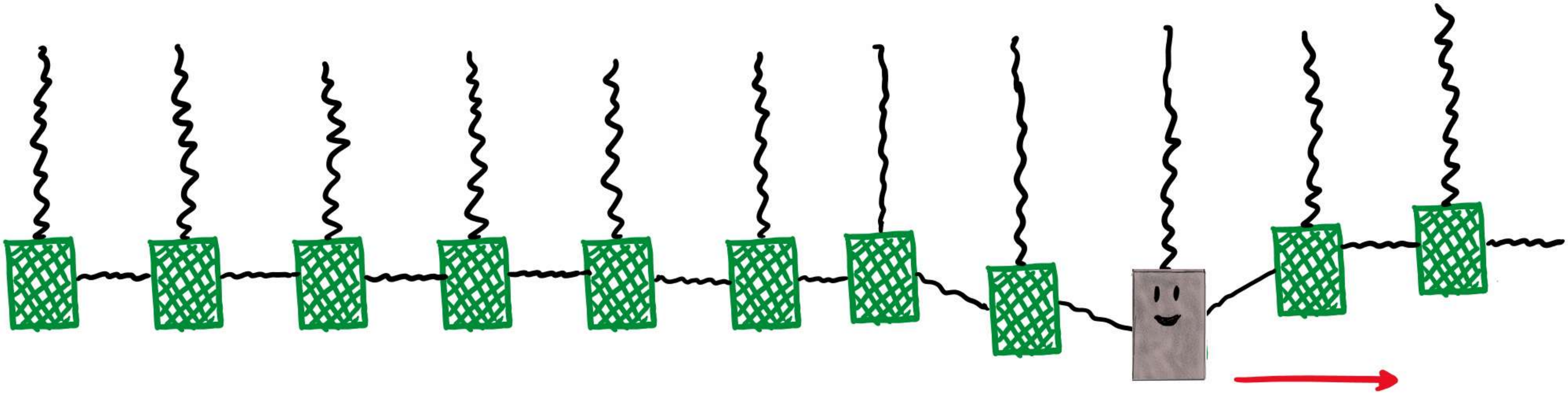
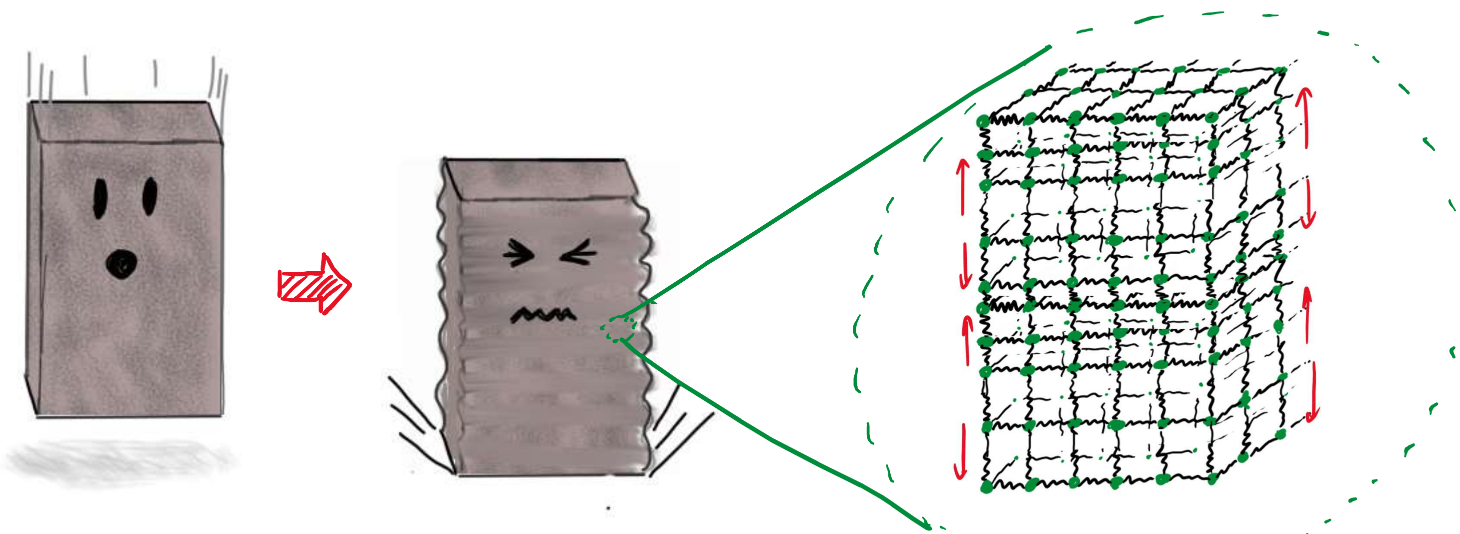
Office hours today: after class (Remo), 3:30-4:30 (or later) in Zoom

Learning goals for today:

- To describe the mathematical description of sinusoidal travelling waves, and explain how the parameters in this mathematical description relate to wavelength, frequency, wave number, period and velocity.
- To relate wave speed with frequency and wavelength for a travelling sinusoidal wave.

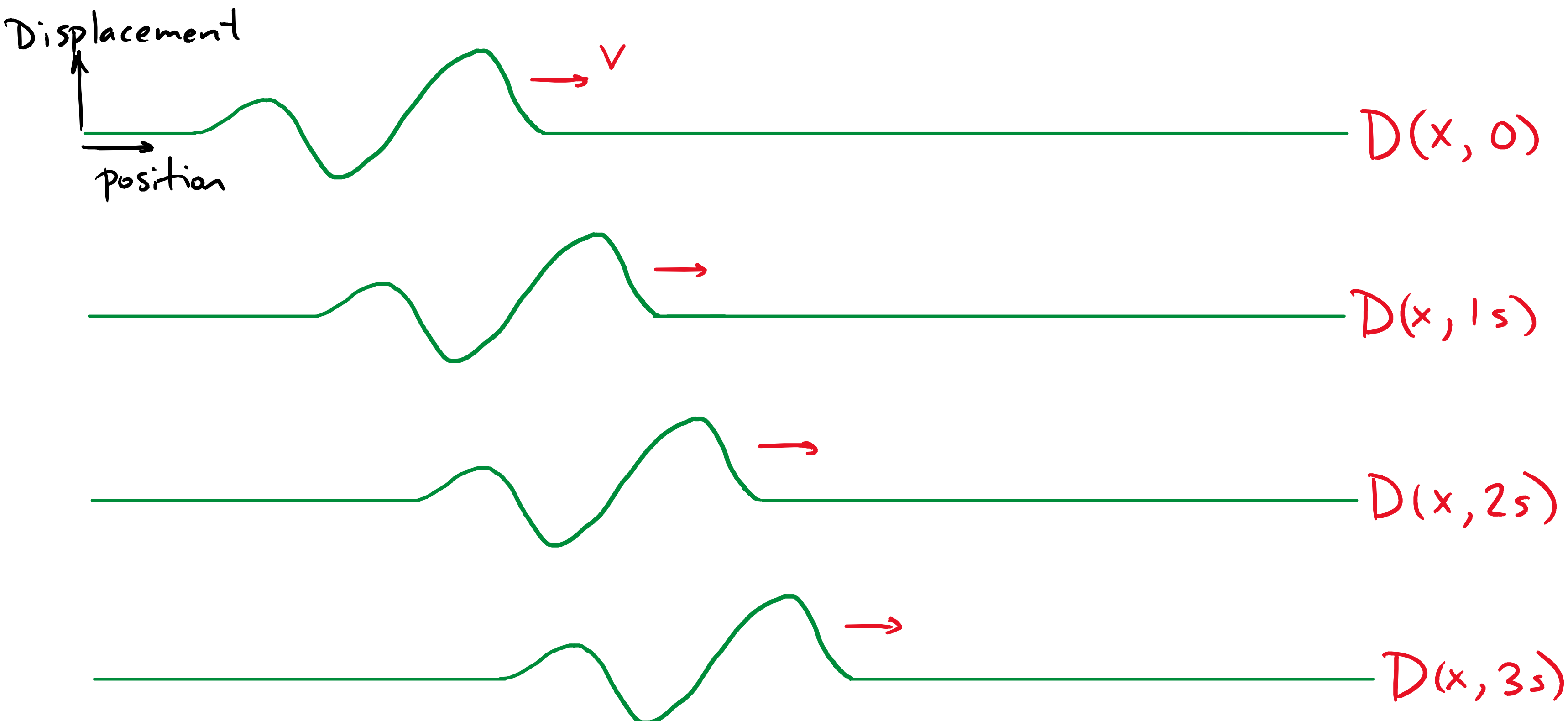


Last time in Physics 157...



