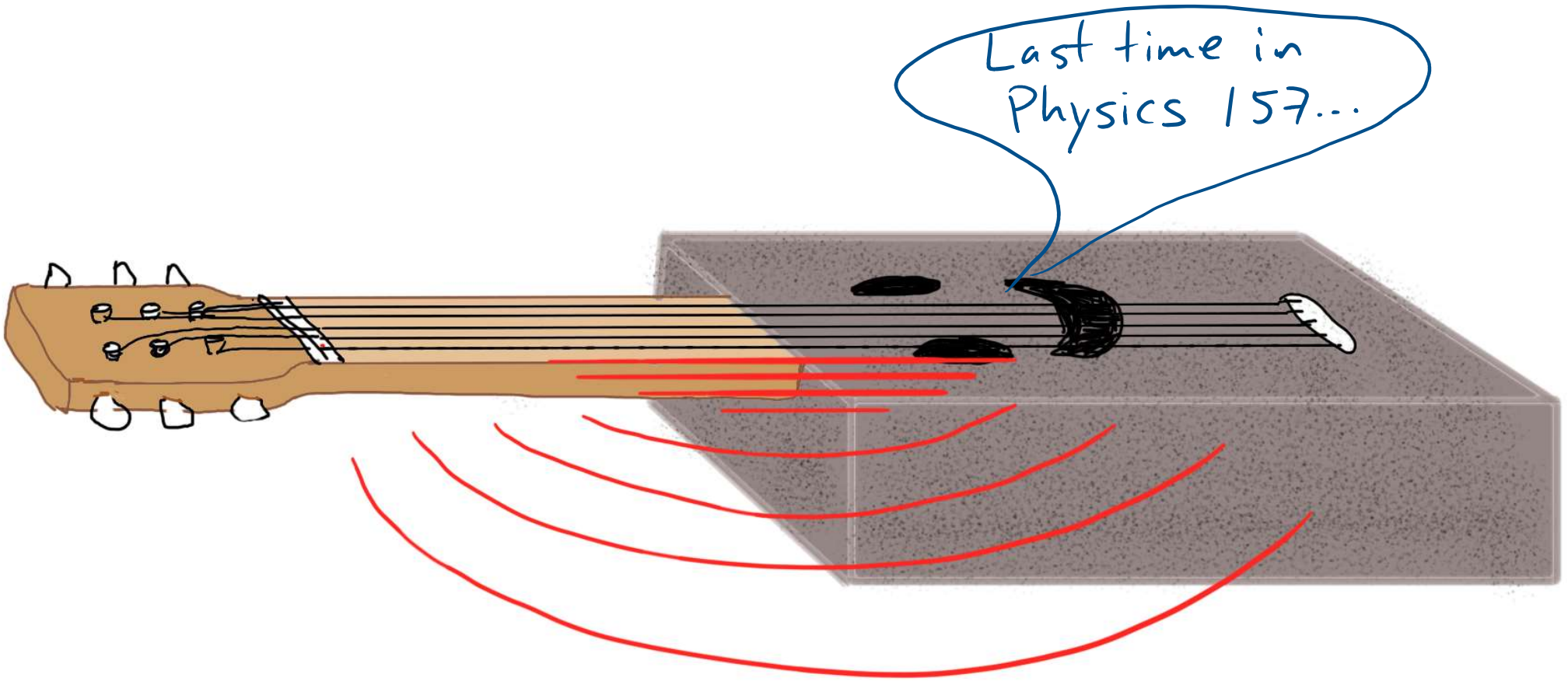
Office hours today: after class in Remo 12-12:30pm, 5-6pm

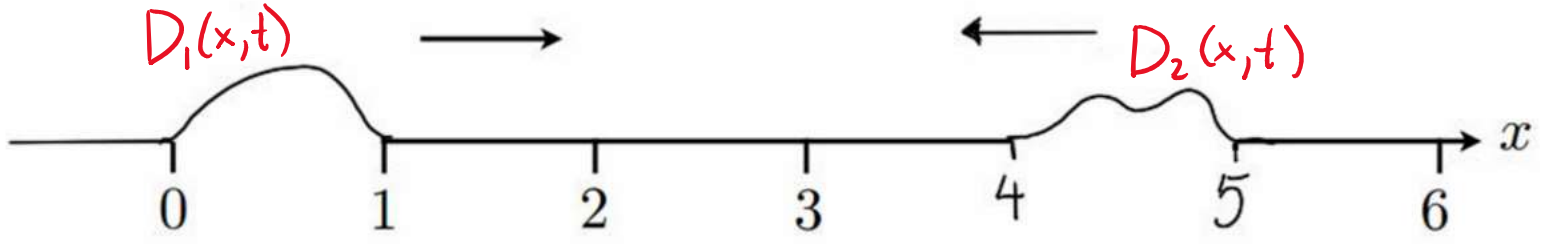
Learning goals for today:

- To calculate the frequencies of notes produced by musical instruments
- To explain why different musical instruments playing the same note sound different
- To describe the phenomena of constructive and destructive interference for waves from two sources, and predict the locations where these will occur
- To describe how interference is used in an interferometer can accurately measure changes in length


Last time in
Physics 157...

The Principle of Superposition

When two or more waves overlap, the net displacement D(x,t) is equal to the sum of the displacements we would have if each wave were present alone.



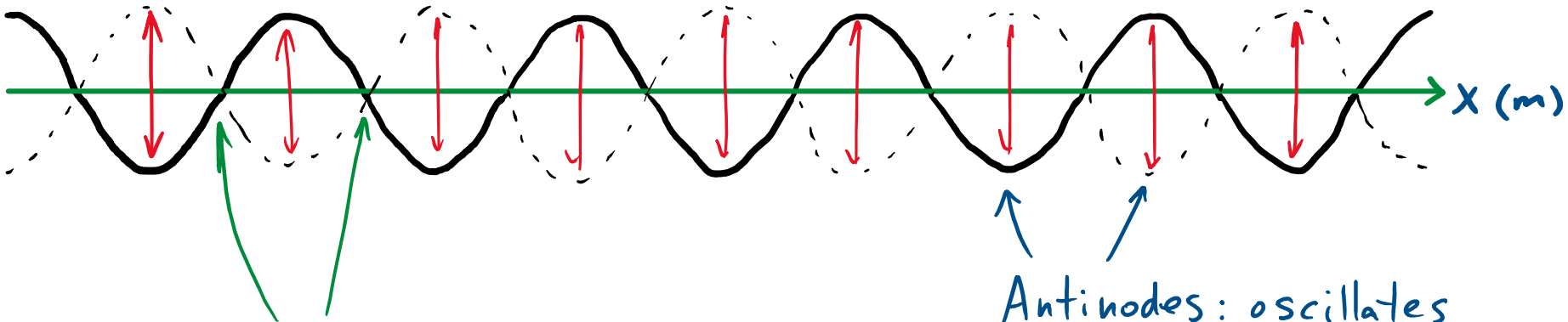
combined wave: $D(x,t) = D_1(x,t) + D_2(x,t)$

★ waves add without disturbing each other ★

Standing Waves

$$D(x,t) = A\cos(kx)\cdot\cos(\omega t)$$

Displacement



Nodes: displacement fixed at 0

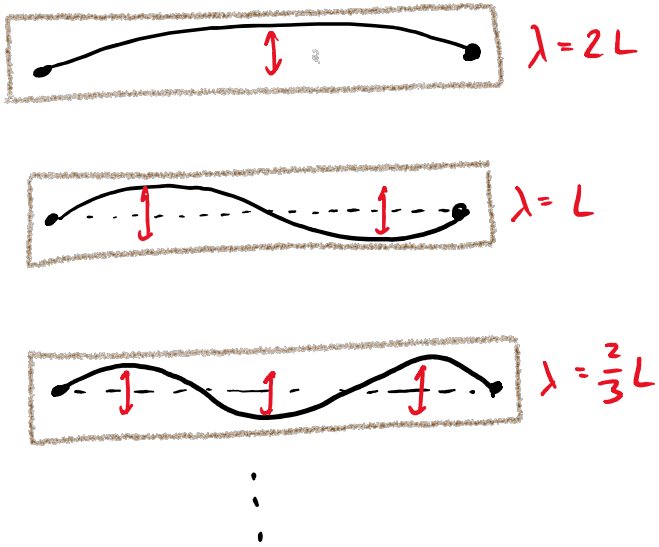
$\cos(kx) = 0$

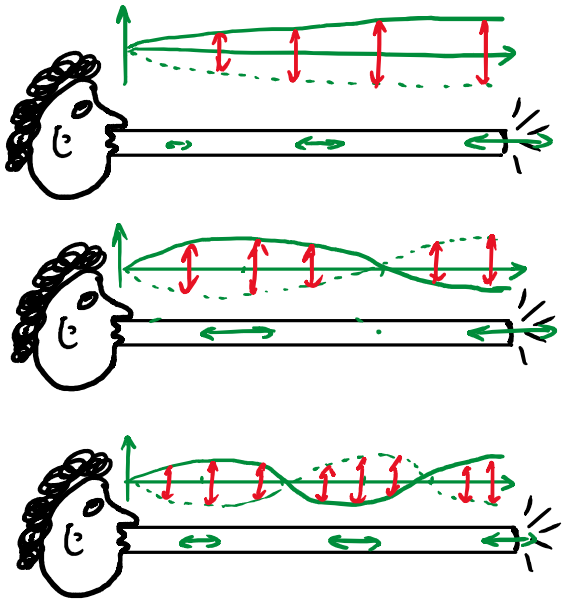
Antinodes: oscillates w. maximum displacement

