Physics 200 Problem Set 5

Problem 1

A black spider weighing 5 grams sits on a frictionless table. In order to move to the other side of the table, he turns on a voice activated 1mW laser to shine directly at him. If the laser is on for a whole day and then turned off, what is the velocity of the spider after the laser is turned off? (Hint: to do this problem exactly, we would have to take into account the fact that the spider's mass increases. However, to a very good accuracy, we can ignore this effect.)

Problem 2

Suppose that the Large Hadron Collider produces a new particle, initially at rest, that decays into a Z particle (mass $91 GeV/c^2$) and a neutrino (which we can take to have zero mass). If the Z particle is measured to have total energy $150 GeV$, what is the mass of the new particle?

By the general relation $E^2 = \vec{p}^2 c^2 + m^2 c^4$, a particle with zero mass has energy and momentum related by $E = |\vec{p}|c$. The formula $E = \gamma mc^2$, implies that the only way for a particle with zero mass to have finite energy is for γ to be infinite, so the particle must travel at the speed of light. For such particles, the velocity doesn't determine the energy and momentum (the formulae $E = \gamma mc^2$ and $p = \gamma mv$ aren't really useful for massless particles). We just have to independently say what the momentum of such a particle is, and then the energy is determined from the momentum by $E = |\vec{p}|c$.

Problem 3

One day, Marge and Homer are out walking their dog when they spot a UFO. The UFO is directly above them traveling directly downwards. When the UFO is only 100m above them, it abruptly turns and starts flying horizontally (parallel to the ground). As the UFO turns, its colour appears to change from green (wavelength 510nm) to orange (600nm). Assuming that the speed of the UFO is the same before and after it turns, how fast is the UFO going?

Problem 4

In this problem, we will estimate the lifetime of a "classical" Hydrogen atom. As we'll discuss in class, the classical model of an electron orbiting a proton is flawed, because accelerating charges radiate energy, so an orbiting electron would produce radiation, lose energy, and spiral into the nucleus.

a) For an electron in a circular orbit around a proton at a radius r, what is the acceleration as a function of r?

b) Using the result from part a), determine the velocity of the electron as a function of r (*remember circular motion?*).

c) Determine the total energy (kinetic plus potential) of the electron as a function of r.

d) Classical electromagnetism tells us that the accelerating charges radiate energy at a rate of

$$P = \frac{e^2 a^2}{6\pi\epsilon_0 c^3} \; .$$

This tells us how much energy the electron will lose per unit time. Using this, and your result for part c), derive an equation for dr/dt in terms of r (hint: you can use the chain rule to relate dr/dt to dE/dt and dE/dr). Assuming the initial of the orbit is $10^{-10} m$, divide the initial radius by $-dr/dt$ to estimate the lifetime of the atom, or solve the equation relating dr/dt to r to get a more precise answer.