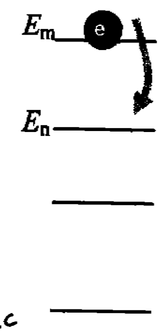
Part I of Phys 200 Exam.
circle your answers in this booklet.

Modern Physics: Quantum Mechanics
(16 questions)

1. The diagram at right shows the electronic energy levels in an atom with an electron at energy level E_m. When this electron moves from energy level E_m to E_n, light is emitted. The greater the energy difference between the electronic energy levels E_m and E_n …



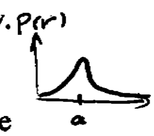
   A. …the more photons emitted.
   B. …the brighter (higher intensity) the light emitted.
   C. …the longer the wavelength (the more red) of the light emitted.
   D. …the shorter the wavelength (the more blue) of the light emitted.
   E. More than one of the above answers is correct.

   $\Delta E = hf = \frac{hc}{\lambda}$ ∴ larger $\Delta E \Rightarrow$ smaller λ

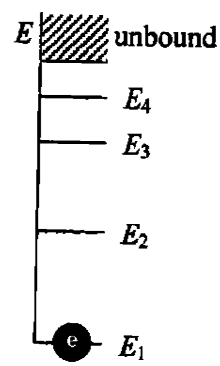
2. The electron in a hydrogen atom is in its ground state. You measure the distance of the electron from the nucleus. What will be the result of this measurement?

   A. You will measure the distance to be the Bohr radius.
   B. You could measure any distance between zero and infinity with equal probability.
   C. You are most likely to measure the distance to be the Bohr radius, but there is a range of other distances that you could possibly measure.
   D. There is an equal probability of finding the electron at any distance within a range from a little bit less than the Bohr radius to a little bit more than the Bohr radius.

   from HW:



3. An electron in an atom has the energy level diagram at right. The electron is in its lowest energy state, as shown in the diagram. What is the lowest energy photon that it can absorb?



   A. It can absorb a photon of any arbitrarily small energy.
   B. E_1
   C. E_2
   D. $E_2 - E_1$
   E. $E_4 - E_3$

4. True or False: In the absence of external forces, electrons move along sinusoidal paths.

   A. True
   B. False

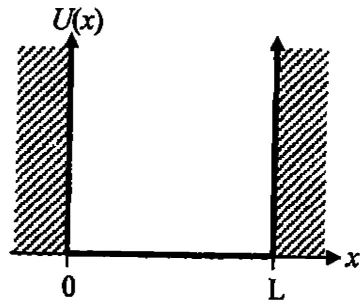
   the wavefunction for a free electron with some p has sinusoidal real & imaginary parts, but these are not related to the path that the electron takes.

5. You see an electron and a neutron moving by you at the <u>same speed</u>. How do their wavelengths λ compare?

A. $\lambda_{neutron} > \lambda_{electron}$
B. $\lambda_{neutron} < \lambda_{electron}$
C. $\lambda_{neutron} = \lambda_{electron}$

$p = mv$: grater for neutron
$\therefore \lambda = \frac{h}{p}$ smaller for neutron

6. Consider an electron with the potential energy
$$U(x) = \begin{cases} 0 & 0 < x < L \\ \infty & x < 0 \;\; or \;\; x > L \end{cases}.$$



This potential energy function, plotted at right, is often referred to as an infinite square well or a rigid box. Your electron is in the lowest energy state of this potential energy, with a wave function $\psi(x) = \psi_1(x)$ and a corresponding energy E_1. Suppose you first measure the position of this electron very precisely, without destroying the electron. *After* measuring the position, you measure the *energy* of the same electron. Which of the following statements describes the result of this energy measurement?

A. The value that you measure will be E_1.
B. The value that you measure could possibly be E_1.
C. The value that you measure will *not* be E_1.

measure x
↳ get position eigenstate

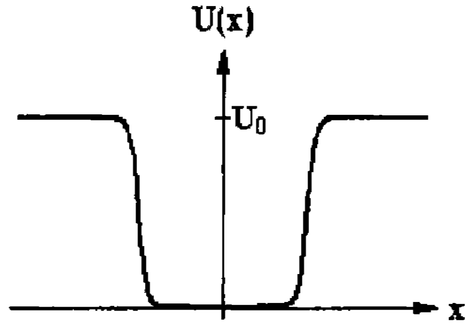

=
sum of energy eigenstates.

