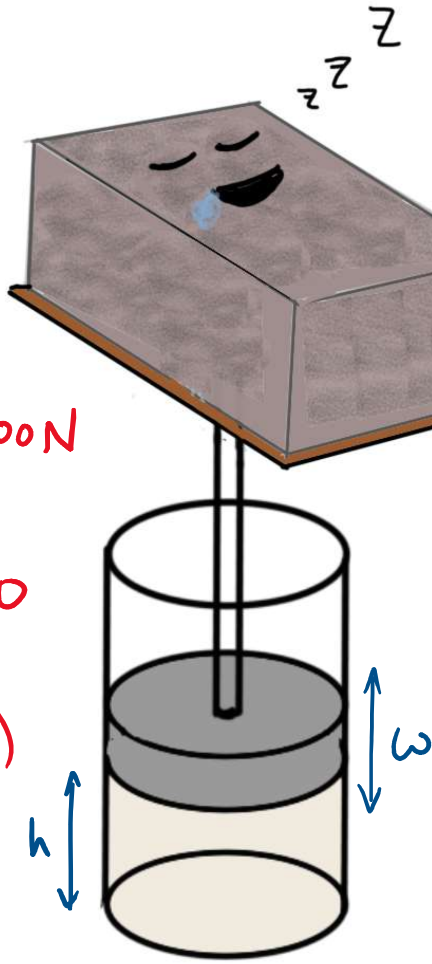
Office hours today: after class (Remo), 3:30-4:30pm (Zoom)

Learning goals for today:

- Describe how kinetic and potential energy vary with time during simple harmonic motion.
- To use conservation of energy to predict amplitude and/or maximum velocity from the displacement and velocity at any given time.
- To describe the relation between the amplitude of an oscillator and the energy stored in the system.
- To explain how oscillating systems losing a fixed fraction of their energy to the environment per oscillation can be described by oscillations with an exponential decaying amplitude

Last time in Physics 157...



$$F_{NET} = \frac{1500\,J}{h} - 5000\,N$$

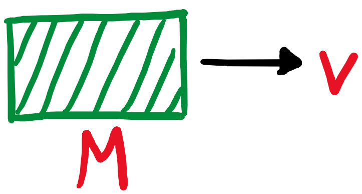
$$h_{eq}:\ F_{NET} = 0$$

$$k = -F_{NET}'(h_{eq})$$

$$\omega = \sqrt{\frac{k}{m}}$$

Energy in S.H.M.

Kinetic energy:



$$\boxed{K.E. = \frac{1}{2} M v^2}$$

Potential Energy relative to equilibrium:

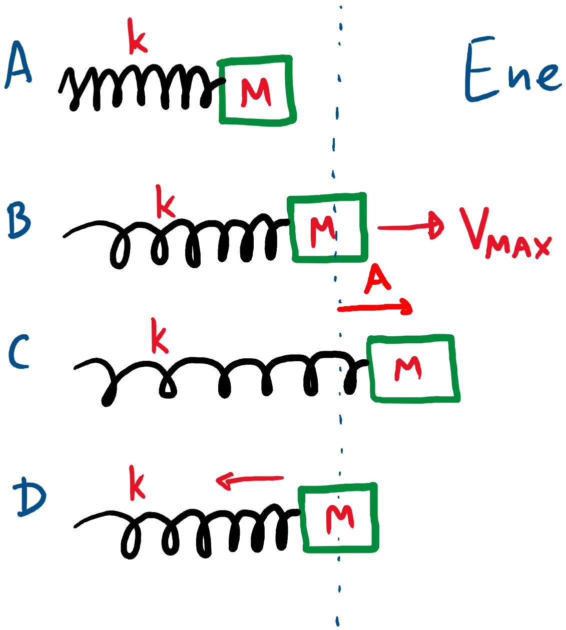
compressed:

normal:

stretched

$$\boxed{P.E. = \frac{1}{2} k (\Delta x)^2}$$

Energy in simple harmonic motion:



