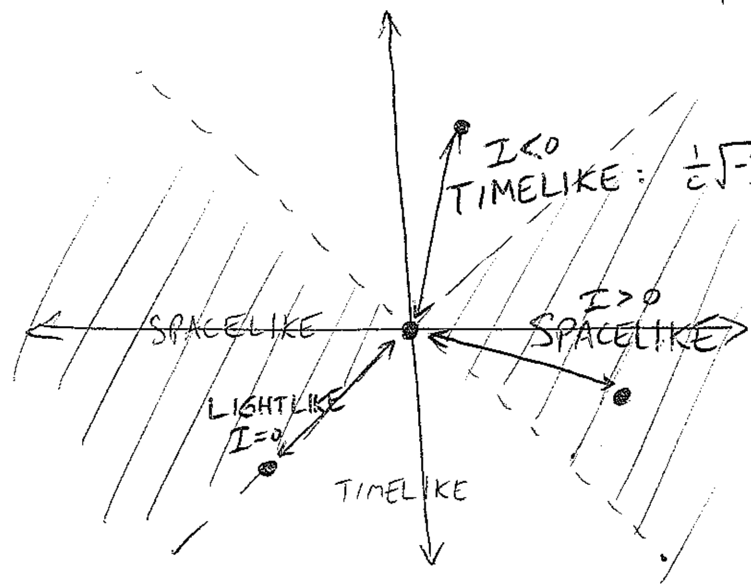
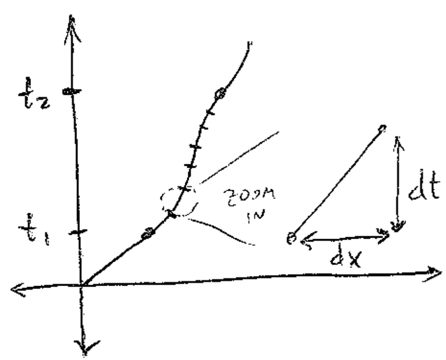
LAST TIME: $I = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - c^2(\Delta t)^2 \equiv -ds^2$ (BOOK)



TIMELIKE: $\frac{1}{c}\sqrt{-I}$ = proper time between events $\equiv \tau$

SPACELIKE: $\sqrt{I}$ = proper distance between events

example:



Time elapsed for observer on trajectory $x(t)$:

break up into segments of approx. constant velocity

$$d\tau = \frac{1}{c}\sqrt{-I}$$

$$= \frac{1}{c}\sqrt{c^2 dt^2 - dx^2}$$

$$= dt\sqrt{1 - \frac{1}{c^2}\left(\frac{dx}{dt}\right)^2}$$

velocity at time t.

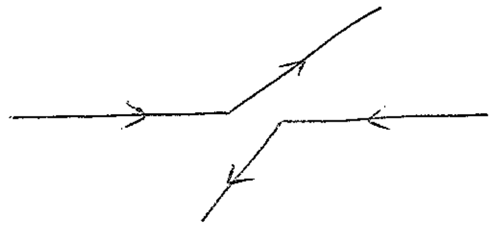
Total time elapsed

$$\tau = \int d\tau = \int_{t_1}^{t_2} dt\sqrt{1 - \frac{1}{c^2}\left(\frac{dx}{dt}\right)^2}$$

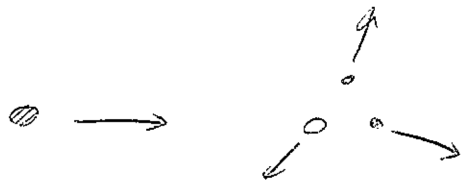
Always less than $t_1 - t_2$

RELATIVISTIC DYNAMICS: want to analyze dynamical processes involving large velocities.

e.g. particle scattering



particle decays



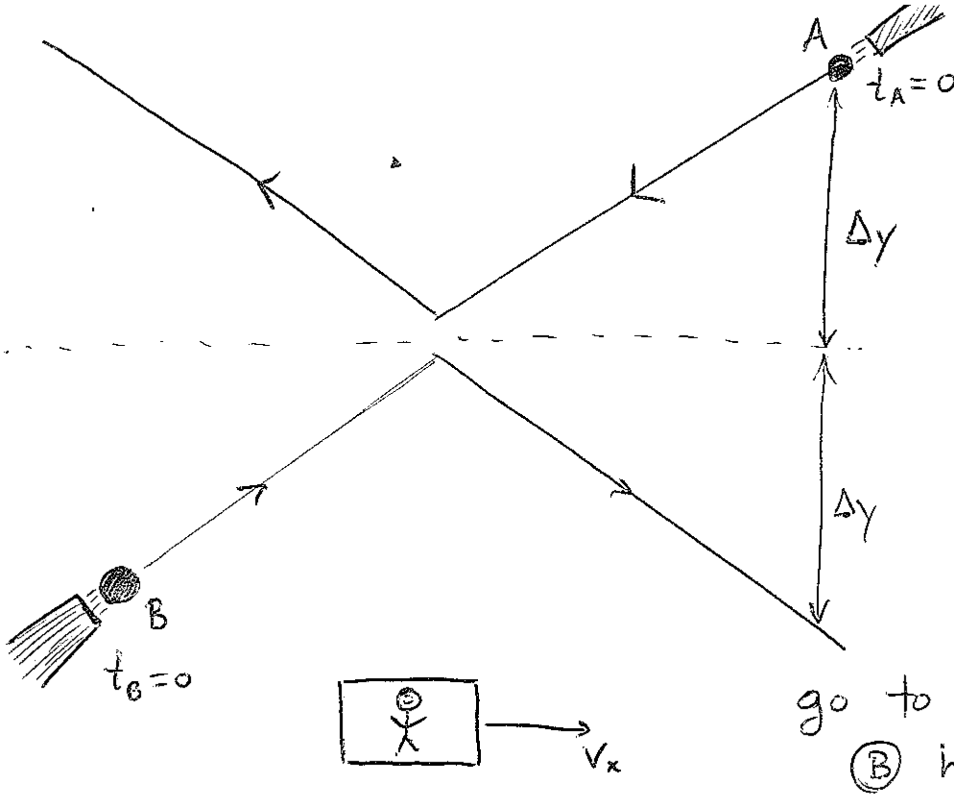
Ordinary velocities: cons. of energy & momentum crucial