7. The total energy of an electron after it tunnels through a potential energy barrier is…

A. …greater than its energy before tunneling.
B. …equal to its energy before tunneling.
C. …less than its energy before tunneling.

energy is conserved

For questions 8-9, consider a particle with the one-dimensional potential energy function plotted at right and total energy E. The value of potential energy remains the same as $x \to +\infty$ or $x \to -\infty$.



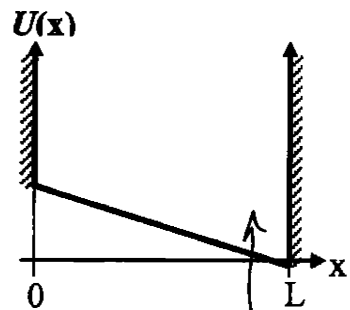
8. Are there any allowed values of the particle's total energy E with $E < U_0$, and if so, are all values allowed, or only a discrete set of energy values?

   A. There are no allowed values of energy in this range.
   B. Only certain discrete values of energy in this range are allowed.
   C. All values of energy in this range are allowed.

9. Are there any allowed values of the particle's total energy E with $E > U_0$, and if so, are all values allowed, or only a discrete set of energy values?

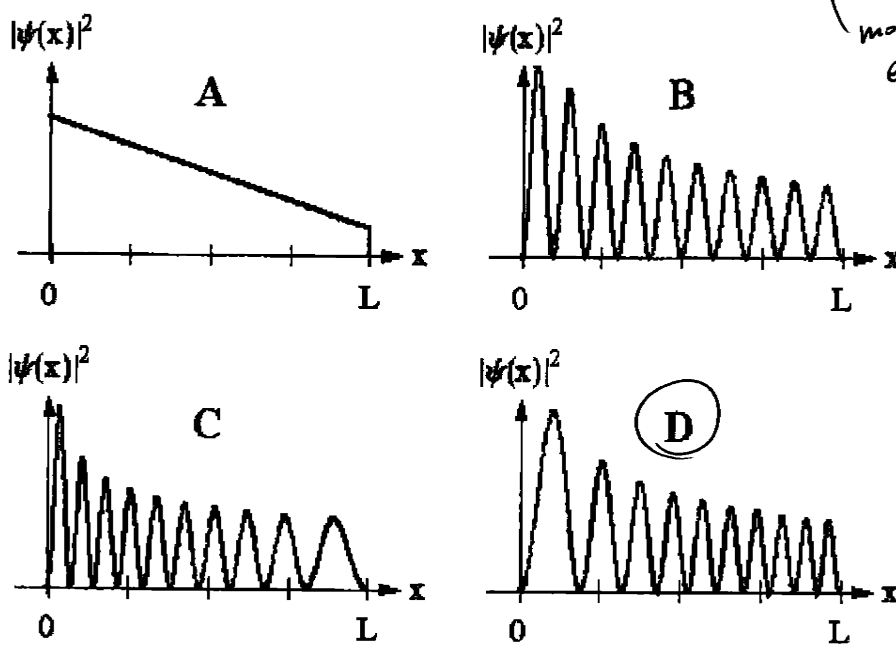
   A. There are no allowed values of energy in this range.
   B. Only certain discrete values of energy in this range are allowed.
   C. All values of energy in this range are allowed.

10. The figure at right shows a potential energy function $U(x)$, where the potential energy is infinite if x is less than 0 or greater than L, and a slanted bottom in between 0 and L, so that the potential well is deeper on the right than on the left. Which of the plots of $|\psi(x)|^2$ vs. x is most likely to correspond to a stationary state of this potential well?



has

more kinetic energy in this region ∴ smaller wavelength

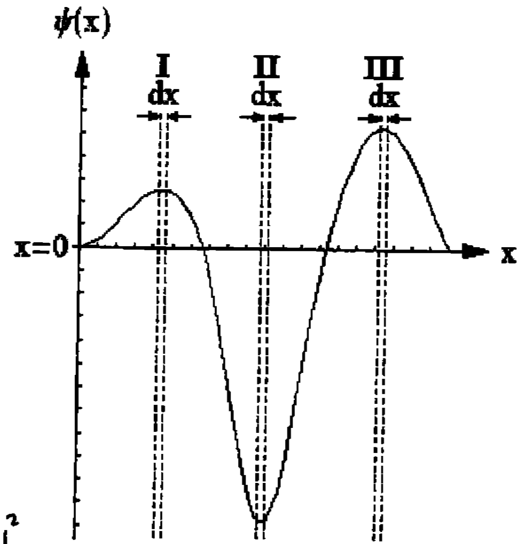


(Also: electron spends more time where U is larger (because it's traveling slower)

