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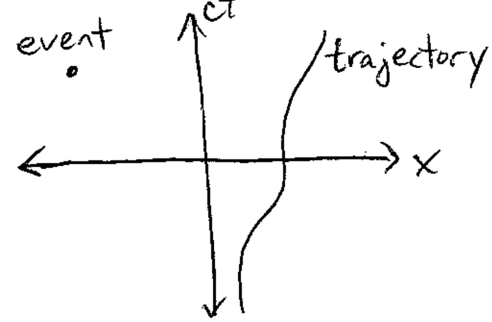
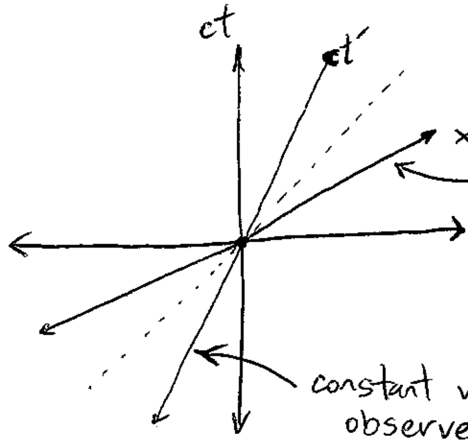
LAST TIME: Spacetime diagrams





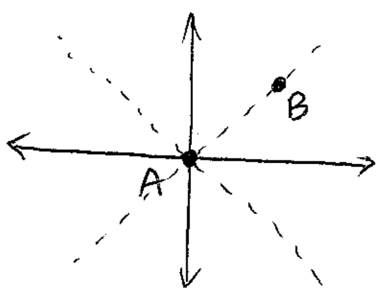
set of events where $t'=0$ = x' axis slope $\frac{v}{c}$

constant velocity observer = set of points where $x'=0$ = t' axis slope $\frac{c}{v}$

examples from tutorial

Use to understand invariant interval

$$I = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - c^2(\Delta t)^2$$



CASE 1: $I = 0$: $|\Delta \vec{x}| = c\,|\Delta t|$

2 events separated by light ray

"LIGHTLIKE SEPARATION"

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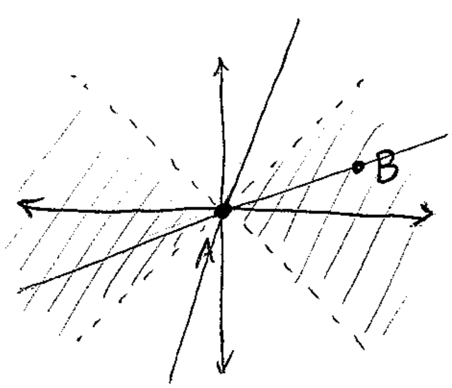
CASE 2: $I > 0$: A & B simultaneous in some frame

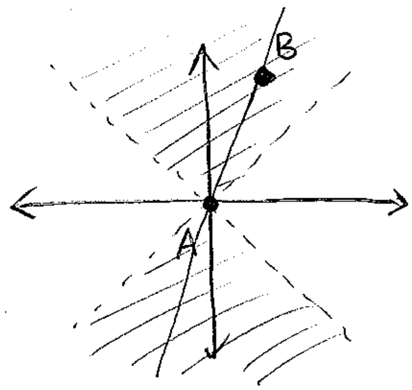


in this frame:

$$I = (\Delta x')^2 + (\Delta y')^2 + (\Delta z')^2 = \text{Distance}^2$$

$\sqrt{I}$ is the distance between A & B in frame where they are simultaneous = PROPER DISTANCE

A & B have SPACELIKE SEPARATION.



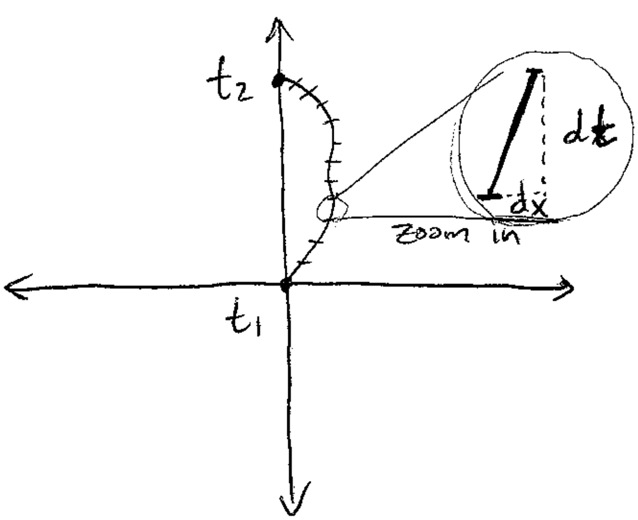
CASE 3: $I < 0$: There is some const. velocity observer who sees both events at same location.

- For this observer: $I = -c^2(\Delta t')^2$
- Define $\tau = \Delta t' = \sqrt{\frac{|I|}{c^2}}$ PROPER TIME

= amount of time between 2 events in frame where they are at same place.

- A + B have ~~[illegible]~~ TIMELIKE SEPARATION
- all observers see B after A.

Example: how much time elapses on a spaceship with trajectory $x(t)$?



- Break up trip into segments of approx const velocity

$$d\tau = \sqrt{\frac{|I|}{c^2}}$$

$$= \sqrt{\frac{c^2 dt^2 - dx^2}{c^2}}$$

$$= dt\sqrt{1 - \frac{1}{c^2}\left(\frac{dx}{dt}\right)^2}$$

velocity at time t

Total time elapsed

$$\tau = \int d\tau = \int_{t_1}^{t_2} dt \sqrt{1 - \frac{1}{c^2}\left(\frac{dx}{dt}\right)^2} \quad \text{Always} < t_2 - t_1$$