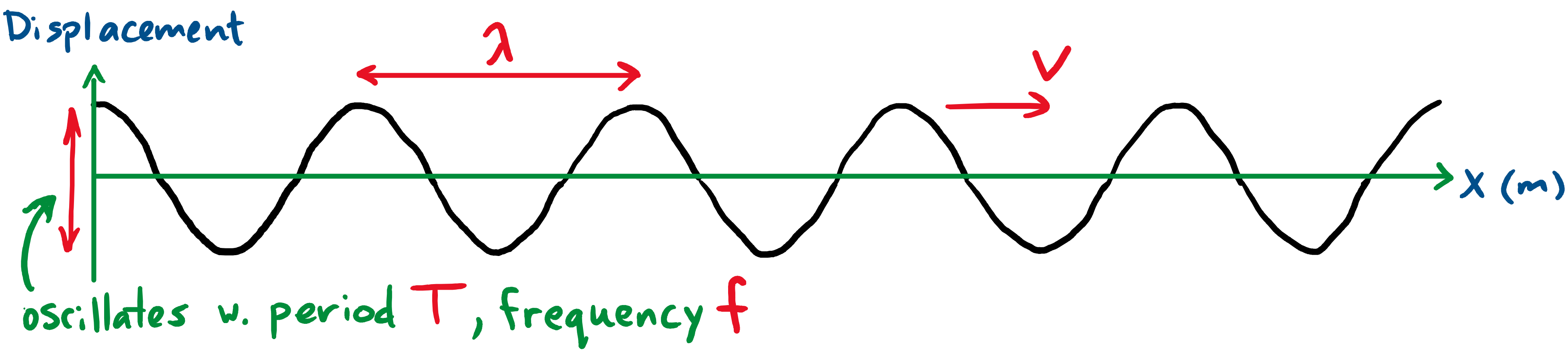
Mathematical description of waves:

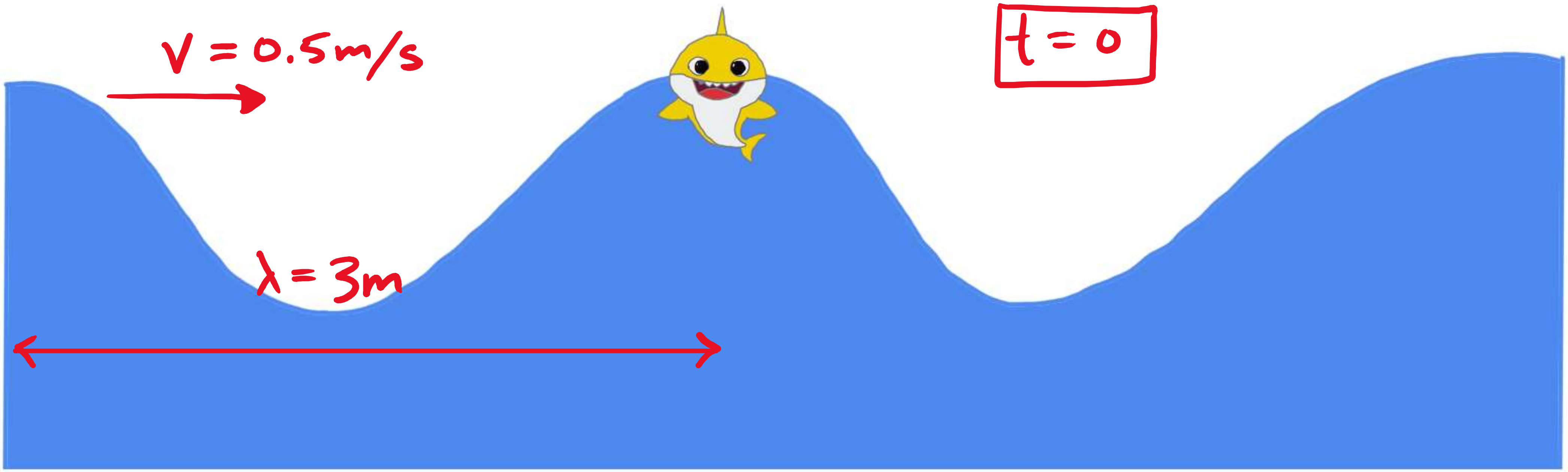
define $D(x,t)$: displacement at position x at time t.



note: some waves change shape over time

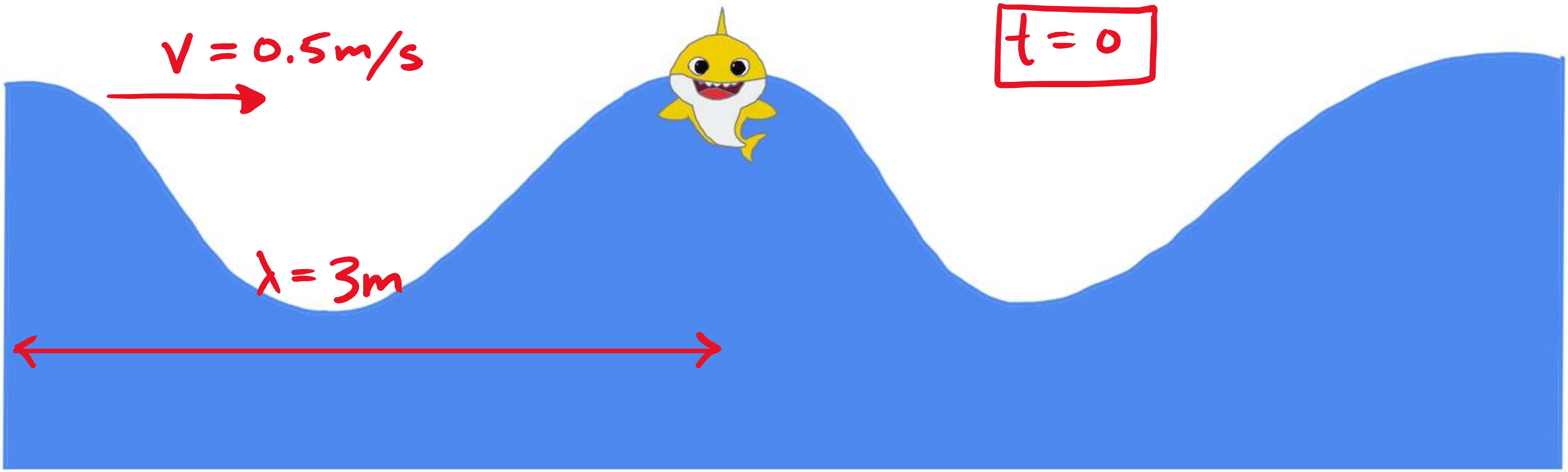
Today: sinusoidal waves: shape of wave at any time is a sinusoidal function





Baby Shark is floating at the surface of the water as waves pass by. At what time will Baby Shark next reach a maximum height?

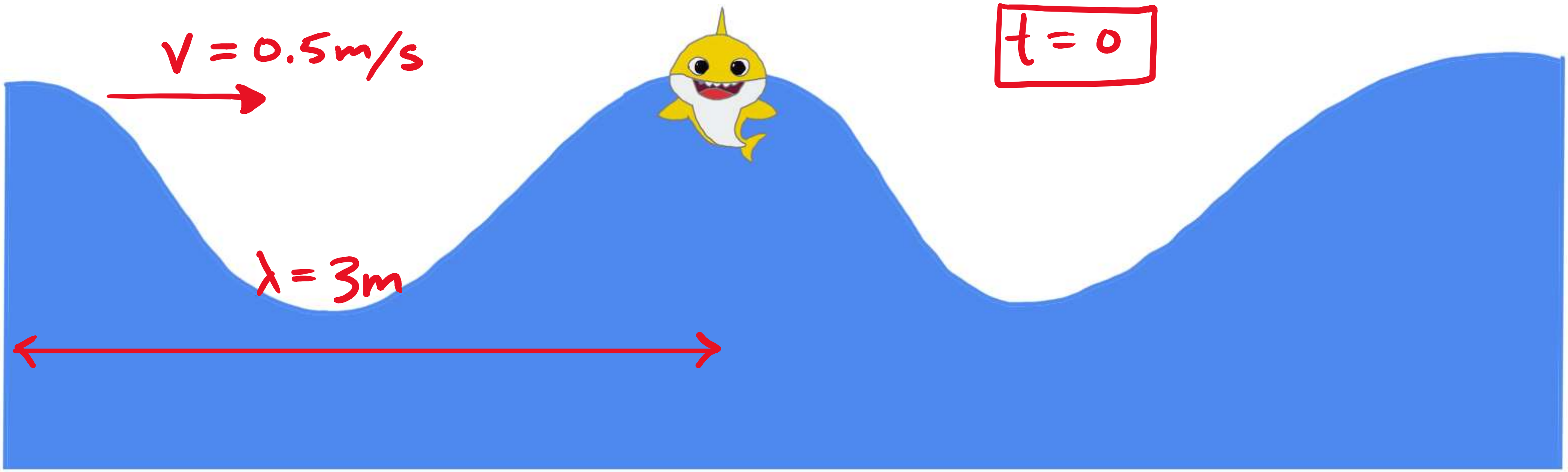
A) 0.17s B) 1.5s B) 3s D) 6s E) 12s



Baby Shark is floating at the surface of the water as waves pass by. At what time will Baby Shark next reach a maximum height?