∴ P is greater here ∴ $|\psi|^2$ larger)

E. More than one of these is a possible stationary state.

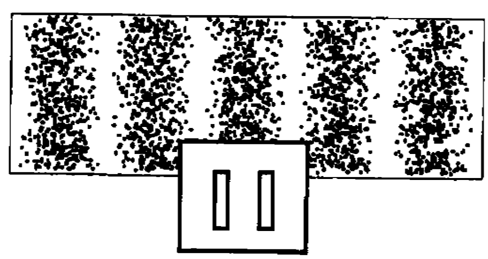
11. The plot at right shows a snapshot of the spatial part of a one-dimensional wave function for a particle, $\psi(x)$, versus x. $\psi(x)$ is purely real. The labels, I, II, and III, indicate regions in which measurements of the position of the particle can be made. Order the probabilities, P, of finding the particle in regions I, II, and III, from biggest to smallest.



A. P(III) > P(I) > P(II)
B. P(II) > P(I) > P(III)
C. P(III) > P(II) > P(I)
D. P(I) > P(II) > P(III)
E. P(II) > P(III) > P(I)

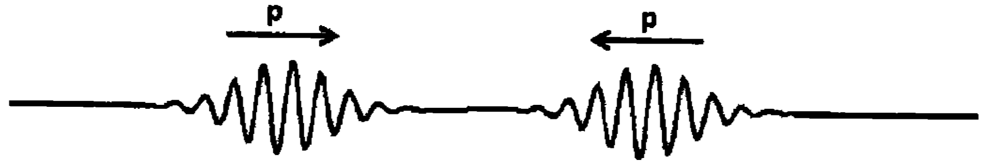
$P \propto |\psi(x)|^2$

12. You shoot a beam of photons through a pair of slits at a screen. The beam is so weak that the photons arrive at the screen one at a time, but eventually they build up an interference pattern, as shown in the picture at right. What can you say about which slit any particular photon went through?



A. Each photon went through either the left slit or the right slit. If we had a good enough detector, we could determine which one without changing the interference pattern.
B. Each photon went through either the left slit or the right slit, but it is fundamentally impossible to determine which one.
C. Each photon went through both slits. If we had a good enough detector, we could measure a photon in both places at once.
D. Each photon went through both slits. If we had a good enough detector, we could measure a photon going through one slit or the other, but this would destroy the interference pattern.
E. It is impossible to determine whether the photon went through one slit or both.

13.

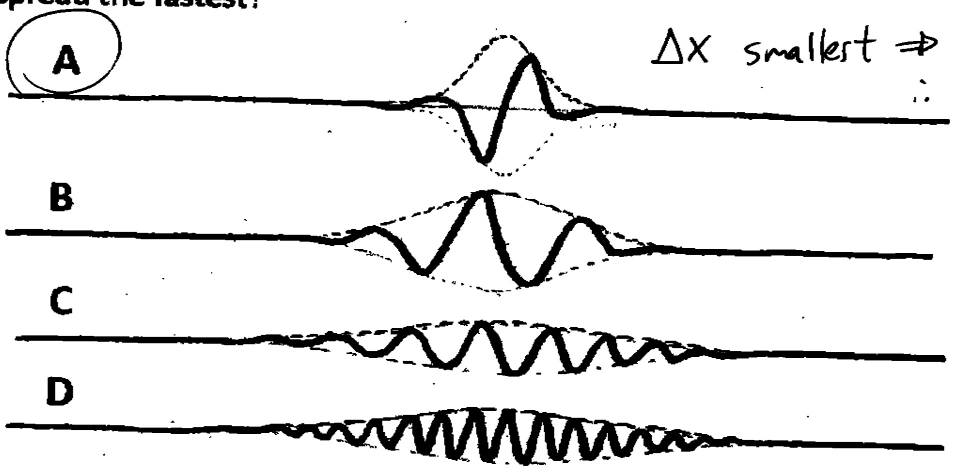


10^6 electrons are initially prepared in the same quantum state -- each electron having the same wavefunction $\Psi(x, t=0)$. The real part of $\Psi(x, t=0)$ is shown above.
In your view, which of the following statement provides the best interpretation for this quantum state?