$\cos(kx) = \pm 1$

= sum of left-moving wave + right-moving wave

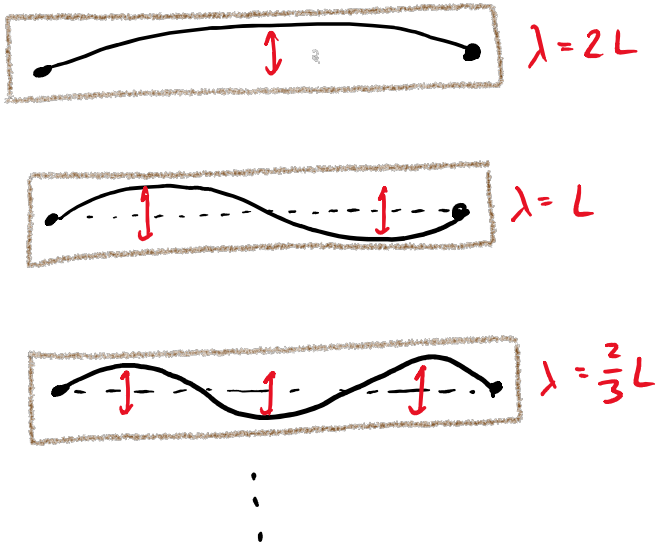
Musical Instruments:

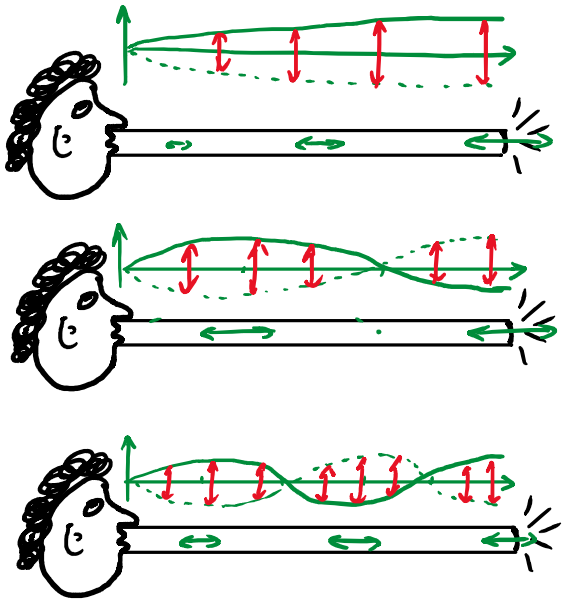




$$f = \frac{v}{\lambda}$$

Musical Instruments:



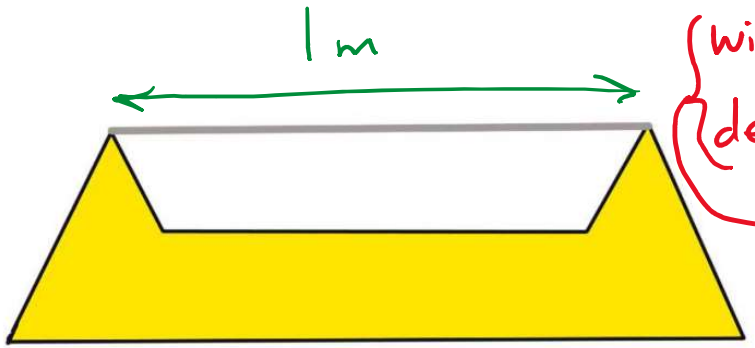


$$f = \frac{v}{\lambda}$$ → determined by properties of the medium

Example : Which note started the Very Serious Skipping+Clapping Race

$T = 800N$

Question 1:



wire diameter : 1mm

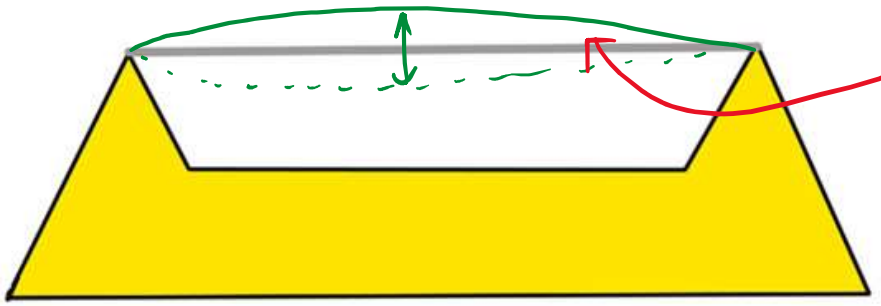
density of platinum : $2.14 \times 10^4 kg/m^3$

gives $\mu = 0.0168\ kg/m$

$* v = \sqrt{\frac{T}{\mu}} *$ $* \lambda \cdot f = v *$

You are the Grand Engineer for the Island Nation of Bthththx (pronounced as written). Each year, on the last day of summer, a new Knightship of Bthththx is awarded to the winner of the Very Serious Skipping and Clapping Race, in which participants (18 years of age and older) must skip and clap through a full lap around the island's perimeter, adhering to the rather strict regulations of the National Skipping and Clapping Federation.

The race begins when the Venerable Leader of Bthththx plucks a single note on the Most Perfect Plucking Instrument, which consists of a single 1mm thick platinum wire stretched between two points on a solid gold frame, as shown in the picture. To achieve the proper note, the wire must be at a tension of 800N. On the morning of the race, you notice the temperature



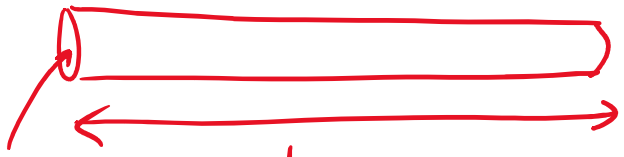
1m = $\frac{1}{2}$ of a wavelength so $\lambda = 2m$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{800N}{0.0168\,kg/m}} = 218\,m/s$$

