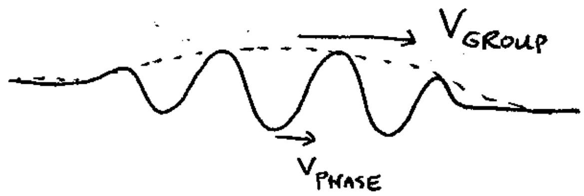
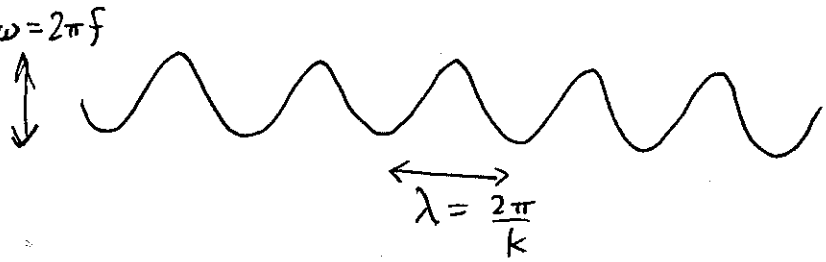
LAST TIME: given $\phi(x,t=0)$ what is $\phi(x,t)$?



- wavepackets have a phase velocity and a group velocity
- these are determined by relation between frequency + wavelength for pure waves: $\omega(k)$



$$V_{PHASE} = \frac{\omega}{k}$$

$$V_{GROUP} = \frac{\partial \omega}{\partial k}$$

evaluate at central value of λ

For electrons w. momentum p: want

① $\boxed{\lambda = \frac{h}{p}}$

② group velocity $= \frac{p}{m}$ $\Rightarrow$ $\boxed{hf = \frac{p^2}{2m} = E_{kin}}$

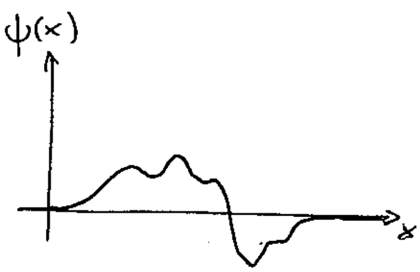
∴ Wavefunction for pure waves (momentum eigenstates):

$$\boxed{\psi(x,t) = e^{i\frac{2\pi}{h}\left(px - \frac{p^2}{2m}t\right)}}$$

SIM

General wavefunctions:

suppose wavefn is $\phi(x)$ at $t=0$



what is ϕ at later time?

Write ψ as superposition of pure waves:

$$\psi(x) = \frac{1}{\sqrt{h}} \int_{-\infty}^{\infty} dp \, A(p) \, e^{i \frac{2\pi p}{h} x}$$

each pure wave evolves independently according to our results above.

At later time t:

$$\psi(x,t) = \frac{1}{\sqrt{h}} \int_{-\infty}^{\infty} dp \, A(p) \, e^{i \frac{2\pi}{h}\left(px - \frac{p^2}{2m} t\right)}$$

have completely solved problem of time dependence.

$\boxed{\text{SIM}} \rightarrow$ results capture spreading of wavepackets with time (DISPERSION) predicted by range of momenta in superposition

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More convenient way to describe time dependence: the Schrödinger equation.

$$\frac{\partial \psi}{\partial t} = i \frac{h}{4\pi m} \frac{\partial^2 \psi}{\partial t^2}$$

check: ① both sides equal to $-i \frac{\pi p^2}{mh} \psi$ for $\psi = e^{i \frac{2\pi}{h}\left(px - \frac{p^2}{2m} t\right)}$

② if ψ_1 and ψ_2 satisfy S.E., so does $a\psi_1 + b\psi_2$

What does S.E. tell us?

recall: $\dfrac{\partial \psi}{\partial t} = \dfrac{\psi(x,t+\delta t) - \psi(x,t)}{\delta t}$ for $\delta t \to 0$

$$\text{S.E.} \Rightarrow \underset{\substack{\uparrow \\ \text{wavefn. at} \\ \text{slightly later time}}}{\psi(x, t+\delta t)} = \underbrace{\psi(x,t) + \delta t \cdot \left(i \frac{h}{4\pi m} \frac{\partial^2 \psi}{\partial x^2} \right)}_{\substack{\text{properties of } \psi \text{ at} \\ \text{time } t \text{, position } x}}$$

S.E. determines time evolution based on initial wavefunction