(A) The velocity of each electron is zero
(B) Half of all the electrons are traveling to the <u>left</u> while the other half are traveling to the <u>right</u>.
(C) Each electron spends half of its time traveling to the right and half of its time traveling to the left
(D) Each electron is traveling right and left at the same time
(E) The wavefunction $\Psi(x, t=0)$ does not represent a physically realizable state

ψ for each electron represents a quantum superposition. ∴ velocity ~~was~~ does not have a definite value.

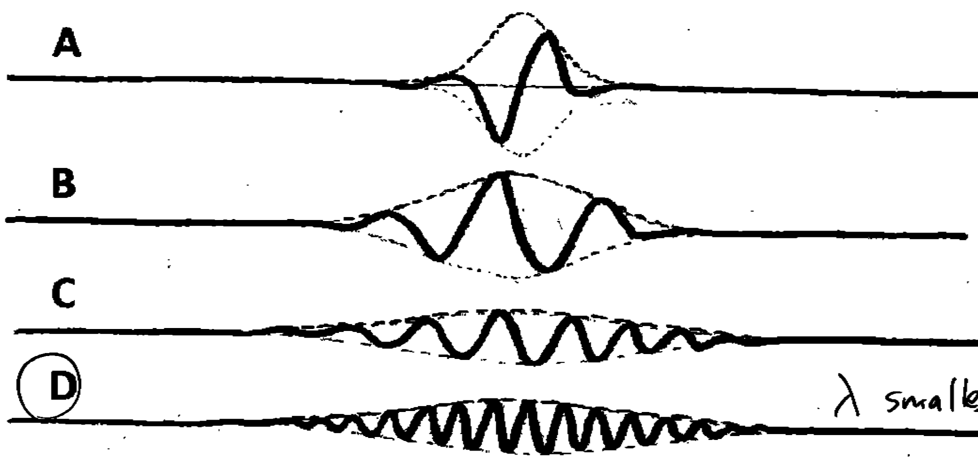
14.
At t=0, 4 electrons are prepared in a different quantum state. All four electrons move in the positive x-direction. The real part of each electron wavefunction is shown below.
In the absence of external forces, which electron wavefunction will **spread the fastest**?



Δx smallest ⇒ Δp largest ∴ largest range of velocities.

E All four electron wavefunctions will spread at the same rate.

15.
At t=0, 4 electrons are prepared in a different quantum state. All four electrons move in the positive x-direction. The real part of each electron wavefunction is shown below.
In the absence of external forces, which electron **travels the fastest?**



λ smallest $\Rightarrow p$ largest
$\Rightarrow$ largest v.

E All four electrons move to the right at the same speed.

16.
An electron can be initially prepared in one of the following quantum state:

(I) $\Psi_I(x, t=0) = \psi_1(x)$
(II) $\Psi_{II}(x, t=0) = 1/\sqrt{2}[\psi_1(x) + \psi_2(x)]$
(III) $\Psi_{III}(x, t=0) = \psi_2(x)$

, where $\psi_1(x)$ and $\psi_2(x)$ are the wavefunctions for the bound states with the lowest energy E_1 and E_2 in the infinite square well.

Which of the three quantum states above will result in a <u>time-dependant</u> probability density? [recall: probability density is defined as $|\Psi(x,t)|^2$]

energy eigenstate
$\Rightarrow$ time-indep. prob. density.

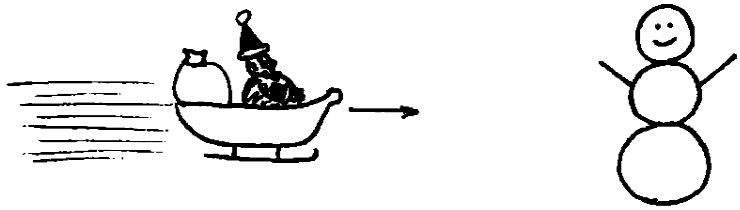
(A) $\Psi_I(x, t=0)$

(B) $\Psi_{II}(x, t=0)$ → not an energy eigenstate

(C) $\Psi_{III}(x, t=0)$

(D) Both $\Psi_{II}(x, t=0)$ and $\Psi_{III}(x, t=0)$

(E) All three quantum states will result in a <u>time dependent</u> probability density



1) Santa Claus is travelling in his hyper-sleigh at velocity $v = \sqrt{3}/4$ c. Which of the pictures below best represents the proportions of Frosty the Snowman as measured by Santa?