For $\lambda = 2m$, we get $f = \frac{v}{\lambda} = 109\,Hz$

Aside: How to derive mass per length

1 m of wire has volume $V = L \cdot A = 1m \cdot \pi\,(0.0005m)^2$



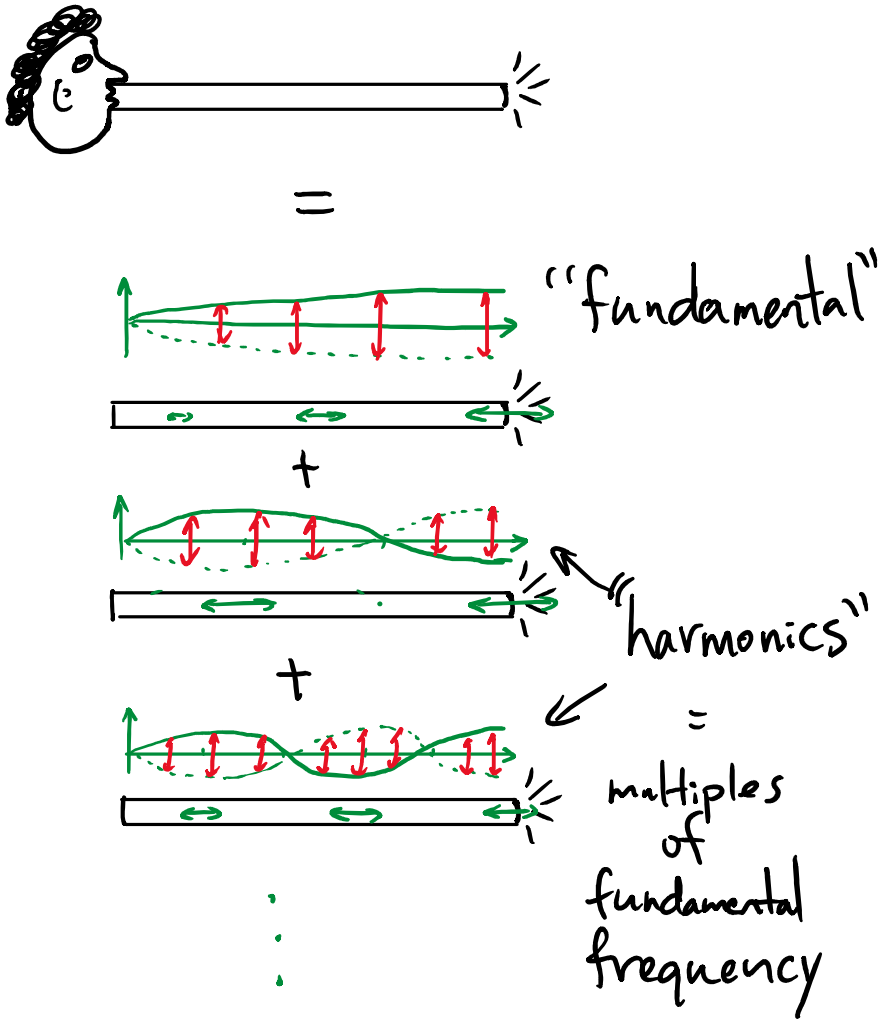
$A = \pi r^2$

mass: $m = \rho \times V$

$= 2.145 \times 10^4\,kg/m^3 \times V$

So $\mu = 0.0168\,kg/m$

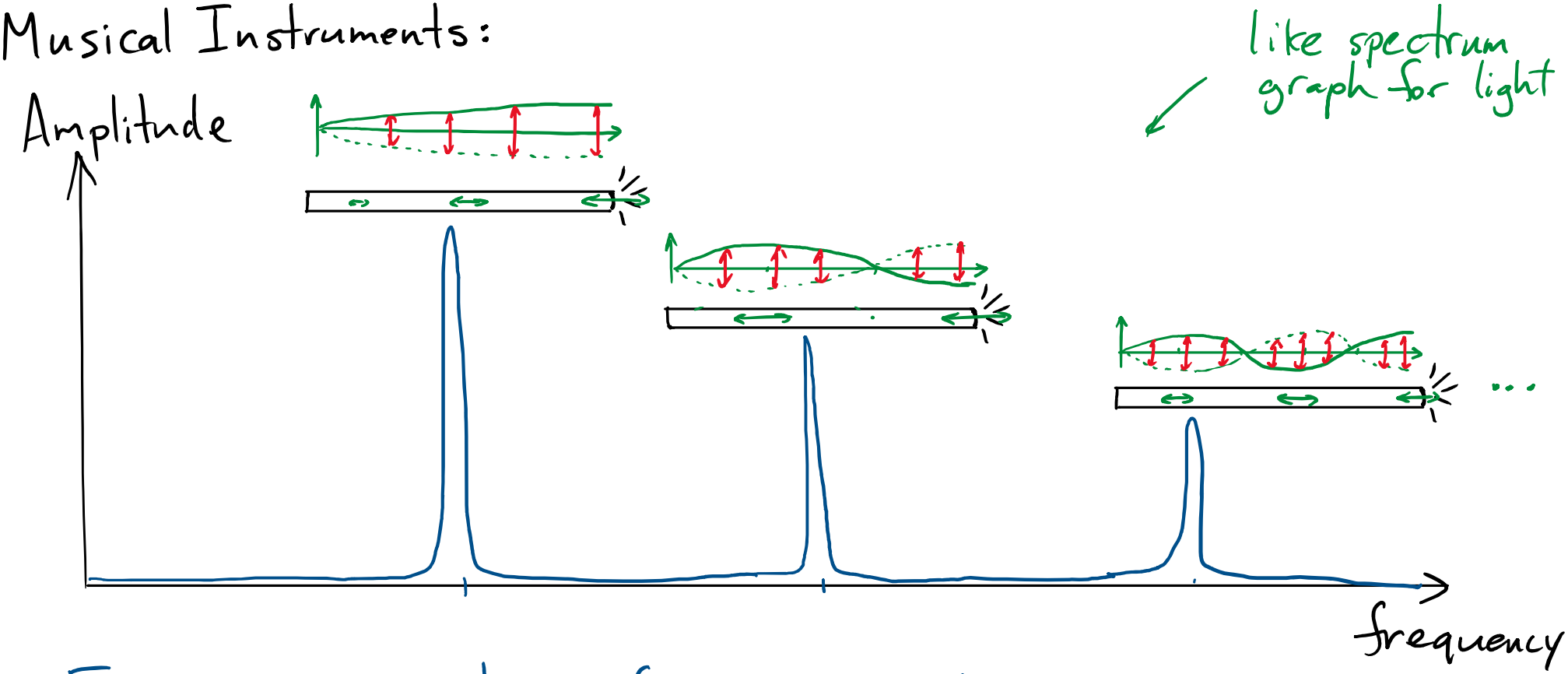
Why do different instruments sound different?



Real musical notes:
superposition of these possibilities with different amplitudes.

Different instruments playing same note:
= same "fundamental" frequency, different amplitudes for "harmonics"

Musical Instruments:



Frequency spectrum for a musical note.

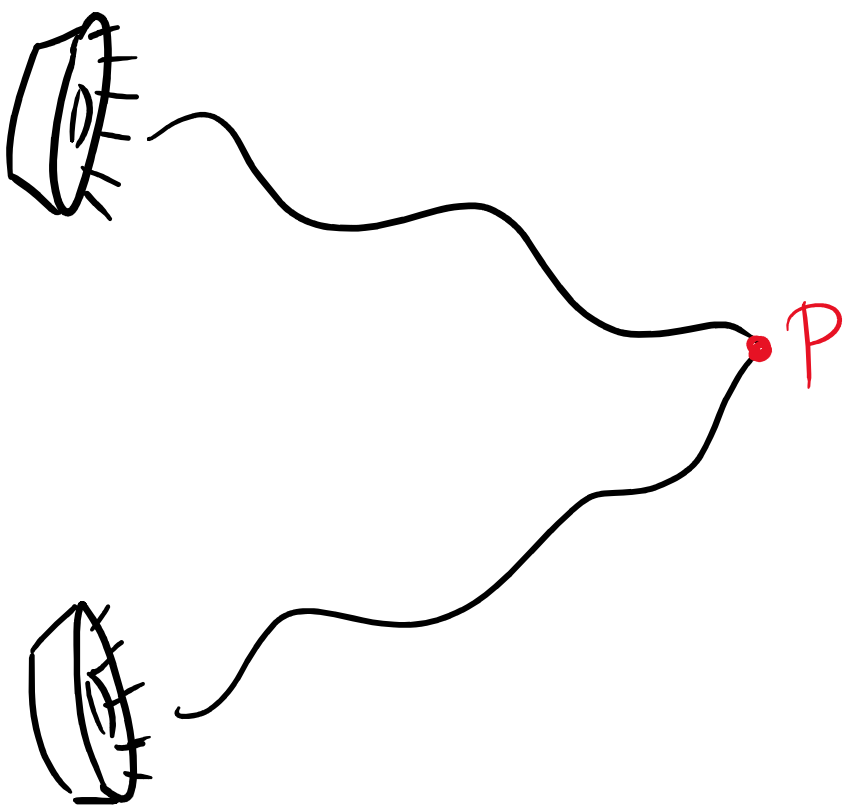
DEMO:

https://youtu.be/RIkvjUQFe7s

Another consequence of the superposition principle:

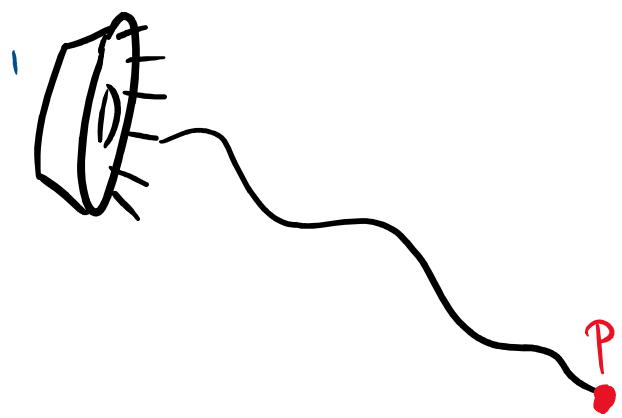
INTERFERENCE of waves

kind of a bad name...

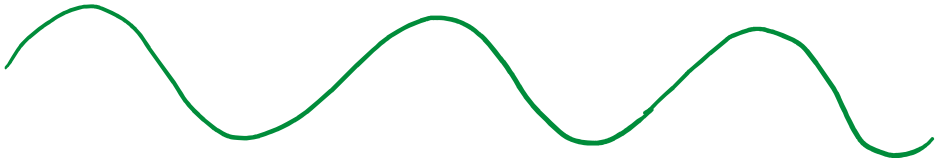


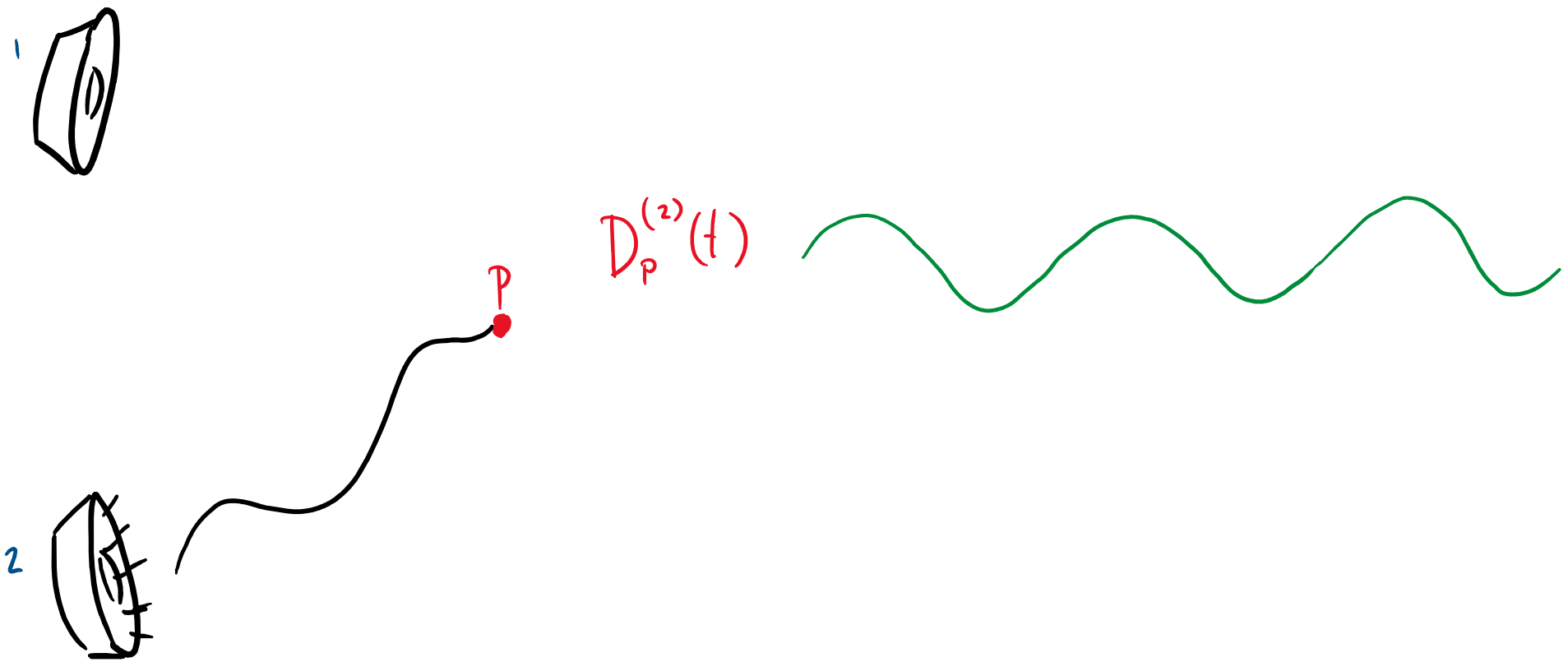
Waves from 2 sources:

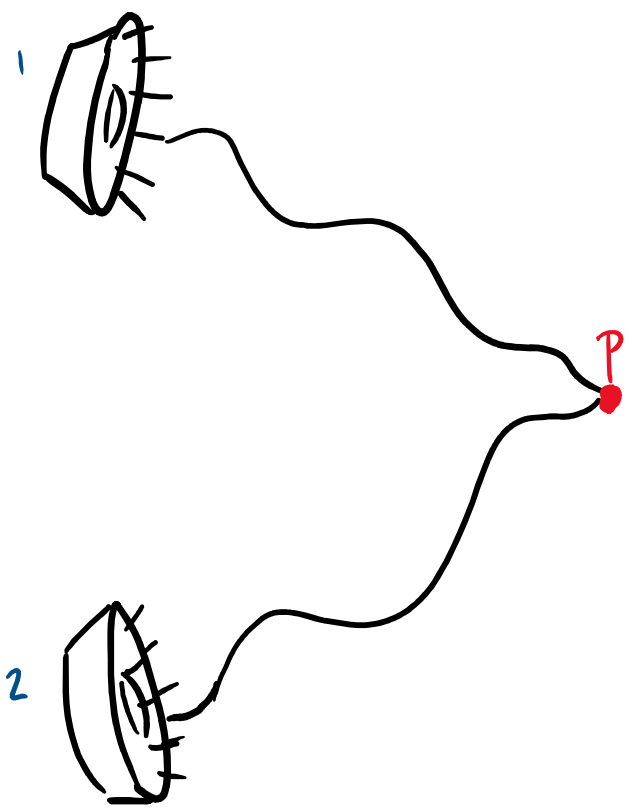
Displacement at point P is the sum of the displacements from the two individual waves.