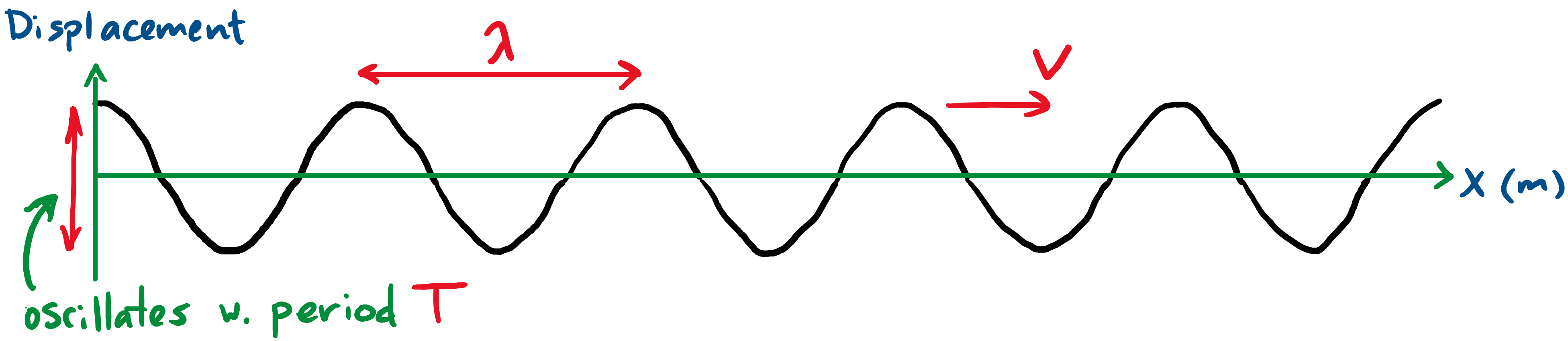
A) 0.17s B) 1.5s B) 3s D) 6s E) 12s

Baby Shark will be at max height again when wave moves distance $\lambda = 3m$. This takes time $T = \frac{\lambda}{v} = \frac{3m}{0.5 m/s} = 6s$

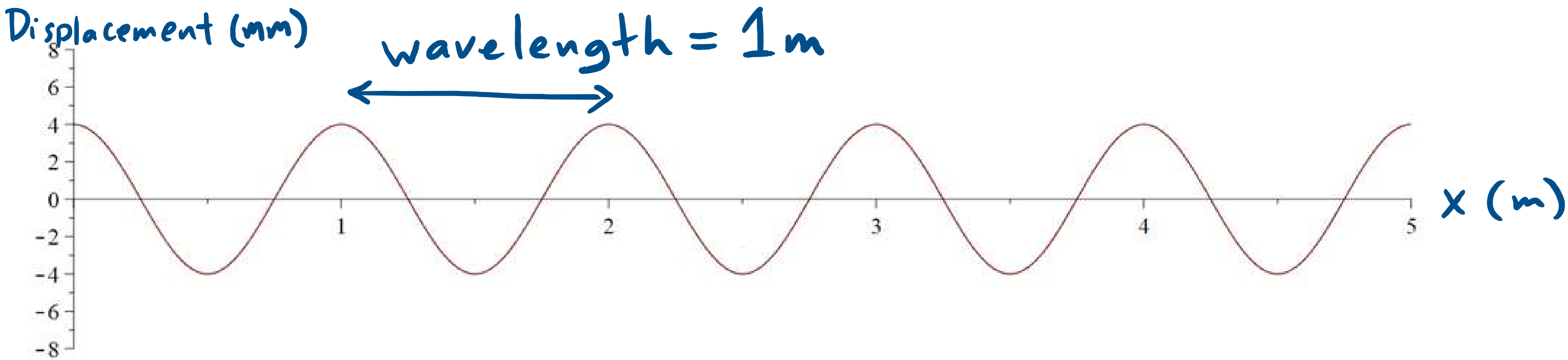


Key point: $T = \frac{\lambda}{v}$ relates period, wavelength, velocity

Velocity, wavelength, and frequency/period

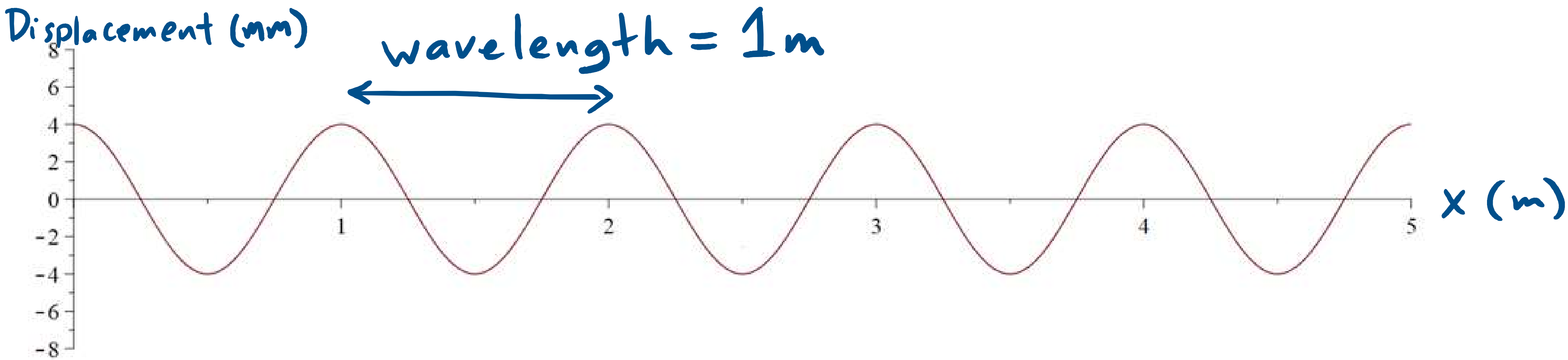


$$v = \frac{\lambda}{T} \quad \text{or} \quad v = \lambda \cdot f$$



The picture shows a wave on a string at some time t=0. Which of the following represents the displacement of the string as a function of position at t=0?

A) $D(x, t=0) = 4\text{mm} \cdot \cos(x / 1\text{m})$
B) $D(x, t=0) = 4\text{mm} \cdot \cos(1\text{m} \cdot x)$
C) $D(x, t=0) = 4\text{mm} \cdot \cos(2\pi / 1\text{m} \cdot x)$
D) $D(x, t=0) = 4\text{mm} \cdot \cos(1\text{m} / 2\pi \cdot x)$
E) $D(x, t=0) = 4\text{mm} \cdot \cos(x - 1\text{m})$

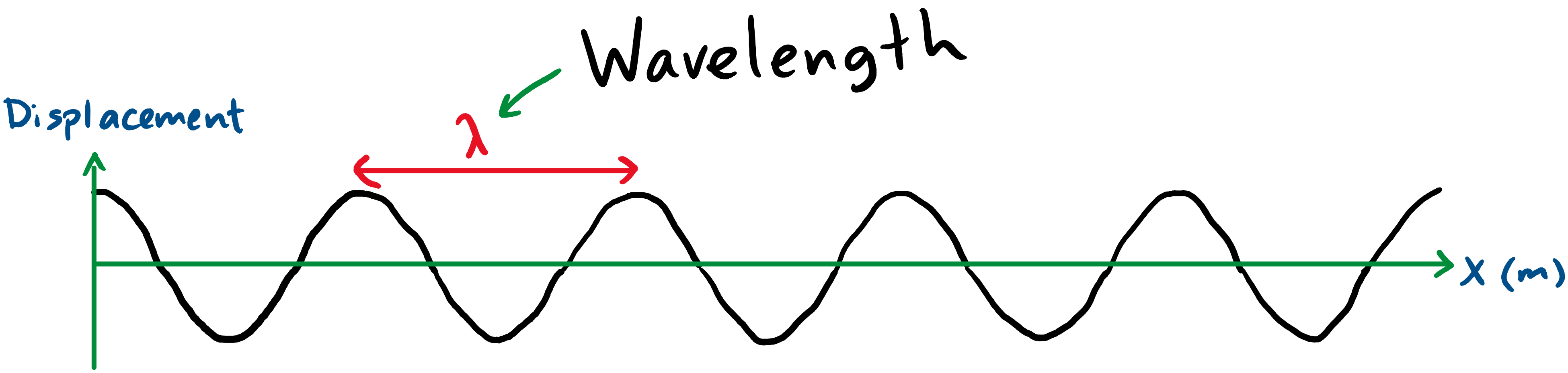


The picture shows a wave on a string at some time t=0. Which of the following represents the displacement of the string as a function of position at t=0?