MOMENTUM: $\vec{P}_{TOT} = \sum m_i \vec{v}_i$ conserved (i.e. same before & after) in all collisions.

KINETIC ENERGY: $E_{kin} = \sum \frac{1}{2} m_i v_i^2$ conserved in elastic collisions

MASS: $M_{TOT} = \sum m_i$ conserved in all collisions

What if velocities are large?

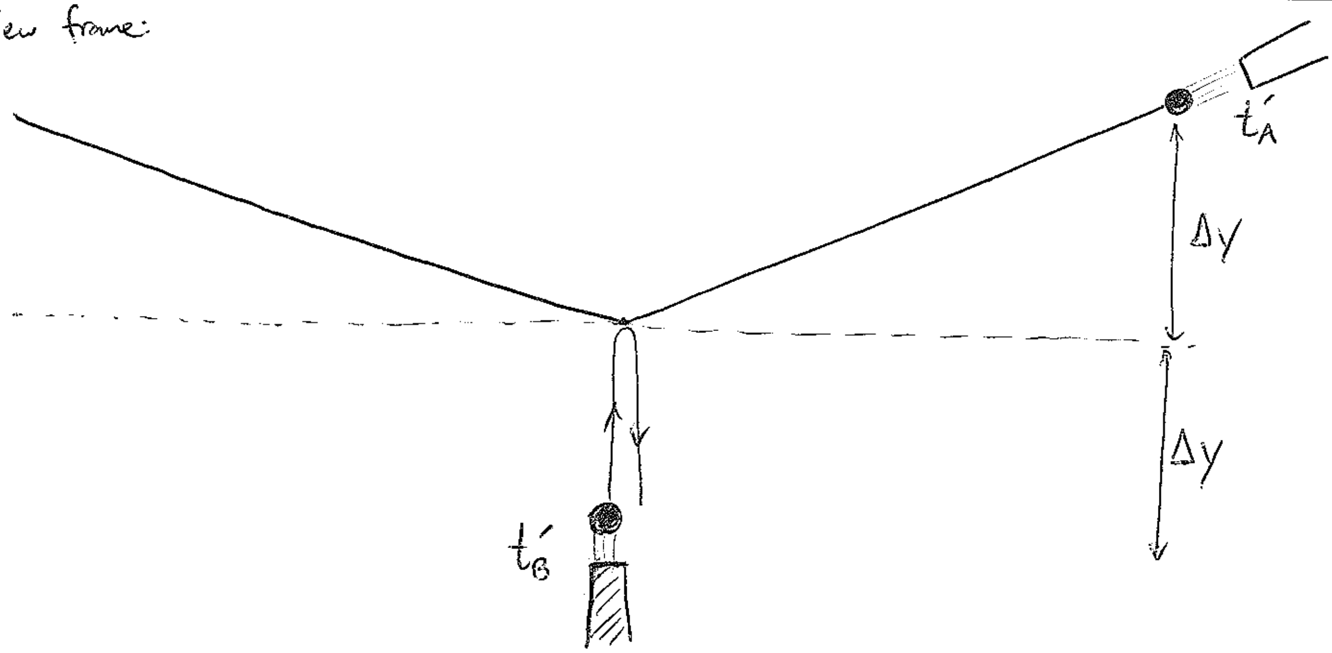


assume
A, B equal mass, speed

$\vec{P}_{before} = \vec{P}_{after} = 0.$

go to frame where
B has no x-velocity

New frame:



CLICKER

New frame: cannon A fires BEFORE cannon B

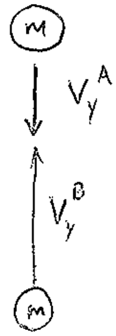
$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

$t_A = t_B$ $\quad$ $x_A > x_B$ $\quad \therefore t'_A < t'_B$

$\therefore$ B travels same Δy in less time: $|v_y^B| > |v_y^A|$

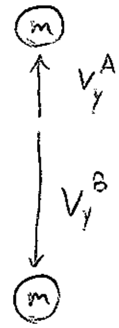
Continued next time...
or see next page.

BEFORE COLLISION:



$mv_y^A + mv_y^B > 0$

AFTER COLLISION:



$mv_y^A + mv_y^B < 0$

y momentum not conserved with ordinary formula $p_y = m\dfrac{\Delta y}{\Delta t}$

PROBLEM: $\Delta t_A, \Delta t_B$ same in one frame, different in other.

SOLUTION: define $p_y = m\dfrac{\Delta y}{\Delta \tau}$ ← same in all frames, equals to Δt for small v

Generally RELATIVISTIC MOMENTUM $\vec{p} = m\dfrac{d\vec{x}}{d\tau} = \gamma m \vec{v}$