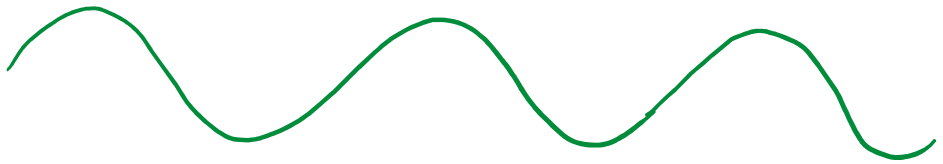
$D_p^{(1)}(t)$:



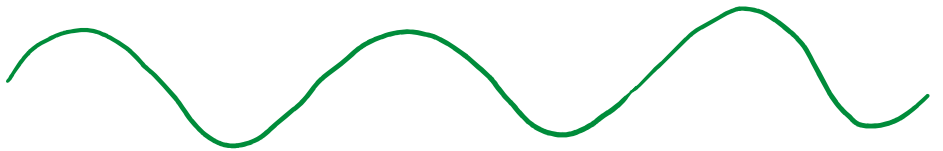

1
P
$D_p^{(2)}(t)$
2

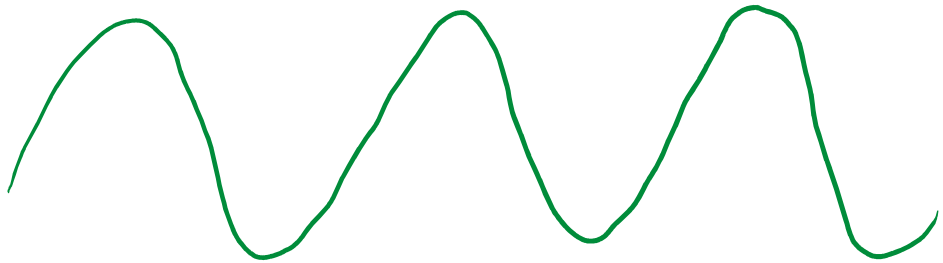


$D_p^{(1)}(t)$:



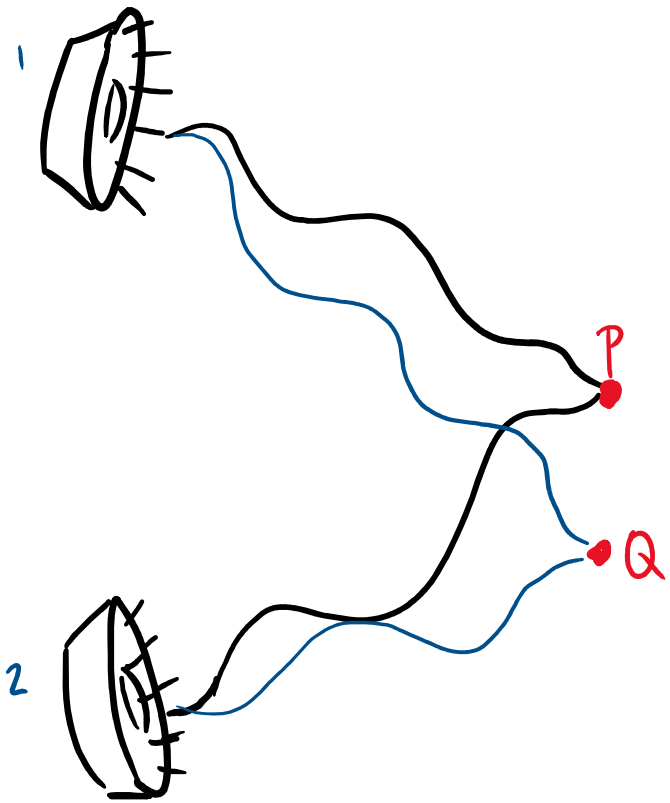
$D_p^{(2)}(t)$



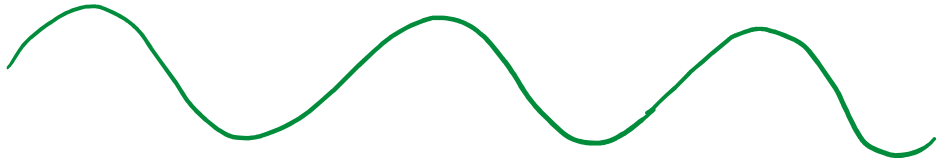


$D_p^{(1+2)}(t) = D_p^{(1)}(t) + D_p^{(2)}(t)$

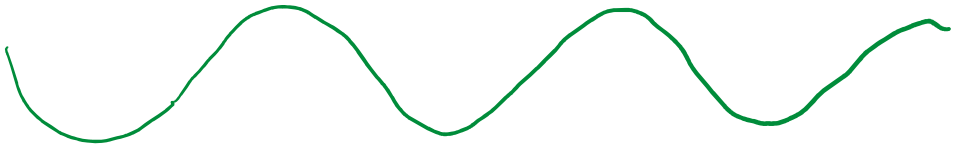
★ CONSTRUCTIVE INTERFERENCE ★



$D_Q^{(1)}(t)$:



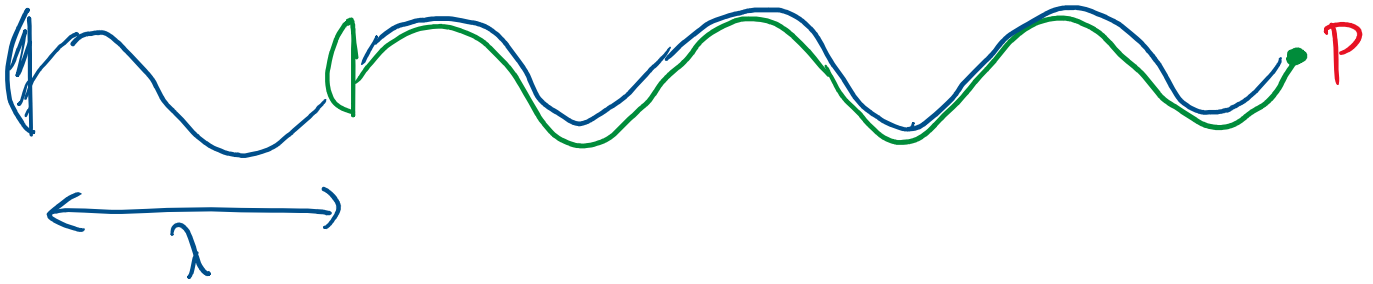
$D_Q^{(2)}(t)$



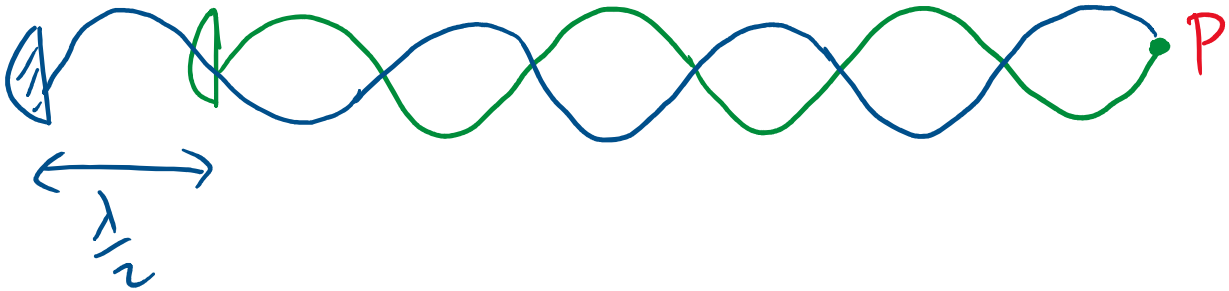
$$D_Q^{(1+2)}(t) = D_Q^{(1)}(t) + D_Q^{(2)}(t)$$

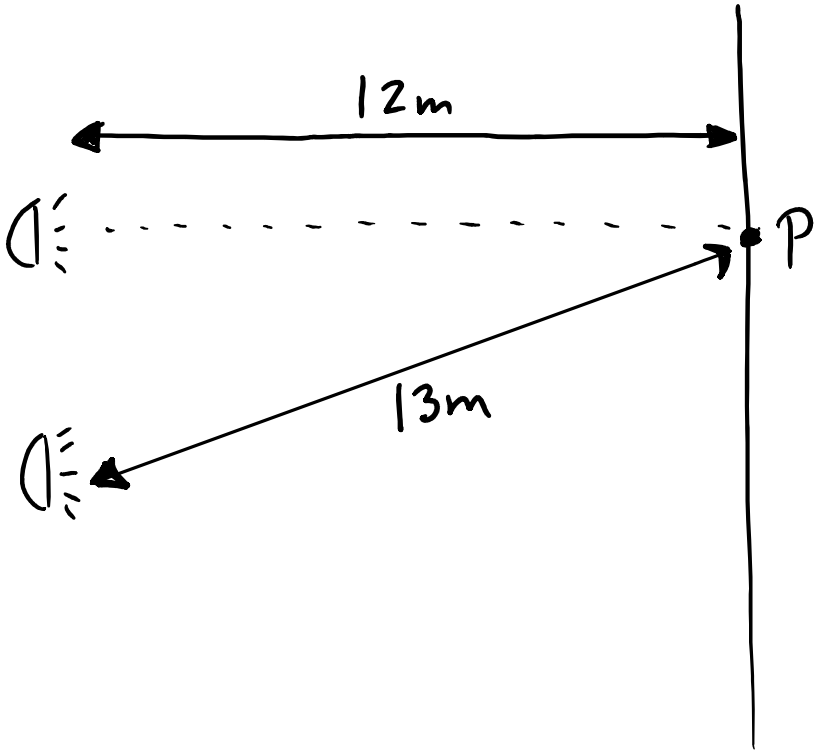
★ DESTRUCTIVE INTERFERENCE ★

For in phase sources: have constructive interference when distances to 2 sources differ by $\lambda, 2\lambda, 3\lambda, \ldots$



Destructive interference when distances to 2 sources differ by $\frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots$





Q: For what frequencies will there be constructive interference at P?