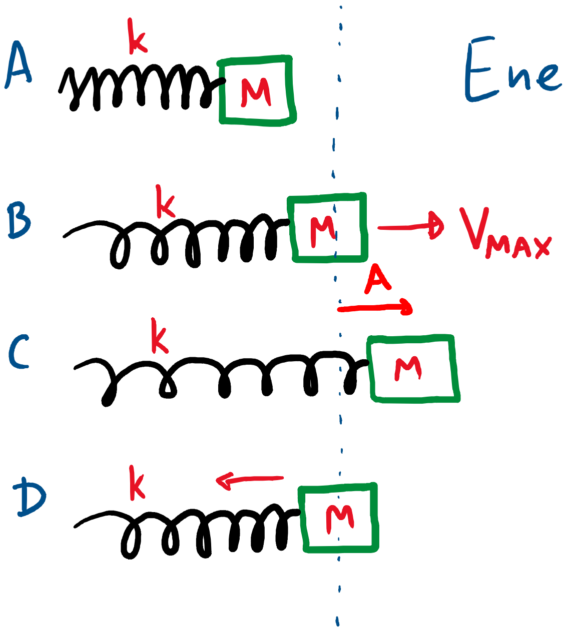
Energy at B or C relative to mass at equilibrium:

$$\frac{1}{2} M V_{MAX}^2$$

Rewrite in terms of k and A:

$$V_{MAX} = A\omega = A\sqrt{\frac{k}{M}}$$

Energy in simple harmonic motion:



Energy at B or C relative to mass at equilibrium:

$$\frac{1}{2} M v_{MAX}^2$$

Rewrite in terms of k and A:

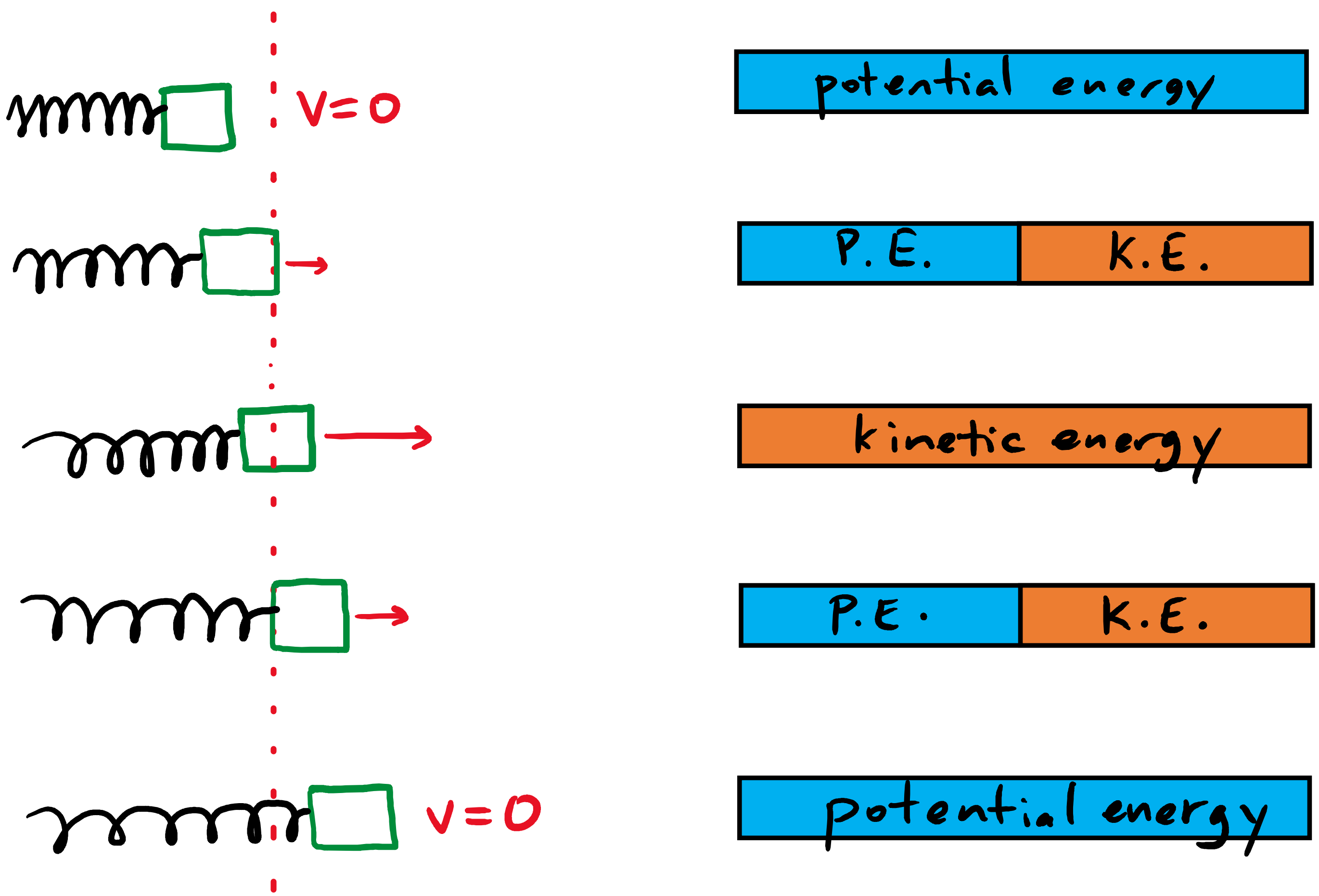
$$V_{MAX} = A\omega = A\sqrt{\frac{k}{M}}$$

