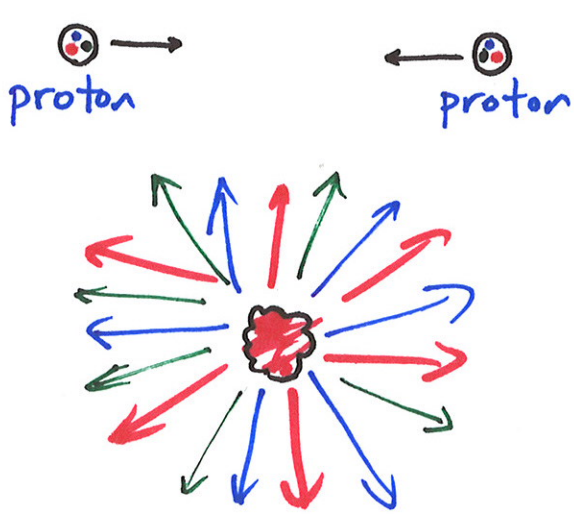
AFTER:



What is $P(\theta)$?

what is probability of absorption? of ejecting electron?

LHC:



What is prob. for various outcomes given some model of nature?

roughly: choice of $U(\vec{x}_1, \vec{x}_2, \ldots)$

TEST EXPERIMENTALLY TO VERIFY/DISPROVE MODEL

THE RULES OF QUANTUM MECHANICS

① For any states ψ_1 and ψ_2, $z_1\psi_1 + z_2\psi_2$ is also a state

② For any physical quantity (x, p, E, etc...) there are special states with definite values for this quantity (EIGENSTATES). Any other state can be written as a superposition of the eigenstates.

③ If we make a measurement of some physical quantity, the state will behave like (& turn into) one of the eigenstates for that quantity. The probability for the various outcomes is determined by the squared magnitude of the coefficients in the superposition.

④ Time evolution of energy eigenstates is

$$\psi \rightarrow e^{-i\frac{E}{\hbar}t}\psi$$

Time evolution for any other state can be determined by writing that state as a superposition of energy eigenstates.

HOLD TRUE in Quantum Mechanics
Quantum Field Theory
String Theory

BASIC FRAMEWORK FOR FUNDAMENTAL PHYSICS