($v_{sound} = 340\ m/s$)

★ $v = \lambda \cdot f$ ★

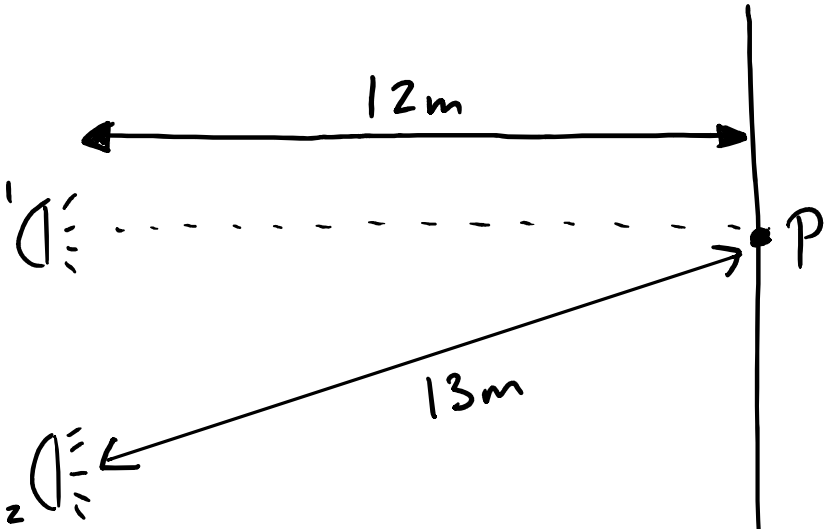
What is the LOWEST freq. for constructive interference?

A) 170 Hz B) 340 Hz

C) 680 Hz D) 510 Hz

E) 68 Hz

Q: For what frequencies will there be constructive interference at P?



Path length difference: 1m

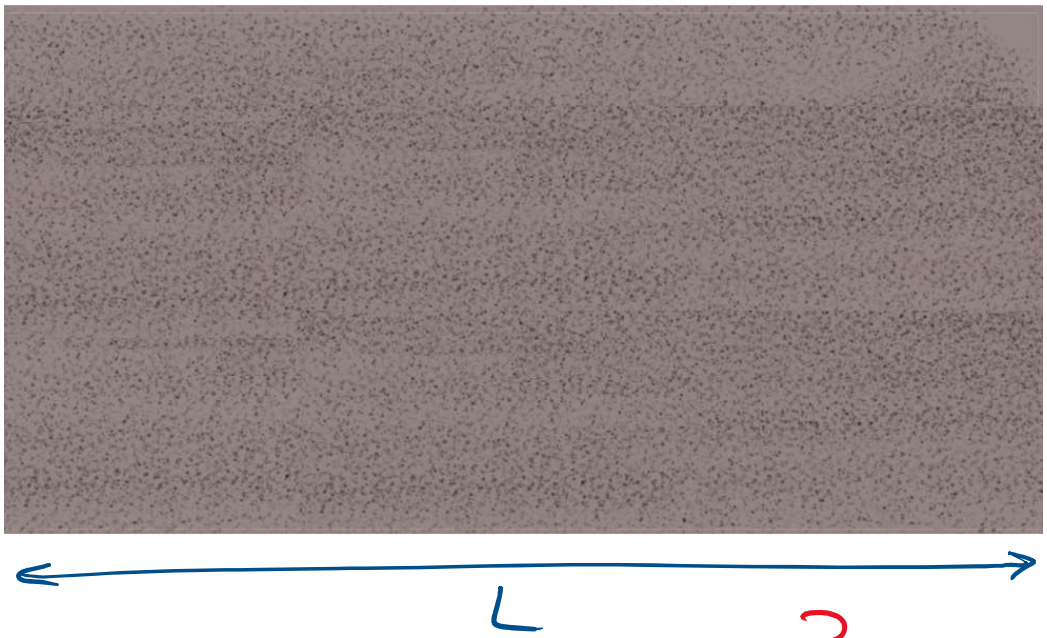
Constructive interference if $1m = \lambda, 2\lambda$, etc...