A) 4mm · cos(x / 1m)
B) 4mm · cos(1m · x)
C) 4mm · cos(2 π / 1m · x)
D) 4mm · cos(1m / 2 π · x)
E) 4mm · cos(x - 1m)

Just like for D vs t in oscillator, but here t is replaced by x, and T is replaced by λ.

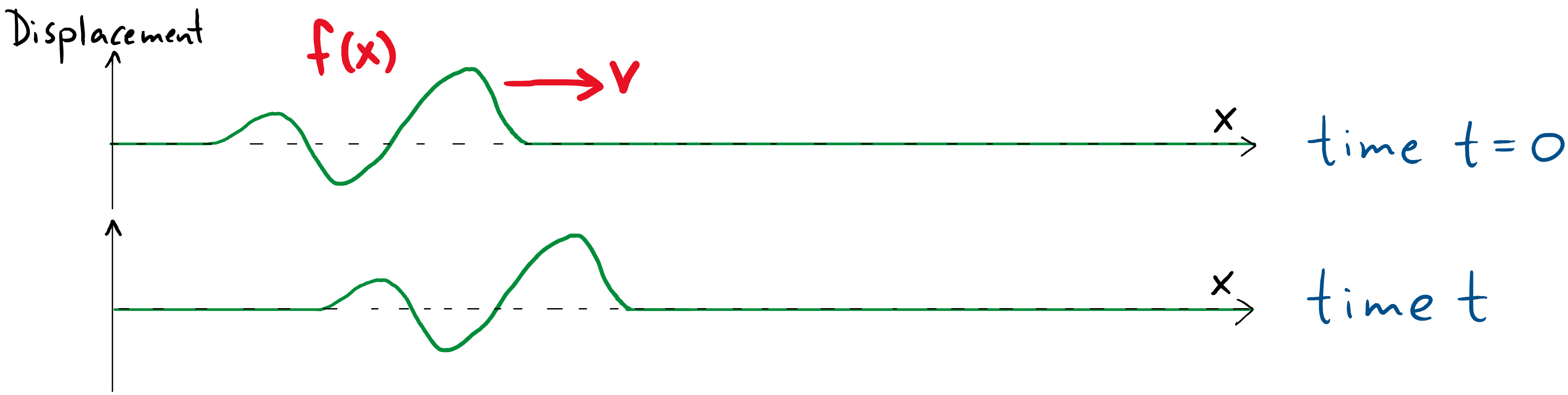
So $A \cdot \cos\left(\frac{2\pi}{\lambda} \cdot x\right)$



"Snapshot" graph: picture of the wave at an instant in time

$$D(x) = A\cos(kx + \phi)$$

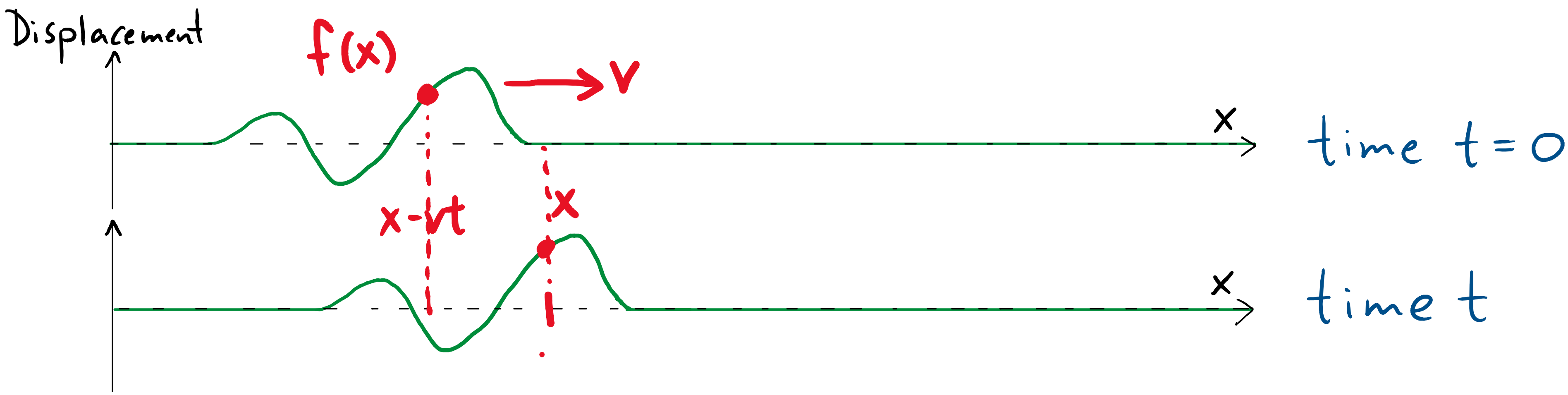
wave number: $k = \frac{2\pi}{\lambda}$



At time t=0, a right-moving wave pulse has displacement D(x,t=0) = f(x) shown in the top picture. At a later time t, the displacement will be described by

* Assume the pulse maintains its shape *

A) $D(x,t) = f(x)$
B) $D(x,t) = f(x) + vt$
C) $D(x,t) = f(x) - vt$
D) $D(x,t) = f(x + vt)$
E) $D(x,t) = f(x - vt)$



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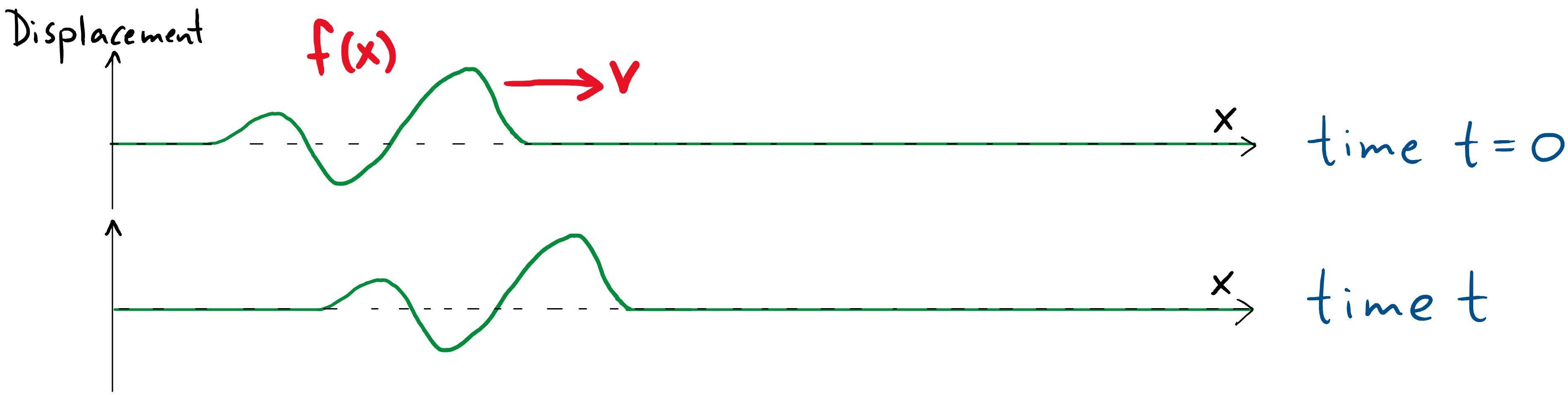
D) D(x,t) = f(x + vt)
E) D(x,t) = f(x – vt)

* Assume the pulse maintains its shape *

in time t : wave shifted by vt to the right
displacement at position x at time t is displacement at position x-vt in original graph.
So new displacement is f(x-vt)



