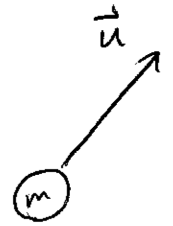
LAST TIME:



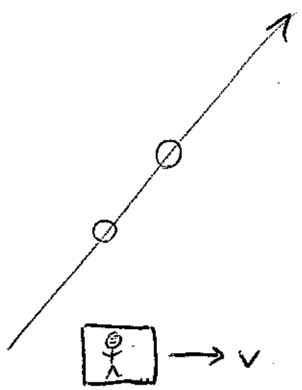
Momentum: $\boxed{\vec{p} = \gamma_{\vec{u}} m \vec{u}} = m \frac{d\vec{x}}{d\tau}$

in any frame, sum of $\vec{p}$ for all objects same before + after collision.

TODAY: Energy: $\boxed{E = \gamma m c^2} = m c^2 \frac{dt}{d\tau}$

Lorentz tforms mix $\vec{x}$ and t

$\therefore$ will mix $m \frac{d\vec{x}}{d\tau}$ with $m \frac{dt}{d\tau}$



$$\begin{aligned} dz' &= dz \\ dy' &= dy \\ dx' &= \gamma_v (dx - v\,dt) \\ dt' &= \gamma_v \left(dt - \tfrac{v}{c^2} dx\right) \end{aligned} \xrightarrow{\times m \cdot \frac{1}{d\tau}} \begin{aligned} p_z' &= p_z \\ p_y' &= p_y \\ p_x' &= \gamma_v \left(p_x - v\left(\tfrac{E}{c^2}\right)\right) \\ E' &= \gamma_v (E - v\, p_x) \end{aligned}$$

$$p_x' = \gamma_v \left(p_x - \frac{v}{c^2} E\right)$$

conserved — conserved — must be conserved.

Small $\vec{u}$:

$$E = \frac{mc^2}{\sqrt{1 - u^2/c^2}} \approx mc^2 + \frac{1}{2} m u^2 + \dots$$

mass $\times\, c^2$ — kinetic energy

classical: - mass always conserved
- kinetic energy only conserved in elastic collisions.

relativity: $E = \underbrace{mc^2}_{\text{MASS ENERGY}} + \underbrace{(\gamma - 1)mc^2}_{\text{RELATIVISTIC KINETIC ENERGY}}$ Conserved in ALL processes.

BUT: can convert mass $\longleftrightarrow$ kinetic energy

$\boxed{\text{CLICKER.}}$

e.g. nuclear reactions

$$^{212}Po \longrightarrow {}^{208}Pb + \alpha$$

$$\underset{M}{(84p,\ 128n)} \longrightarrow \underset{m_1}{(82p,\ 126n)} + \underset{m_2}{(2p,\ 2n)} \qquad M > m_1 + m_2$$

$(\Delta M)c^2 \approx 1.4 \times 10^{-12}\,J \longrightarrow$ goes into kinetic energy of products.

$\longrightarrow$ enormous if multiplied by Avogadro's #.

Analyze via cons. of energy + momentum:

BEFORE:

(M)

Momentum cons:

$$0 = -\gamma_1 m_1 v_1 + \gamma_2 m_2 v_2$$

AFTER:

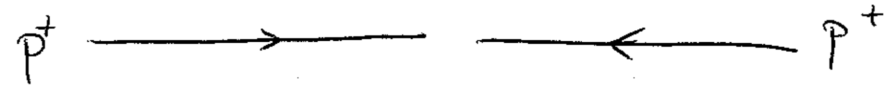
$v_1 \longleftarrow (m_1) \quad (m_2) \longrightarrow v_2$

Energy cons:

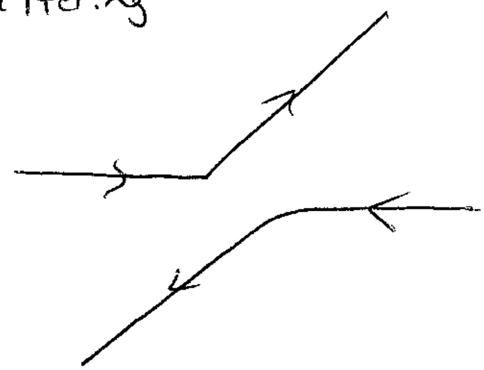
$$Mc^2 = \gamma_1 m_1 c^2 + \gamma_2 m_2 c^2$$

- Can solve for v_1 + v_2

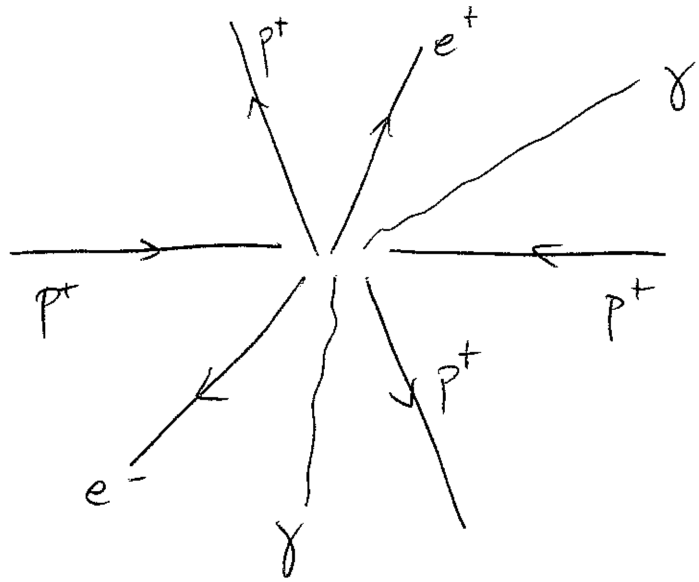
Another example: particle collisions



low E: scattering



high E:



can produce new particles from kinetic energy!

LHC: each proton has $E \approx 7000$ times its rest energy

$\longrightarrow$ could produce 14000 protons/antiprotons.