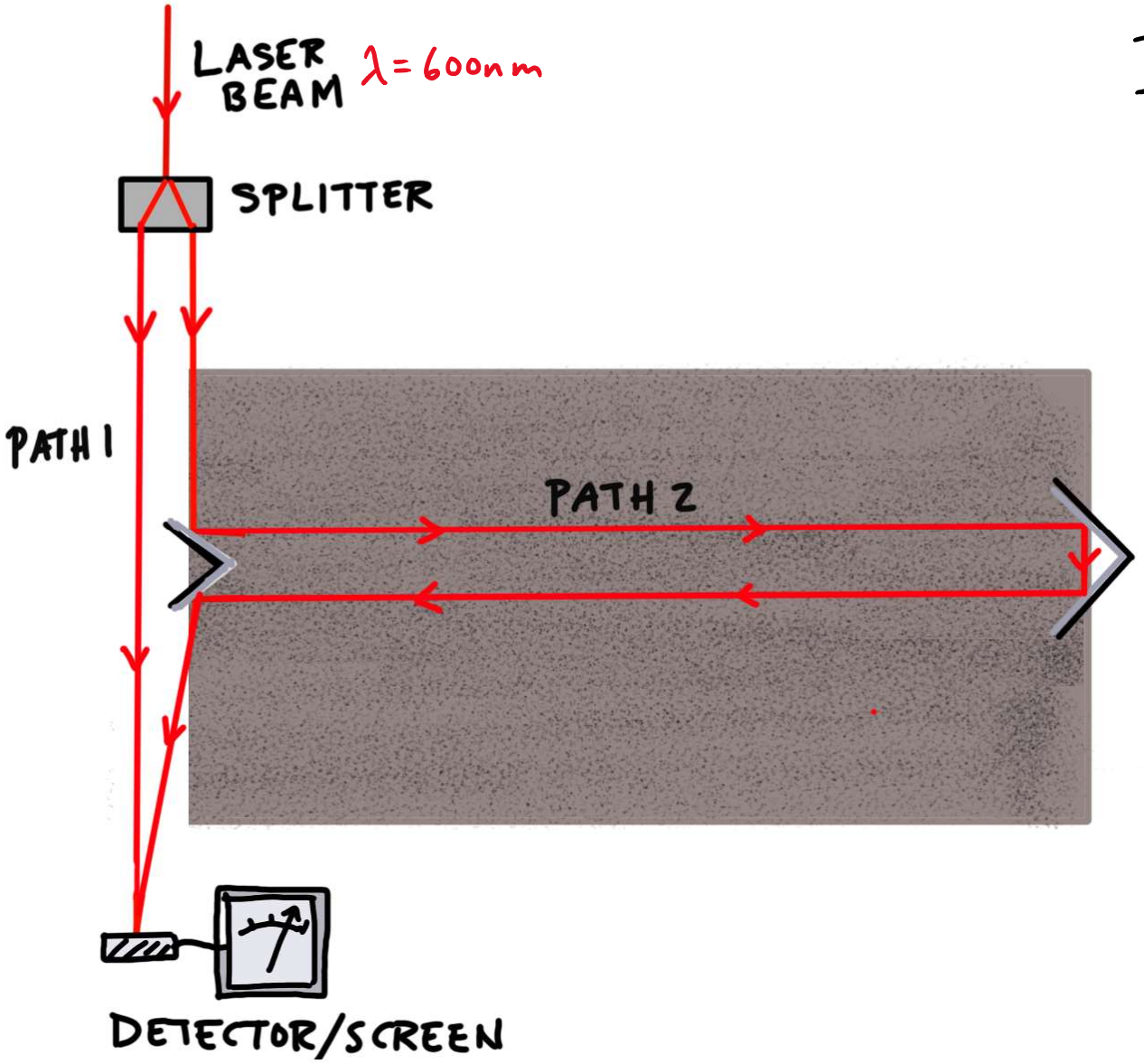
So: $\lambda = \frac{1m}{n}$

$$f = \frac{v}{\lambda} = 340\,Hz \times n$$

Application: interferometer

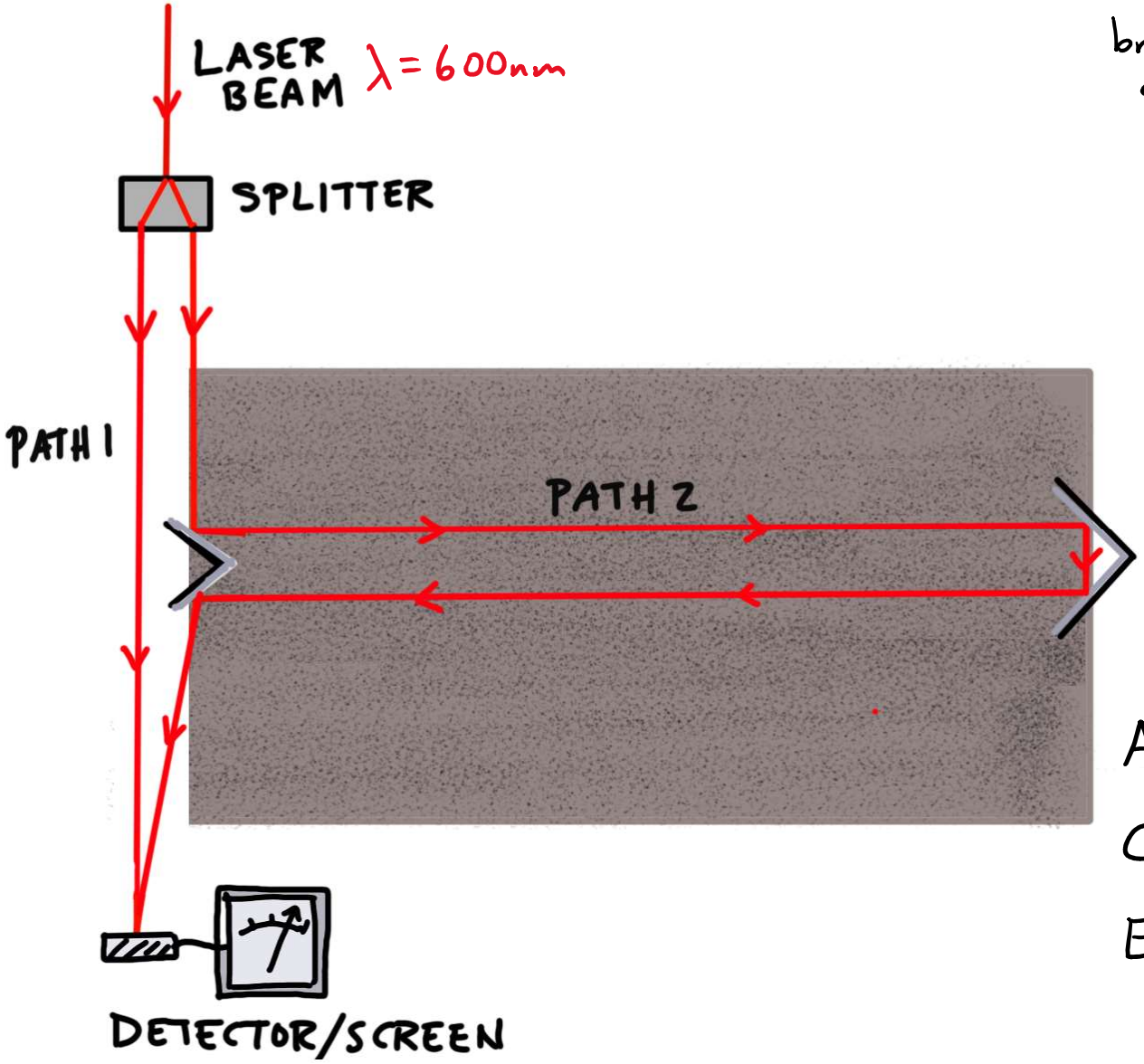
How can we measure change in length when we heat an object?

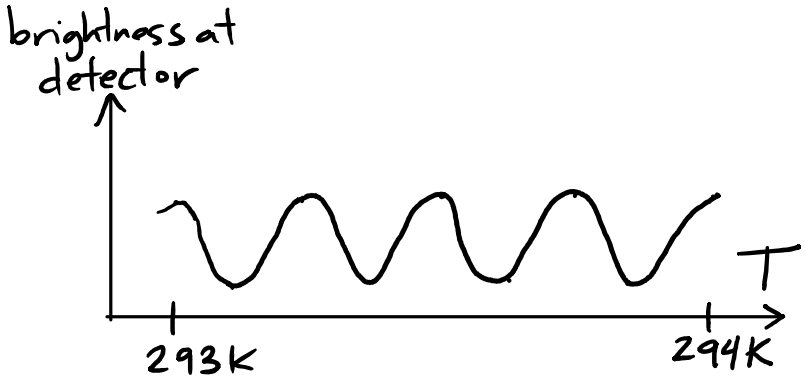




INTERFEROMETER:

As length changes, path length difference changes, and we go from constructive → destructive → constructive etc.. interference.



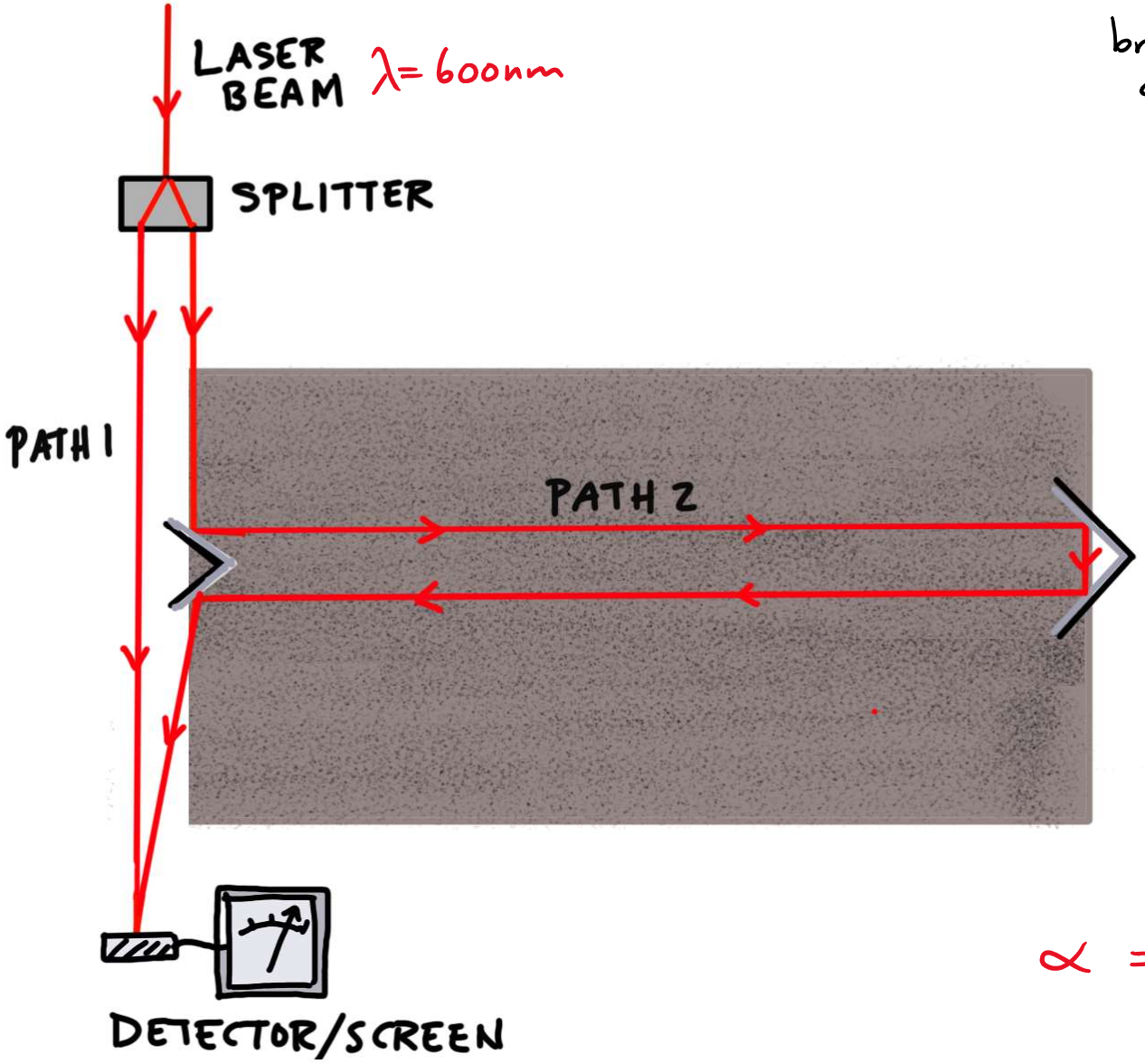


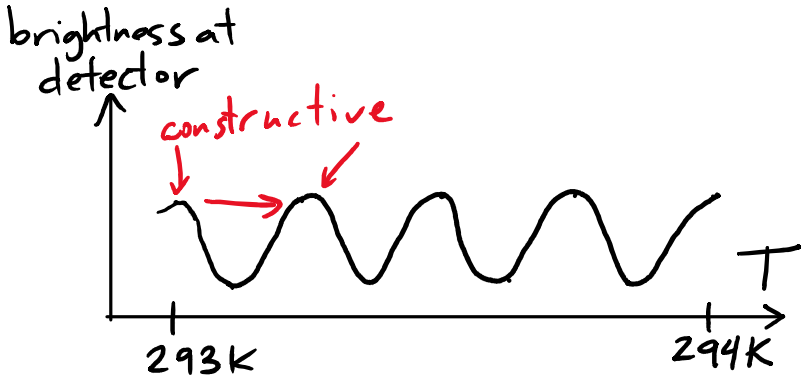
Q: By how much did the brick's length change between 293K and 294K?

A) 600nm B) 1200nm

C) 2400nm D) 4800nm

E) 150nm



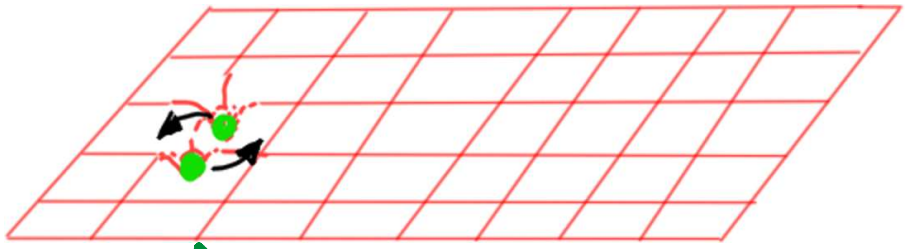


path length changes by 1 wavelength each time we go from constructive to constructive interference.