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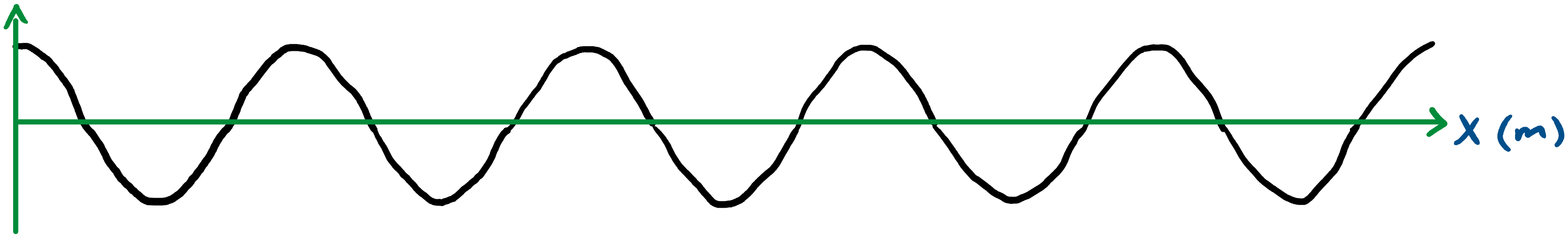
shape at $t=0$: $f(x)$

Right moving wave: $D(x,t) = f(x-vt)$

Left moving wave: $D(x,t) = f(x+vt)$

SINUSOIDAL CASE:



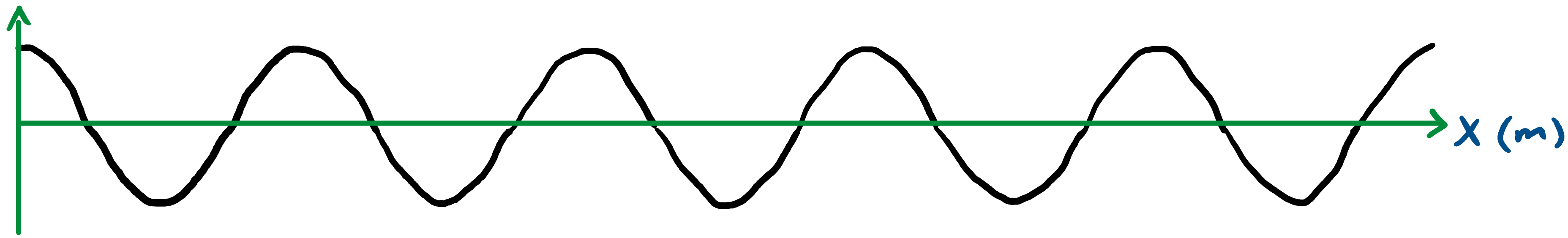
$$D(x, t=0) = A\cos\left(\frac{2\pi}{\lambda}\cdot x\right)$$

right moving wave: $D(x,t) = A\cos\left(\frac{2\pi}{\lambda}(x - vt)\right)$

left moving wave: $D(x,t) = A\cos\left(\frac{2\pi}{\lambda}(x + vt)\right)$

speed (i.e. this is a positive #)

SINUSOIDAL CASE:



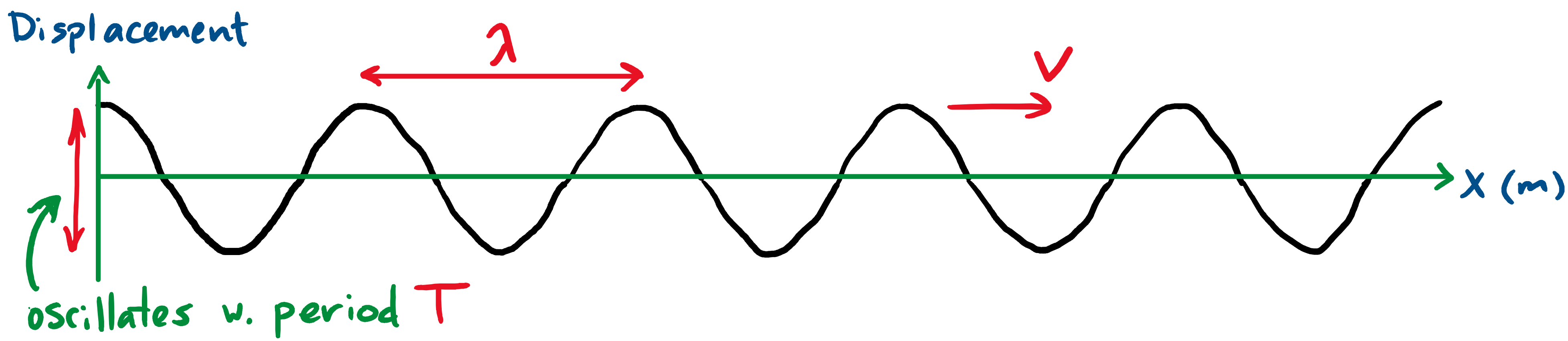
$$D(x, t=0) = A\cos\left(\frac{2\pi}{\lambda}\cdot x\right)$$

right moving wave: $D(x,t) = A\cos\left(\frac{2\pi}{\lambda}(x - vt)\right)$

$$= A\cos\left(\frac{2\pi}{\lambda}x - 2\pi\frac{v}{\lambda}t\right)$$

$$= A\cos\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{T}\cdot t\right)$$

$$= A\cos(kx - \omega t)$$



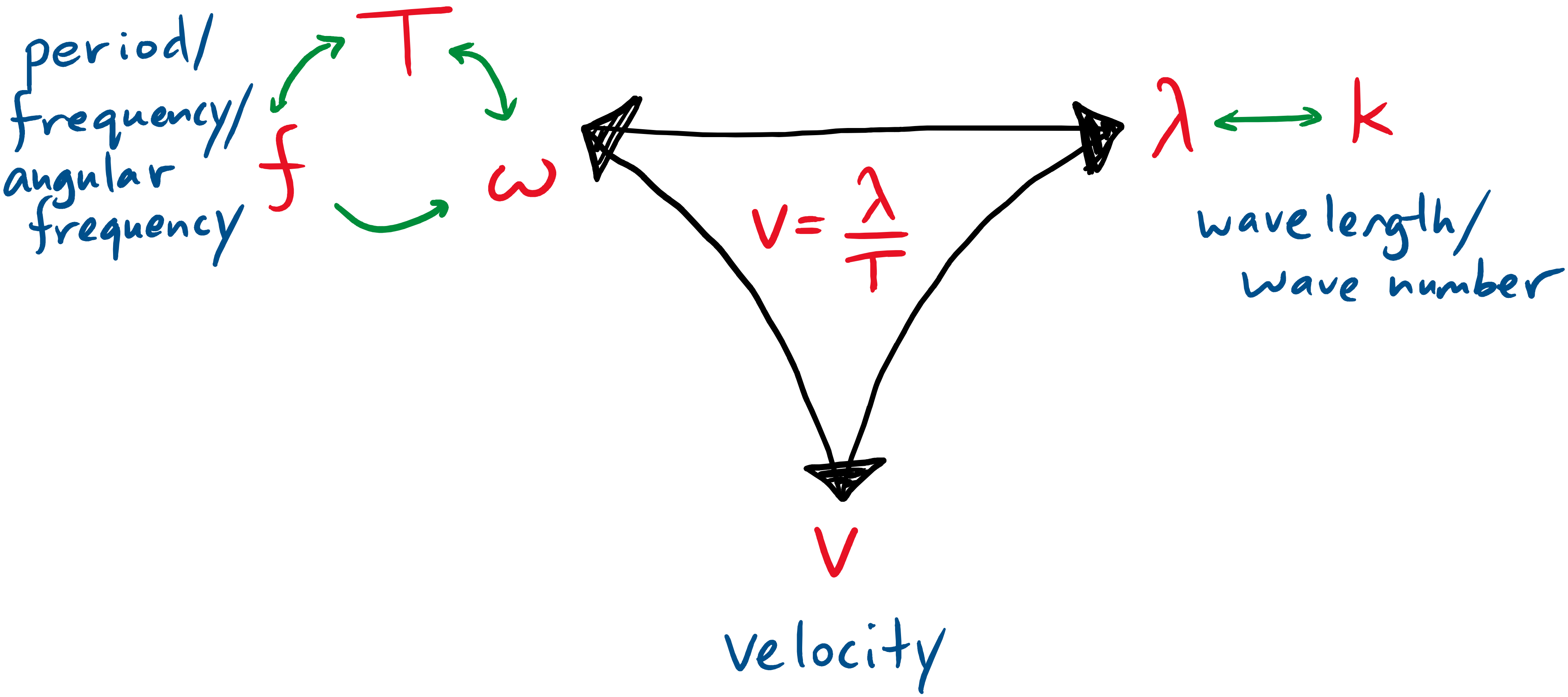
Right moving wave: $D(x,t) = A\cos(kx - \omega t)$