A)

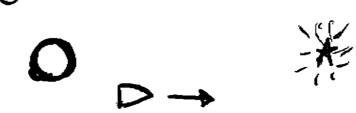
Santa measures length contraction in direction of relative motion.

C)

B)

D)

2) A distant supernova explodes just as the Physics 200 exam begins. In the frame of reference of a rocket travelling towards the supernova,



A) the supernova explodes after the Physics 200 exam begins.

$t' = \gamma(t - \frac{v}{c^2}x)$

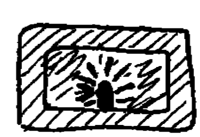
B) the supernova explodes before the Physics 200 exam begins.

(larger x, same t) $\Rightarrow$ smaller t'

C) the supernova explodes at the same time as the Physics 200 exam begins.

3) Inside a space probe, a tank of hydrogen gas explodes, heating up all the air inside the probe. Assuming there is no radiation emitted from the space probe during this time, we can say that

A) The total mass of the probe is greater after the explosion

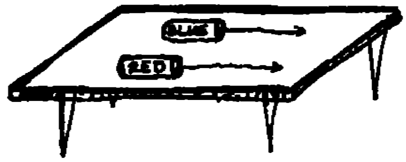


B) The total mass of the probe is less after the explosion

C) The total mass of the probe is the same after the explosion

Mass $= \frac{1}{c^2} \times$ (total energy of system) ← conserved

↑ since no net velocity



4) A blue laser and a red laser, with an identical total power of 1mW, sit motionless on a frictionless table. The lasers are each turned on for one minute and then turned off again. We can say that

A) The two lasers have emitted the same number of photons.

B) The blue laser has emitted more photons.

C) The red laser has emitted more photons.

5) After the lasers in problem 3 are turned off, we can say that

(assume the lasers have equal mass)

A) the blue laser will be moving faster.

B) the red laser will be moving faster.

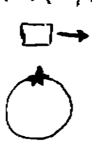
C) both lasers will be moving at the same (nonzero) velocity.

D) the lasers will move while the beams are on but both lasers will be motionless after they are turned off.

6) In order to make the GPS (global positioning system) work correctly, engineers make the clocks on the orbiting satellites run at a different rate from clocks on Earth so that they appear to be ticking at the same rate as the Earth clocks. In order for this to work, the engineers should make the orbiting clocks in orbit run

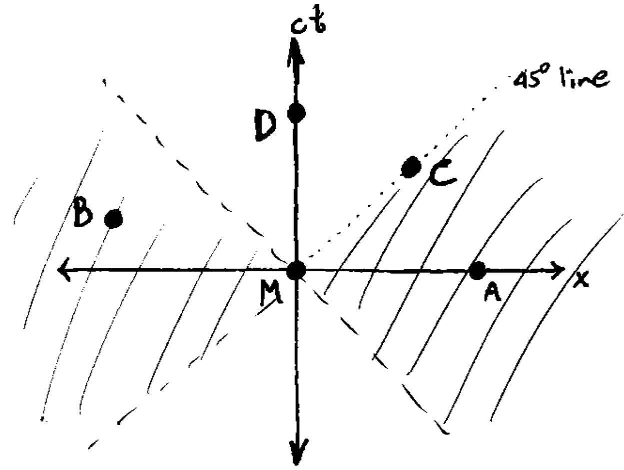
A) Faster than the clocks on Earth

observed ticking is slower than actual ticking, so need actual rate faster if observed rate is the same as for Earth clocks



B) Slower than the clocks on Earth

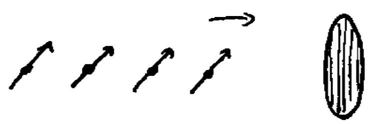
(You should ignore gravitational time dilation for this question, though in reality it cannot be ignored!)



7) In the spacetime diagram above, which event is simultaneous with the event M in some frame of reference?

any event in shaded region is spacelike separated from M.