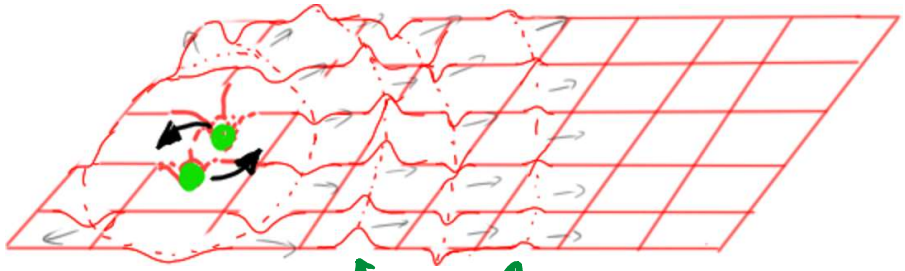
From 293K → 294K, path 2 changes by 4λ. Length of brick changes by $2\lambda = 1200nm$

$$\alpha = \frac{\Delta L}{L \Delta T} = \frac{1200nm}{0.2m \times 1K} = 6 \times 10^{-6} K^{-1}$$

GRAVITATIONAL WAVES (predicted by Einstein's theory of general relativity)

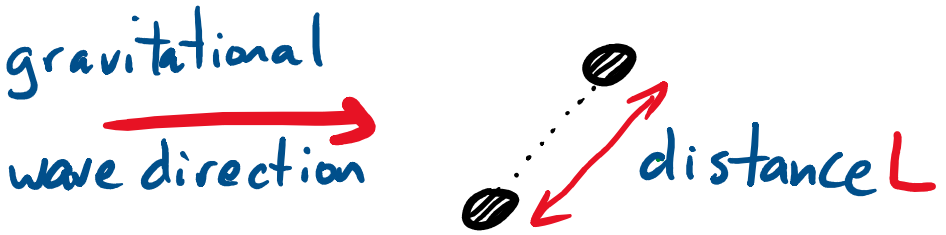


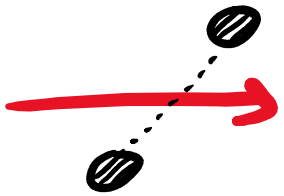
oscillating matter system
e.g. stars orbiting



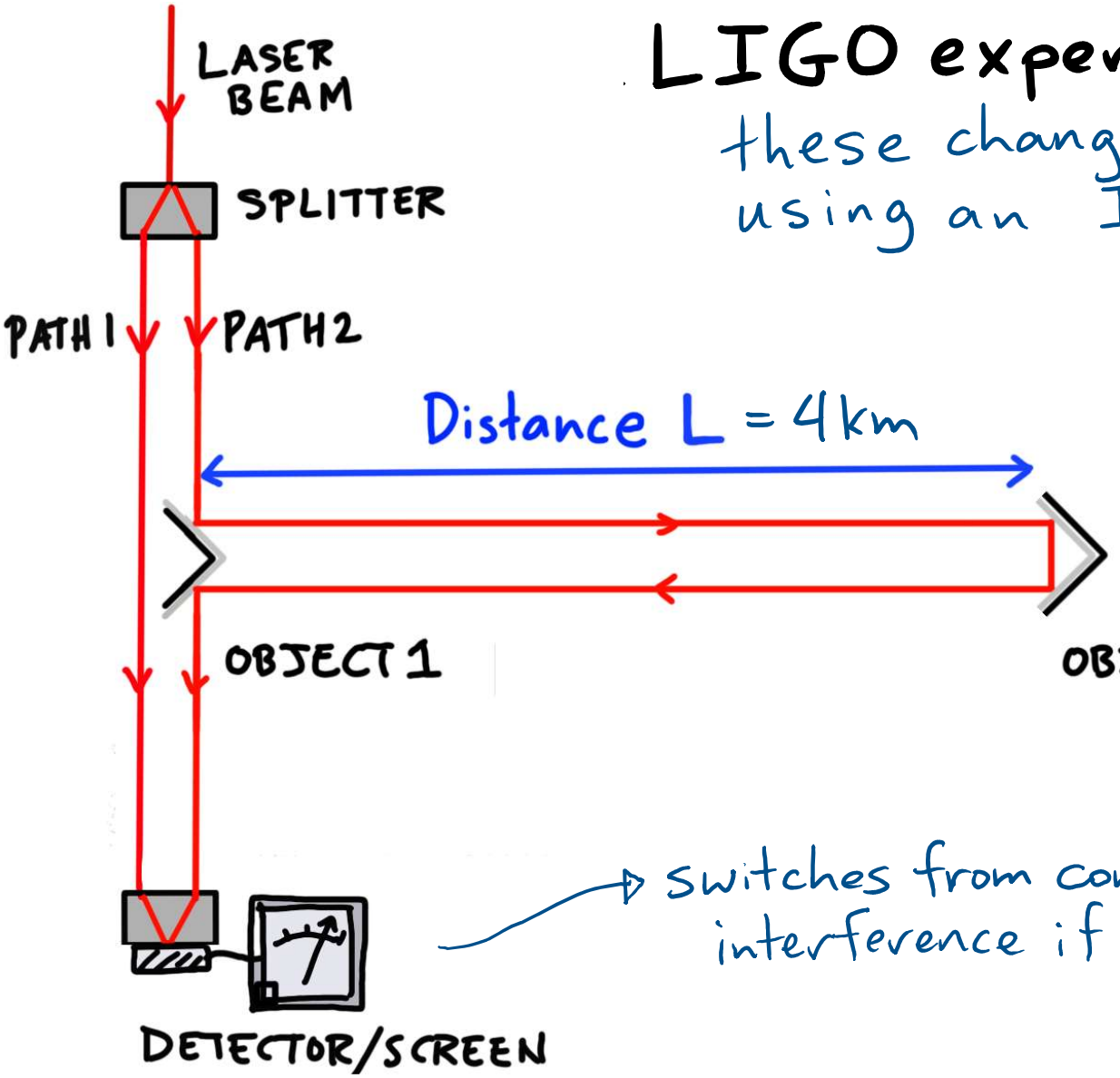
energy carried away by ripples of space/time

How can we "see" a gravitational wave?





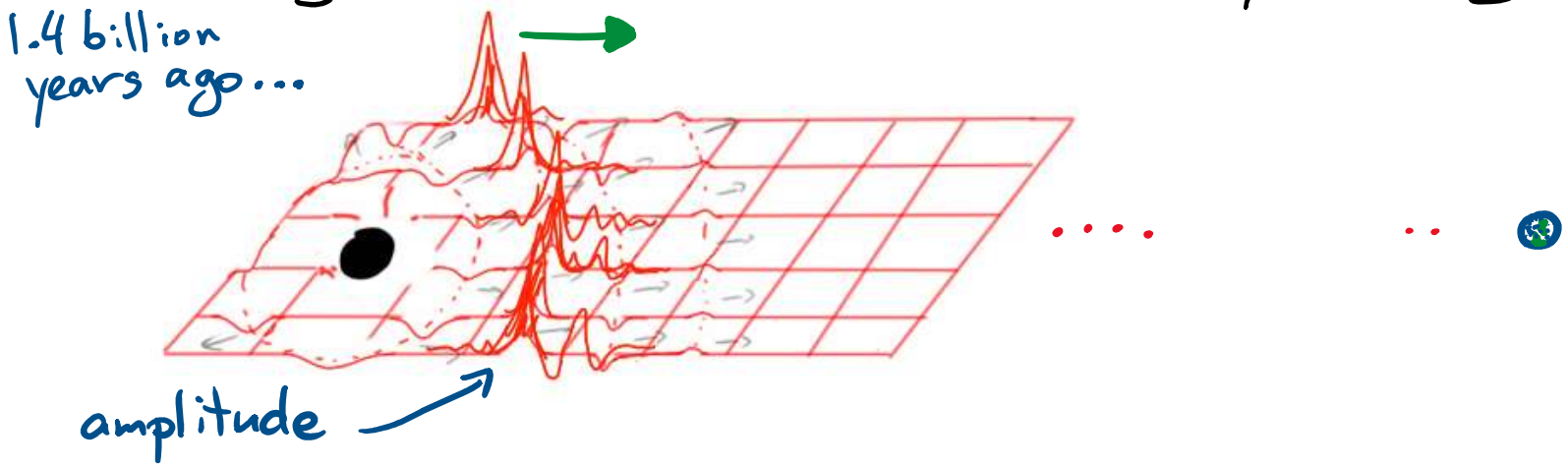
distance oscillates as gravitational wave passes.



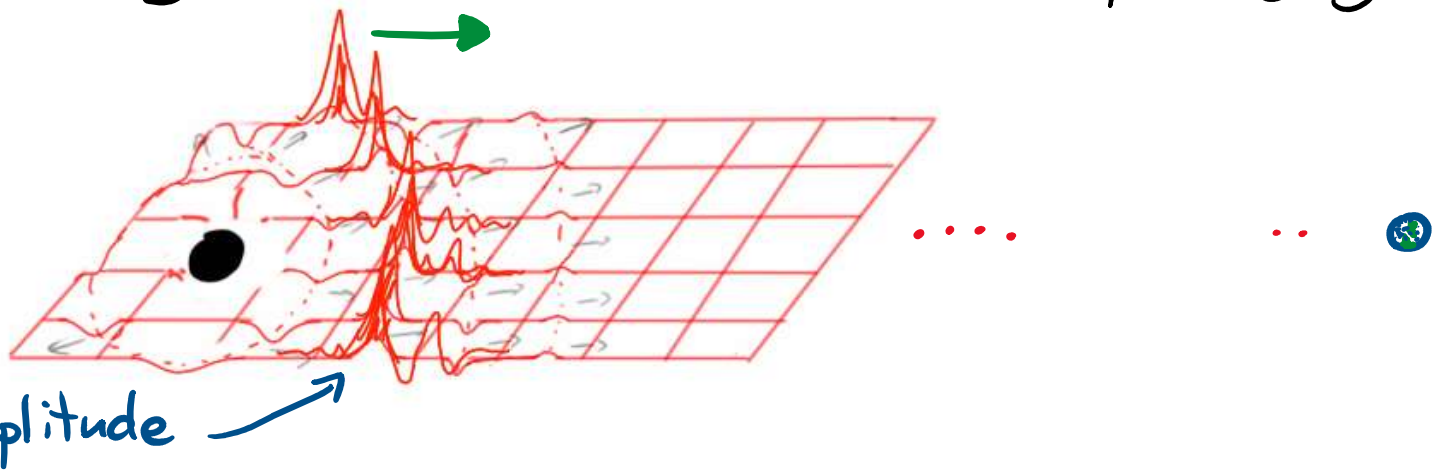
LIGO experiment: try to measure these changes in length directly using an INTERFEROMETER

→ switches from constructive to destructive interference if L changes by $\frac{\lambda}{4}$