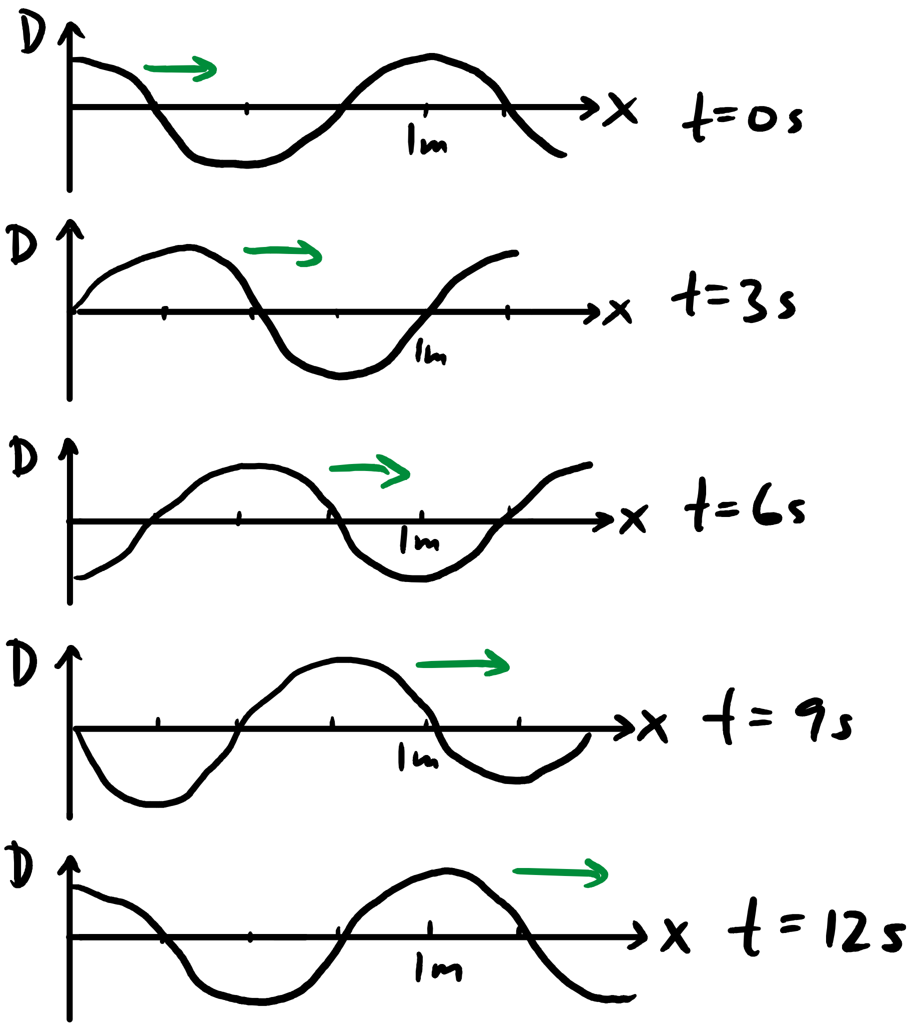
Left moving wave: $D(x,t) = A\cos(kx + \omega t)$

$$k = \frac{2\pi}{\lambda} \qquad \omega = \frac{2\pi}{T}$$

Properties of waves:

A : amplitude





Which of the following represents the displacement of the wave shown as a function of position

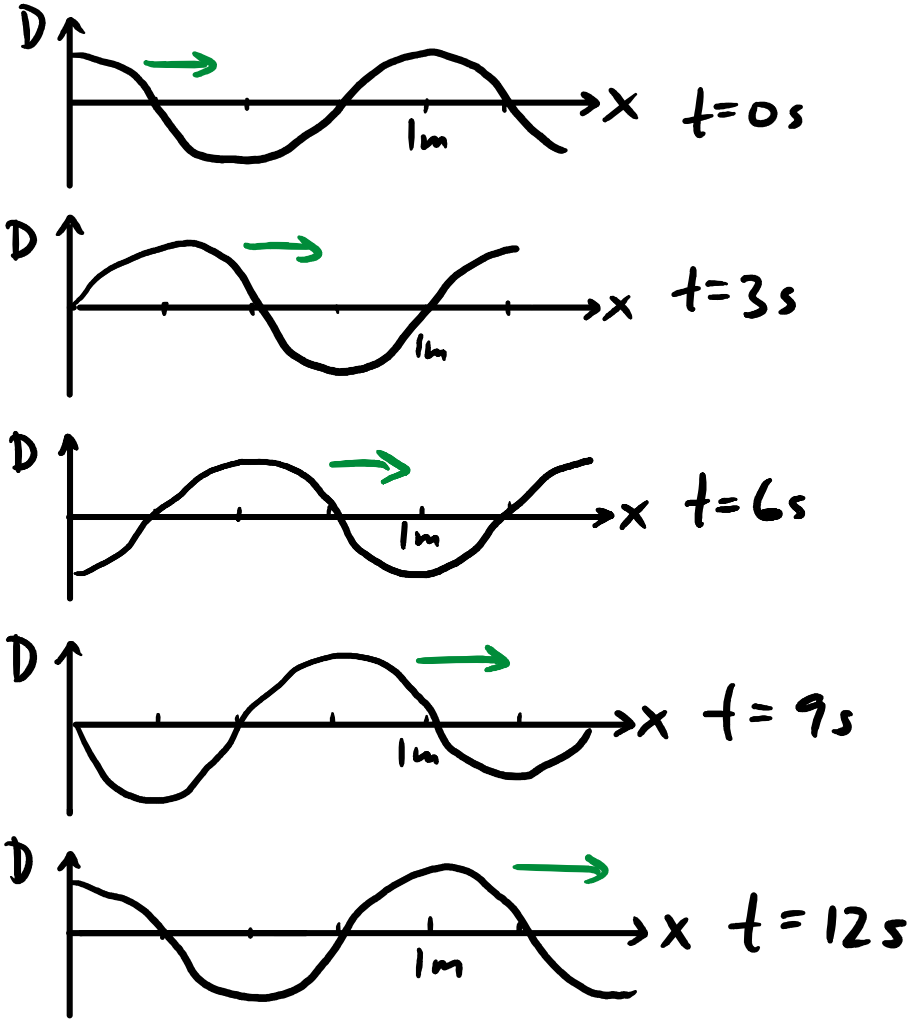
A) $D = A\cos\left(\frac{2\pi}{1\,m}\cdot x - \frac{t}{12\,s}\right)$

B) $D = A\cos\left(\frac{2\pi}{1\,m}\cdot x - 12\,s\cdot t\right)$

C) $D = A\cos\left(\frac{2\pi}{1\,m}\cdot x - \frac{2\pi}{12\,s}\cdot t\right)$

D) $D = A\cos\left(\frac{2\pi}{1\,m}\cdot x - \frac{12\,s}{2\pi}\cdot t\right)$

E) $D = A\cos\left(\frac{2\pi}{1\,m}\cdot x - \frac{\pi}{2}\cdot t\right)$



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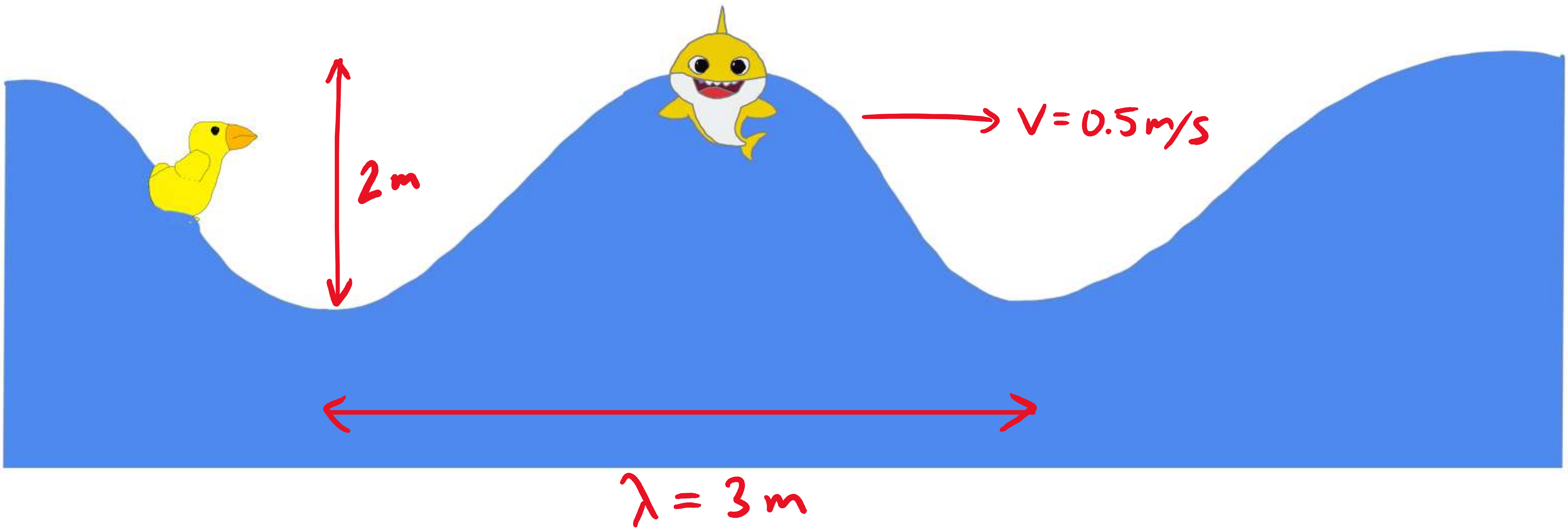
C) $D = A\cos\left(\frac{2\pi}{1m}\cdot x - \frac{2\pi}{12s}\cdot t\right)$