A) A B) B C) C D) D E) Both A and B



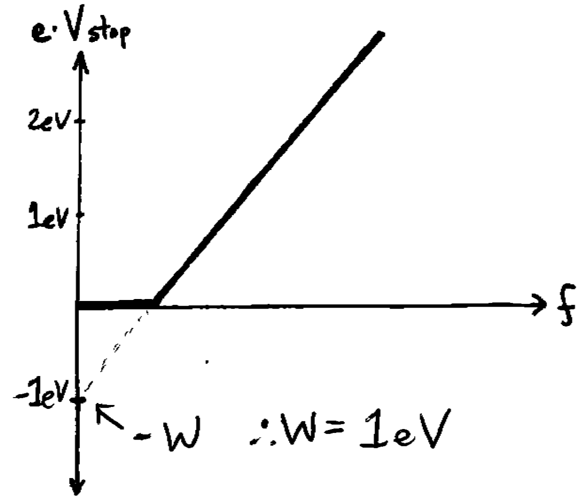
8) Four photons polarized at 45° are incident on a vertical polarizer as shown. Which of the pictures below represents the most likely outcome of this experiment? (i.e. the most likely of the ones shown)

A)  Polarization after is aligned w. polarizer.

B) 

C) 

D) None of these is possible



$$eV_{stop} = hf - W$$

9) The graph above shows the stopping voltage plotted against frequency in a photoelectric effect experiment. For the metal used in the experiment, what is the minimum amount of energy required to remove an electron?

A) -2 eV B) -1 eV C) 0 eV D) 1eV E) 2 eV

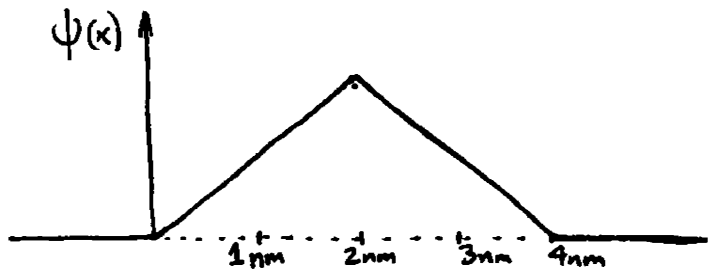
(10) Describe the photoelectric effect and explain why it provides evidence for the photon picture of light. Answer as concisely as possible.
(4 points)

The photoelectric effect describes the ejection of electrons from a metal (or other material) when light with a high enough frequency is used to illuminate it. The key observation is that if the frequency is too small, no electrons are ejected regardless of how high the intensity is. On the other hand, for frequencies high enough to eject electrons, electrons are ejected for any non-zero intensity.

These observations cannot be explained by the classical picture of light, since in that picture, the flow of energy into the metal is continuous and is exactly the same for any frequency as long as the intensity is the same.

On the other hand, the photon picture says that light of frequency f comes in discrete lumps of energy $E = hf$. If we assume that there is some minimum energy W required to eject an electron from the metal and that electrons are ejected by absorbing the energy from a single photon, then we must have $hf > W$ to eject an electron. This explains the minimum frequency that is observed in the experiment. Also, the photon picture explains why the intensity doesn't matter for whether or not electrons are ejected, since it is only the individual photons that are involved.

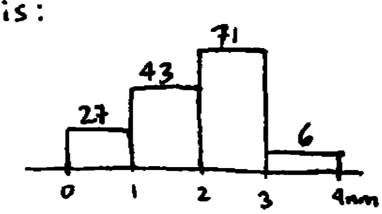
(A shorter answer than this could have received full credit)



(11) a) The wavefunction for an electron in a short wire is shown above. A team of physicists prepares 16,000 electrons in this state and then measures the position of each of the electrons. They make a histogram showing the number of electrons that they find in each of the four intervals [0nm,1nm], [1nm,2nm], [2nm,3nm], and [3nm,4nm]. Draw what you expect their histogram to look like and indicate the expected number in each bin. Show your calculations and explain your work.

(5 points)

your answer should look something like this:



The probability density for measuring an electron at x is $|\psi(x)|^2$. For the four intervals, we have:

$$P_{[0,1)} = \int_{0nm}^{1nm} |\psi(x)|^2$$

$$P_{[1,2]} = \int_{1nm}^{2nm} |\psi(x)|^2$$

And since the wavefunction is symmetrical $P_{[2,3]} = P_{[1,2]}$, $P_{[3,4]} = P_{[0,1]}$.