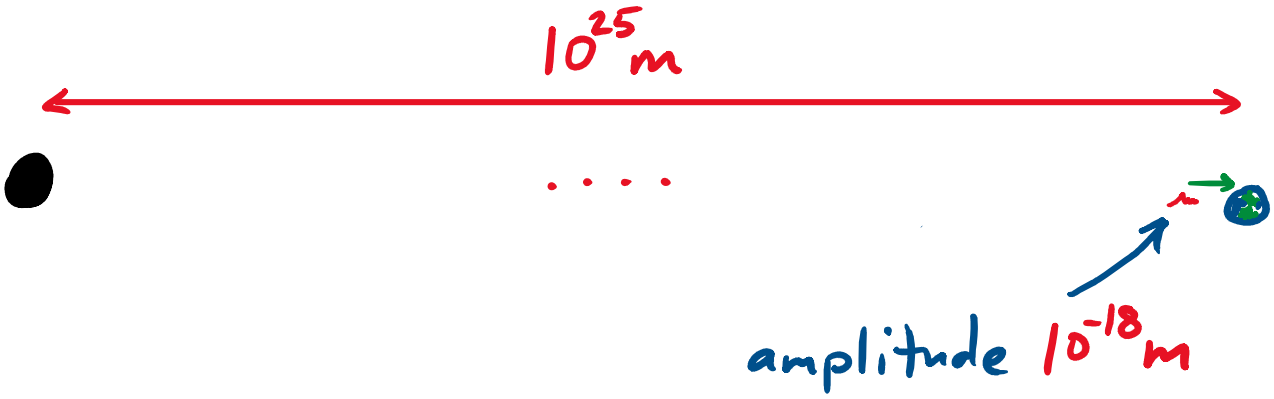
Largest gravitational waves produced by colliding black holes



Largest gravitational waves produced by merging black holes

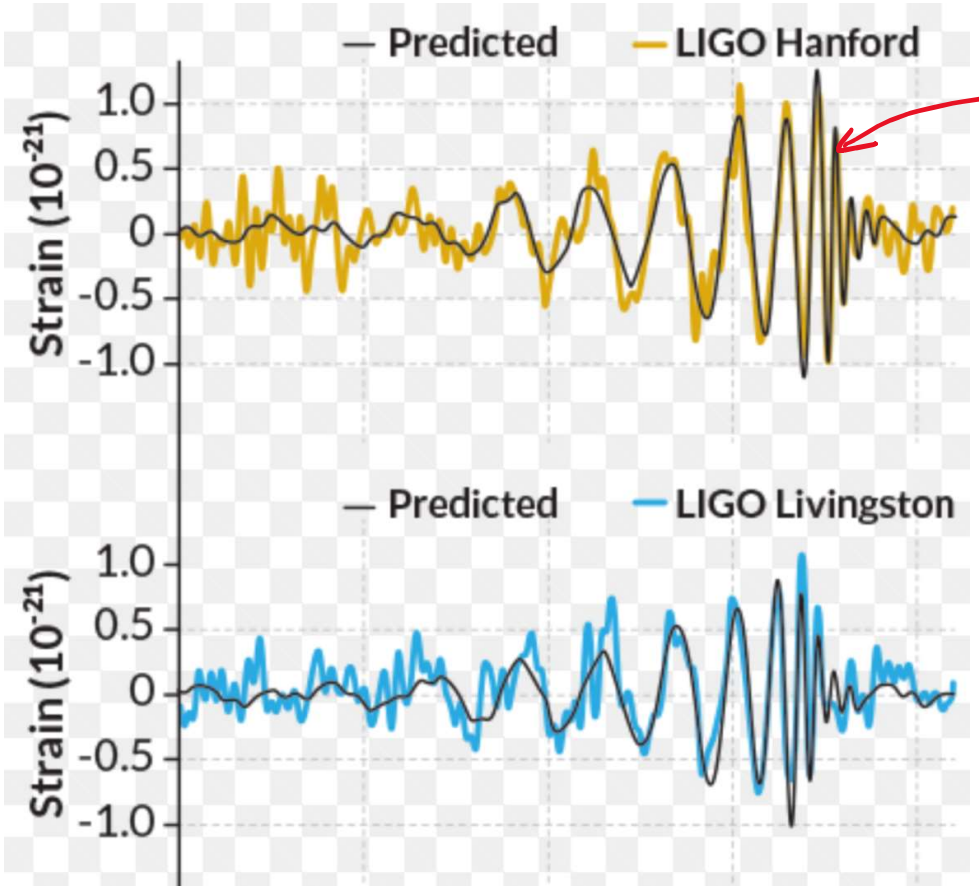


1.4 billion years later...



Observed signal: September 14th, 2015

(reported 2016)



detectors at different location

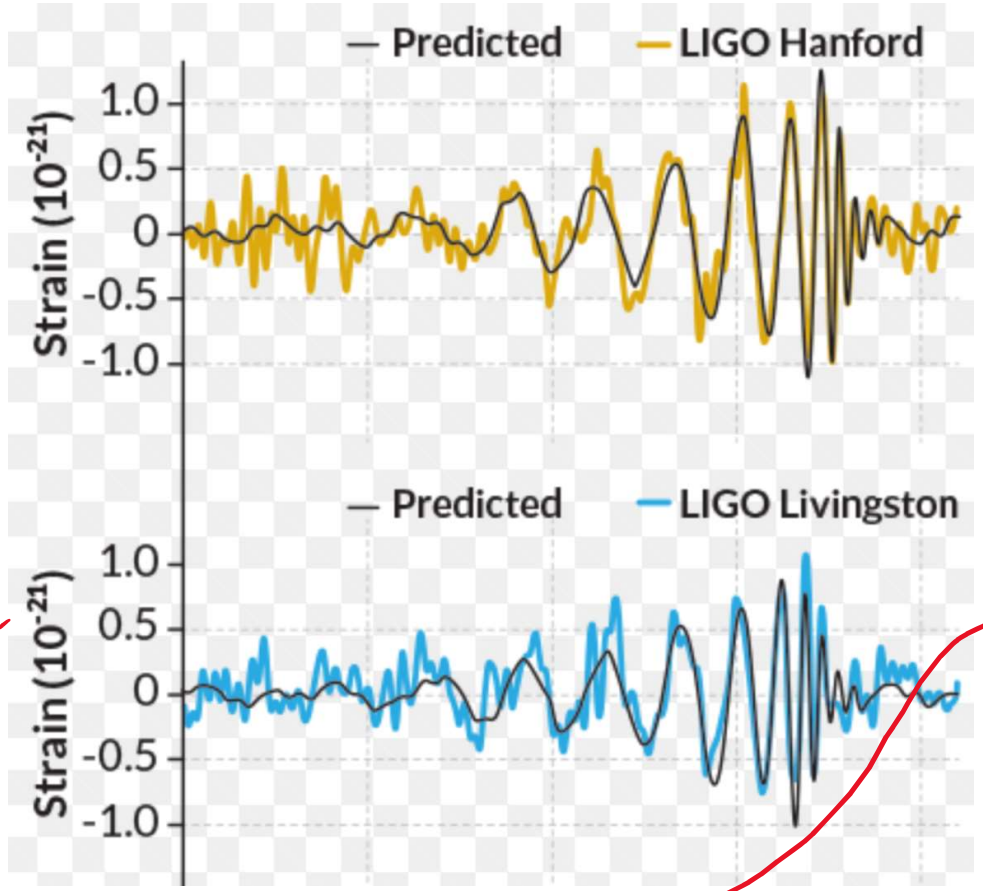
predicted wave pulse shape for collision of black holes with masses:

$M_1 = 36\,M_{sun}$

$M_2 = 29\,M_{sun}$

Observed signal: September 14th, 2015

(reported 2016)

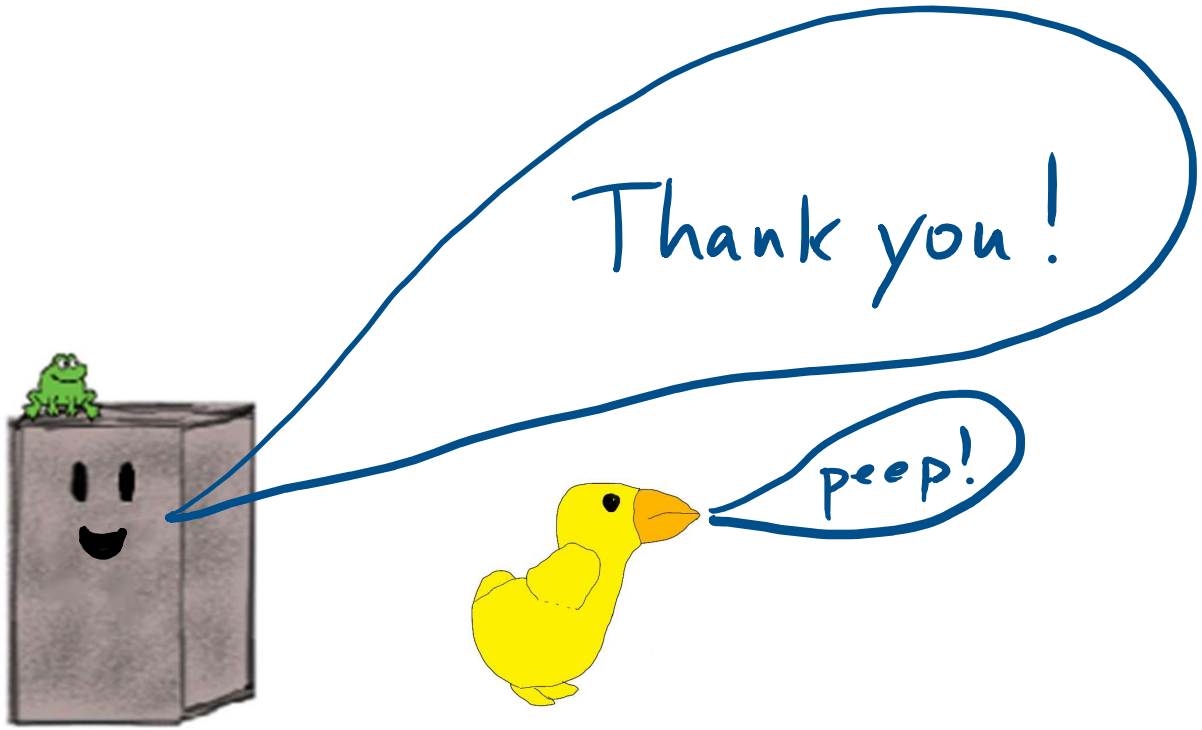


detectors at different location

$\frac{\Delta L}{L} = 10^{-21}$

$L = 4\,km$

$\Delta L \approx 10^{-18} m = \frac{1}{1000} \times$ proton size


Thank you!
peep!