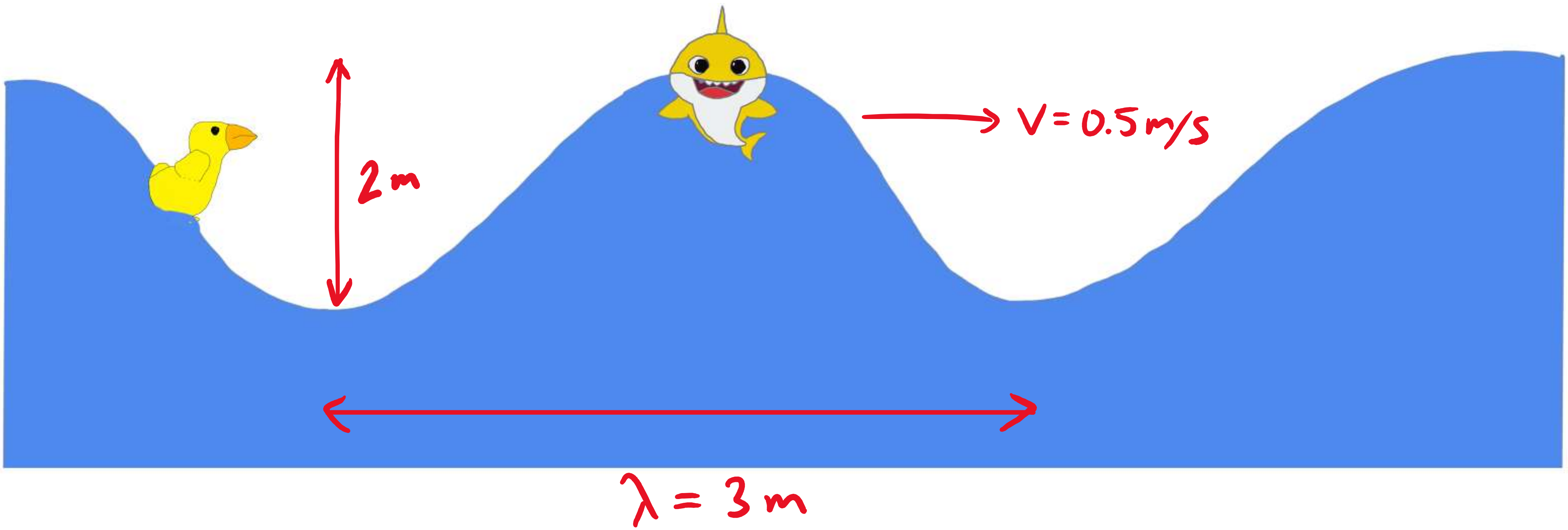
D) $D = A\cos\left(\frac{2\pi}{1m}\cdot x - \frac{12s}{2\pi}\cdot t\right)$

E) $D = A\cos\left(\frac{2\pi}{1m}\cdot x - \frac{\pi}{2}\cdot t\right)$

Shift by full period in 12s, so want phase -2π for $t=12s$



Discussion question: what will be Baby Shark's maximum vertical velocity?

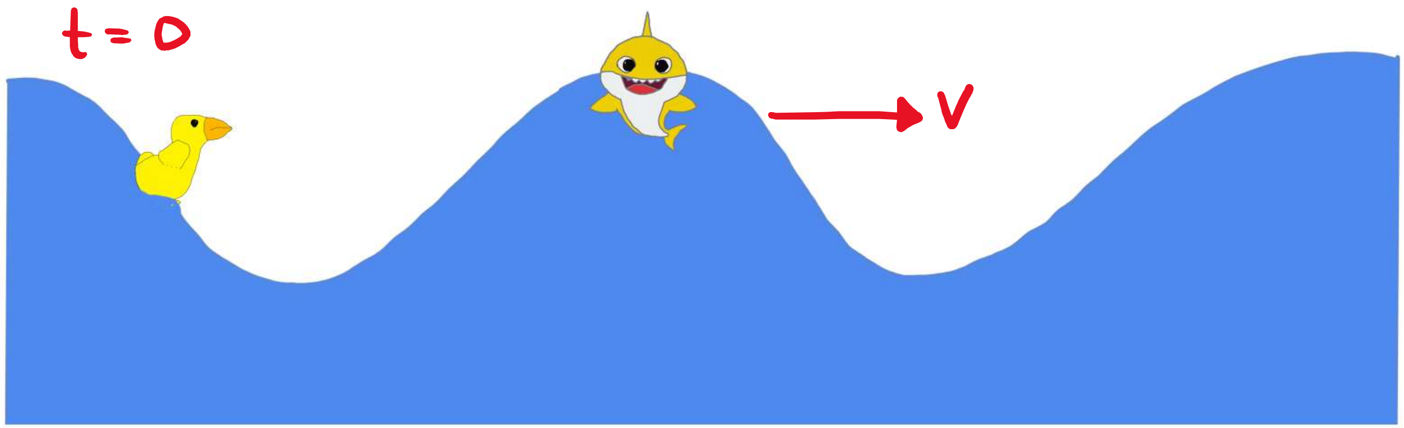


Discussion question: what will be Baby Shark's maximum vertical velocity?

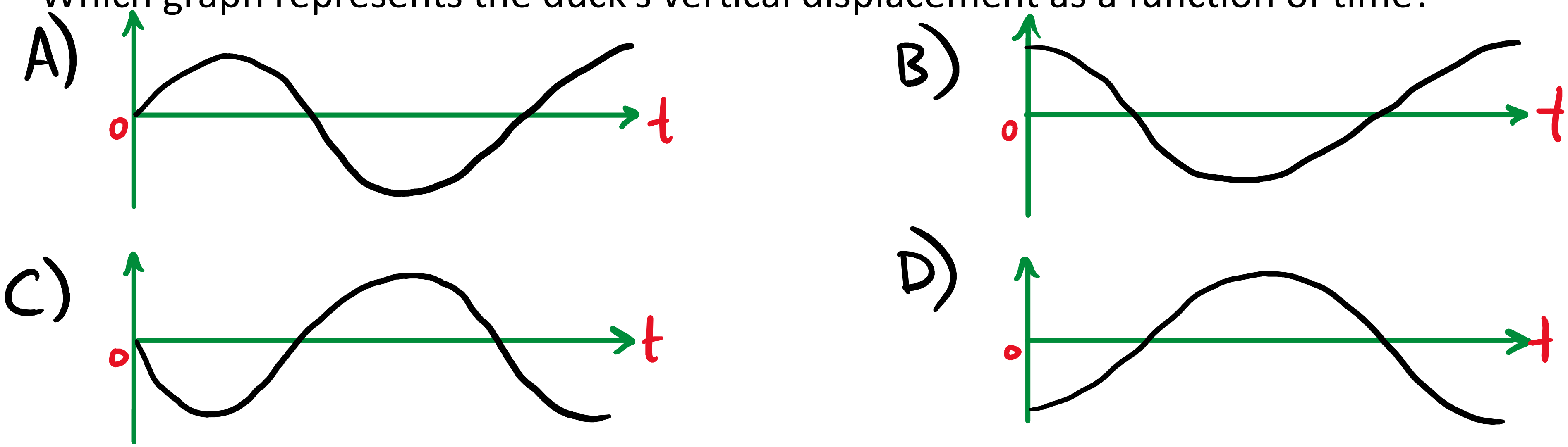
Shark is in simple harmonic motion, $D = A\cos(\omega t + \phi)$. Velocity is $\frac{dD}{dt} = -A\omega\sin(\omega t+\phi)$. Max v is $A\omega = A\cdot\frac{2\pi}{T} = A\cdot\frac{2\pi}{\lambda/v}$