The total probability must be 1, and we can use this to normalize the wavefunction. In the region $0 \leq x \leq 2nm$, we have $\psi(x) = A \cdot x$ for some constant A. So:

$$P_{[0,1)} = \int_{0nm}^{1nm} A^2x^2 = \frac{1}{3}A^2x^3\Big|_{0nm}^{1nm} = \frac{1}{3}A^2\ (nm)^3$$

$$P_{[1,2]} = \int_{1nm}^{2nm} A^2x^2 = \frac{1}{3}A^2x^3\Big|_{1nm}^{2nm} = \frac{7}{3}A^2\ (nm)^3$$

Also $P_{[2,3]} = \frac{7}{3}A^2\ (nm)^3$, $P_{[3,4)} = \frac{1}{3}A^2\ (nm)^3$

More room for #11 a)

For the total probability to be 1, we need:

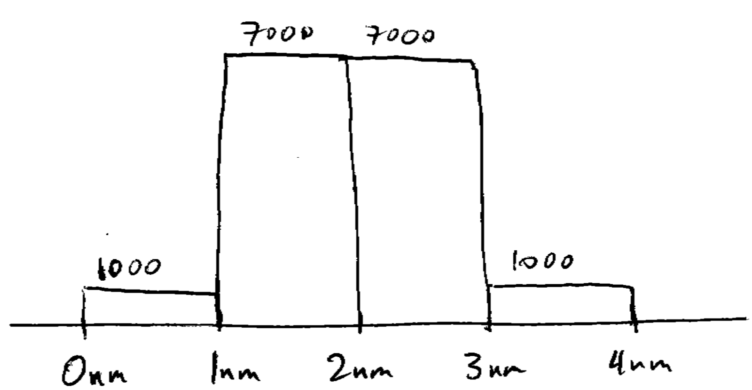
$$P_{[0,1)} + P_{[1,2]} + P_{[2,3]} + P_{(3,4]} = 1$$

$$\Rightarrow \frac{1}{3}A^2(nm)^3 + \frac{7}{3}A^2(nm)^3 + \frac{7}{3}A^2(nm)^3 + \frac{1}{3}A^2(nm)^3 = 1$$

$$\Rightarrow A^2 = \frac{3}{16}(nm)^{-3}$$

So: $P_{[0,1)} = P_{(3,4]} = \frac{1}{16}$ and $P_{[2,3]} = P_{[1,2]} = \frac{7}{16}$

If we ~~see~~ measure 16,000 electrons in this same initial state, we therefore expect around 1000 to be measured in the first and fourth regions (each) and 7000 in each of the second and third regions. Thus, the histogram should look like:



11 b) If the experimenters had measured the velocity for these electrons instead of the position, what can you say about the range of values they would likely find? *(I'm not expecting a precise answer here)* (2 points)

We can see from the graph that Δx (the position uncertainty) is around 2nm. Thus, the momentum uncertainty must obey

$$\Delta p \geq \frac{h}{4\pi} \cdot \frac{1}{2\,nm}$$

In terms of velocity, the range must be at least

$$\Delta v \geq \frac{h}{4\pi m} \cdot \frac{1}{2\,nm}.$$

In this case, the average value will be zero, since the wavefunction is completely symmetrical about (2nm) so doesn't distinguish between the $+x$ and $-x$ directions.

(Actually any real wavefunction is an equal combination of e^{ipx} and e^{-ipx} functions, so has zero average velocity).

(12)

In a radioactive gamma decay, a nucleus that is initially at rest decays by emitting a photon. The resulting nucleus is observed to have half the mass of the original nucleus. What is the speed of the new nucleus after the decay?

(5 points)

We use energy conservation & momentum conservation. Let the energy of the photon be E and the mass of the original nucleus be M. Then energy cons. gives:

$$\underset{E_{before}}{Mc^2} = \underbrace{\tfrac{1}{2}\gamma Mc^2 + E}_{E_{after}} \qquad (*)$$