$$\frac{1}{2} M v_{MAX}^2 = \frac{1}{2} M \times \left(A \times \sqrt{\frac{k}{m}}\right)^2 = \frac{1}{2} k A^2$$

this must be the formula for potential energy when $\Delta x = A$

Total energy is conserved $\frac{1}{2}Mv^2 + \frac{1}{2}kx^2 = E$ constant

equal to initial energy



A 0.5 kg mass is attached to a horizontal spring of spring constant 200 N/m. If the spring is initially compressed by 0.1m, and the mass is then released, what is the speed of the block when the spring is at its equilibrium length?

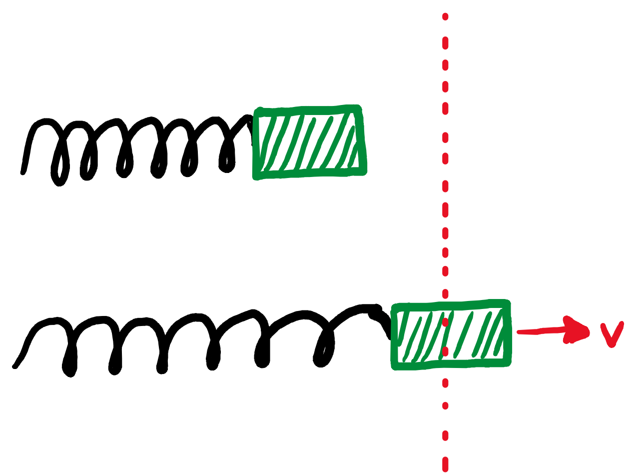
A. 1 m/s

B. 2 m/s

C. 3 m/s

D. 4 m/s

E. 5 m/s



M k

A 0.5 kg mass is attached to a horizontal spring of spring constant 200 N/m. If the spring is initially compressed by 0.1m, and the mass is then released, what is the speed of the block when the spring is at its equilibrium length?