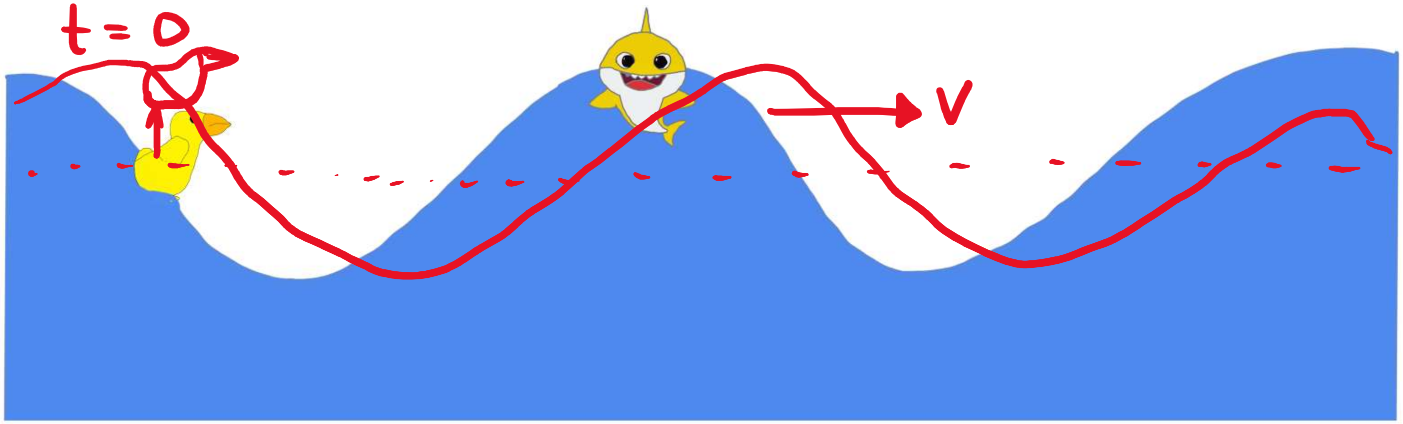
$= 1\text{m}\cdot\frac{2\pi}{6\text{s}} = \frac{\pi}{3}\,\text{m/s}$

Bonus. Slides.



Which graph represents the duck's vertical displacement as a function of time?

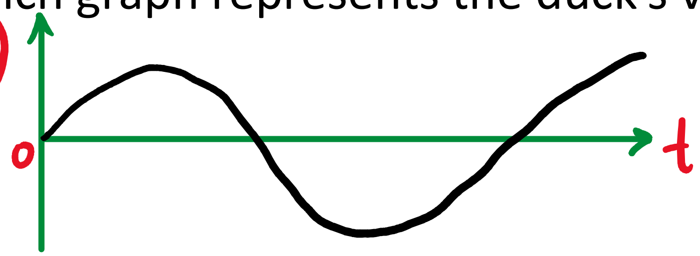




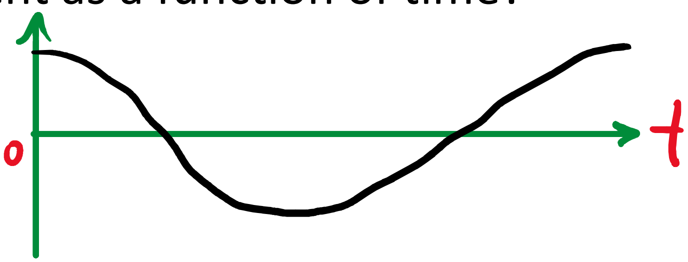
After short amount of time, duck moves up. Eventually, will be lower than original height.

Which graph represents the duck's vertical displacement as a function of time?

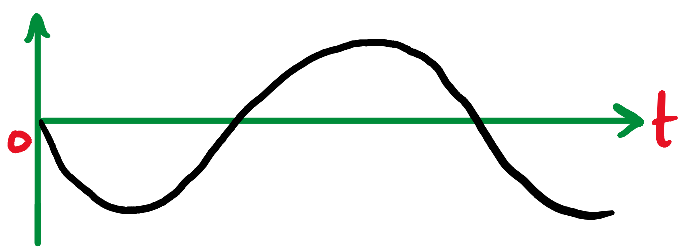
A)



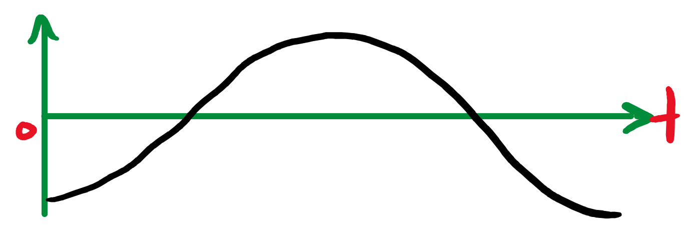
B)

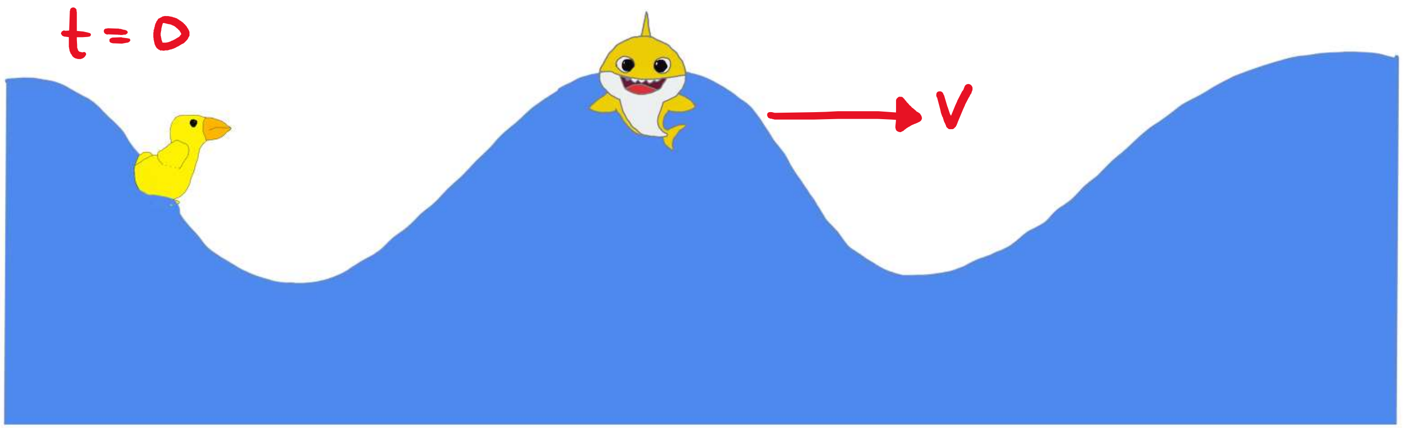


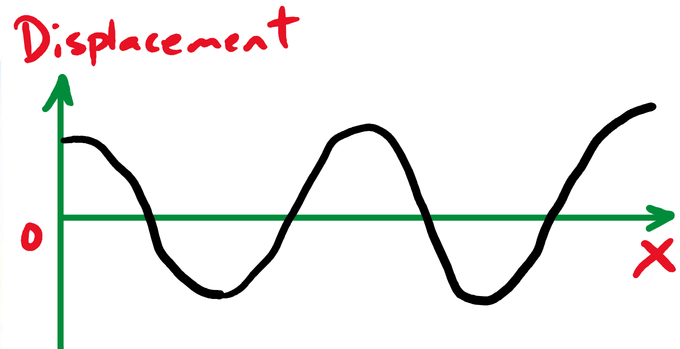
C)



D)

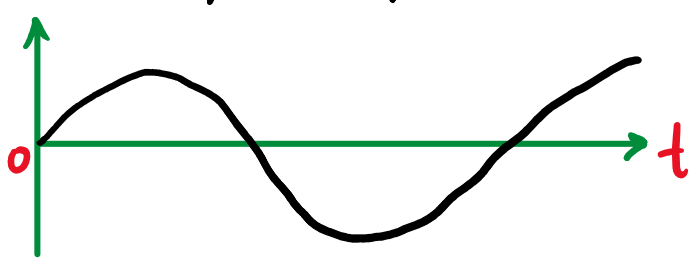




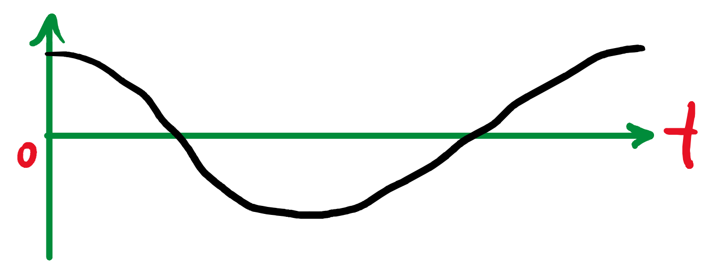


★ Snapshot Graph ★

★ History Graphs ★



Duck



Baby Shark