Momentum conservation gives:

$$0 = -\tfrac{1}{2}\gamma Mv + \frac{E}{c}$$

$$\Rightarrow \quad E = \tfrac{1}{2}\gamma Mvc$$

Plugging this into $(*)$ gives:

$$Mc^2 = \tfrac{1}{2}\gamma Mc^2 + \tfrac{1}{2}\gamma Mvc$$

$$\Rightarrow \quad 1 = \tfrac{1}{2}\gamma + \tfrac{1}{2}\gamma\frac{v}{c}$$

$$\Rightarrow \sqrt{1-\frac{v^2}{c^2}} = \tfrac{1}{2}\left(1+\frac{v}{c}\right)$$

$$\Rightarrow \left(1+\frac{v}{c}\right)\left(1-\frac{v}{c}\right) = \tfrac{1}{2}\left(1+\frac{v}{c}\right)^2$$

$$\Rightarrow \left(1-\frac{v}{c}\right) = \tfrac{1}{2}\left(1+\frac{v}{c}\right)$$

$$\Rightarrow \frac{3}{2}\frac{v}{c} = \frac{1}{2}$$

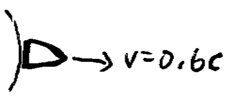
$$\Rightarrow \boxed{v = \tfrac{1}{3}c}$$

(13)

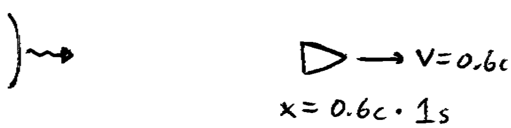
A rocket leaves the Earth at v=0.6c. After 1 second, a radio wave signal is sent towards the rocket from Earth.

a) If the rocket's clock is set to zero when it leaves the Earth, what time does the rocket's clock read when the signal is received? (3 points)

$t=0$:



$t=1s$:



In the Earth's frame, the time for the light signal to reach the rocket is:

$$\Delta t = \frac{\text{initial separation}}{\text{speed at which separation is changing}}$$

$$= \frac{0.6c \cdot 1s}{0.4c}$$

$$= \frac{3}{2}s$$

Thus, the light signal reaches the rocket at $t = \frac{5}{2}s$ in Earth's frame. The rocket's clock runs slow as observed by Earth, so the time on the rocket's clock when ~~the~~ the radio wave hits it is

$$t_{rocket} = \frac{1}{\gamma} \cdot \frac{5}{2}s = \frac{4}{5} \cdot \frac{5}{2}s = 2s$$

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or, using Lorentz transforms: $x' = 0$ for this event, so:
$t = \frac{5}{2}s$

$$t = \gamma\left(t' + \frac{v}{c^2} \cdot x'\right) \rightarrow t' = \frac{1}{\gamma}t = 2s.$$

b) In the rocket's frame of reference, at what time does the radio signal leave Earth? (2 points)

For this event: $x = 0$ and $t = 1s$. In the rocket's frame,

$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

$$= \frac{5}{4} \cdot 1s$$

$$\Rightarrow \boxed{t' = 1.25s}$$

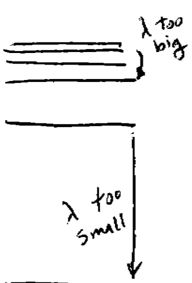
(14) The allowed energies for an electron in a hydrogen atom are given by $E_n = -13.6\ \text{eV} / n^2$. For a hot gas of hydrogen atoms, a discrete spectrum is observed, with light only at particular wavelengths. Determine the two longest wavelengths that are observed in the visible range (380nm to 750nm). (4 points)

The observed wavelengths in the emission spectrum are:

$$\frac{hc}{\lambda} = E_n - E_m$$

energy of emitted photon $E = hf = \frac{hc}{\lambda}$

difference in energy between 2 electron states.



So: $\frac{hc}{\lambda} = 13.6\text{eV}\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$ with $n > m$

$$\Rightarrow \lambda = \frac{hc}{13.6\text{eV}} \frac{1}{\frac{1}{m^2} - \frac{1}{n^2}}$$

$$= 91.2\text{nm} \times \frac{1}{\frac{1}{m^2} - \frac{1}{n^2}}$$

For $m = 1$, the possible wavelengths are all too small:

e.g. $\lambda_{2\to1} = 122\text{nm}$ and all other $\lambda_{n\to1}$ are smaller

For $m > 2$, the ~~[illegible]~~ wavelengths are all to large, since the smallest would be for $\lambda_{\infty\to3} = 820.8\text{nm}$.

So only $m = 2$ gives visible wavelengths. These are:

$\lambda_{3\to2} = 656\text{nm}$, $\lambda_{4\to2} = 486\text{nm}$, $\lambda_{5\to2} =$ smaller

So the two largest visible wavelengths are 486nm and ~~[illegible]~~ 656nm.