Δx

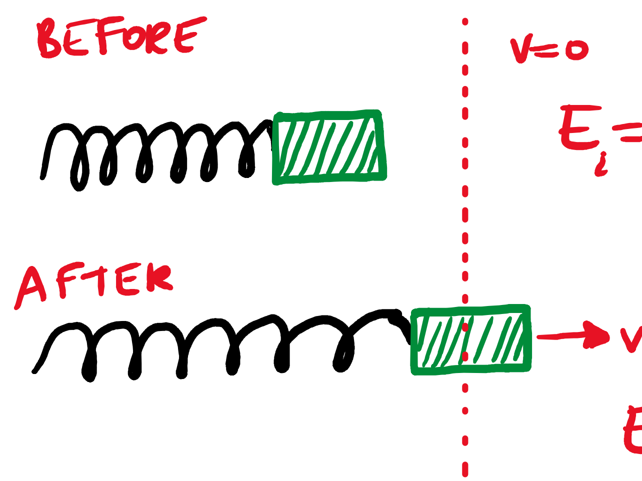
A. 1 m/s

B. 2 m/s

C. 3 m/s

D. 4 m/s

E. 5 m/s

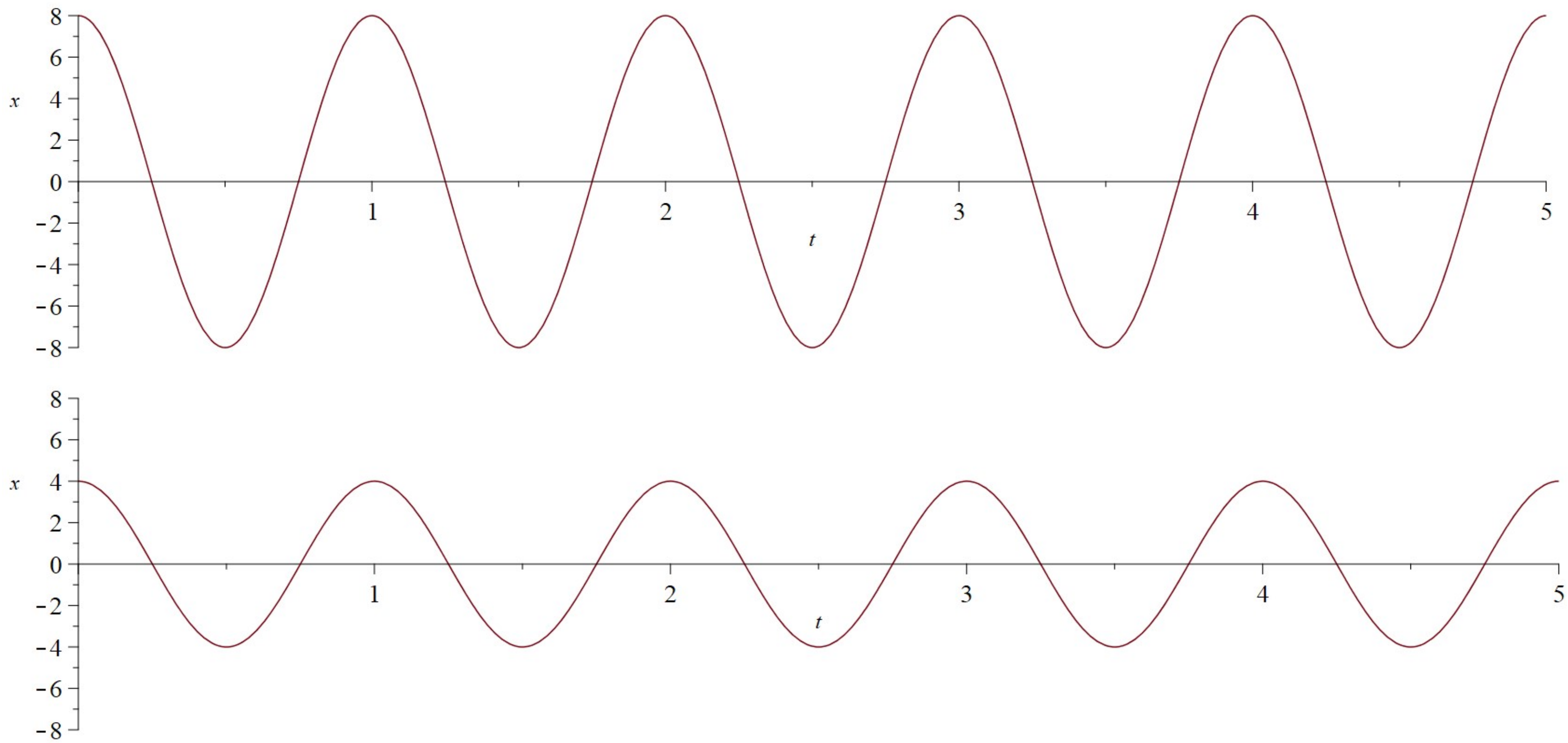


$E_i = \frac{1}{2} k(\Delta x)^2$

$\Delta x = 0$

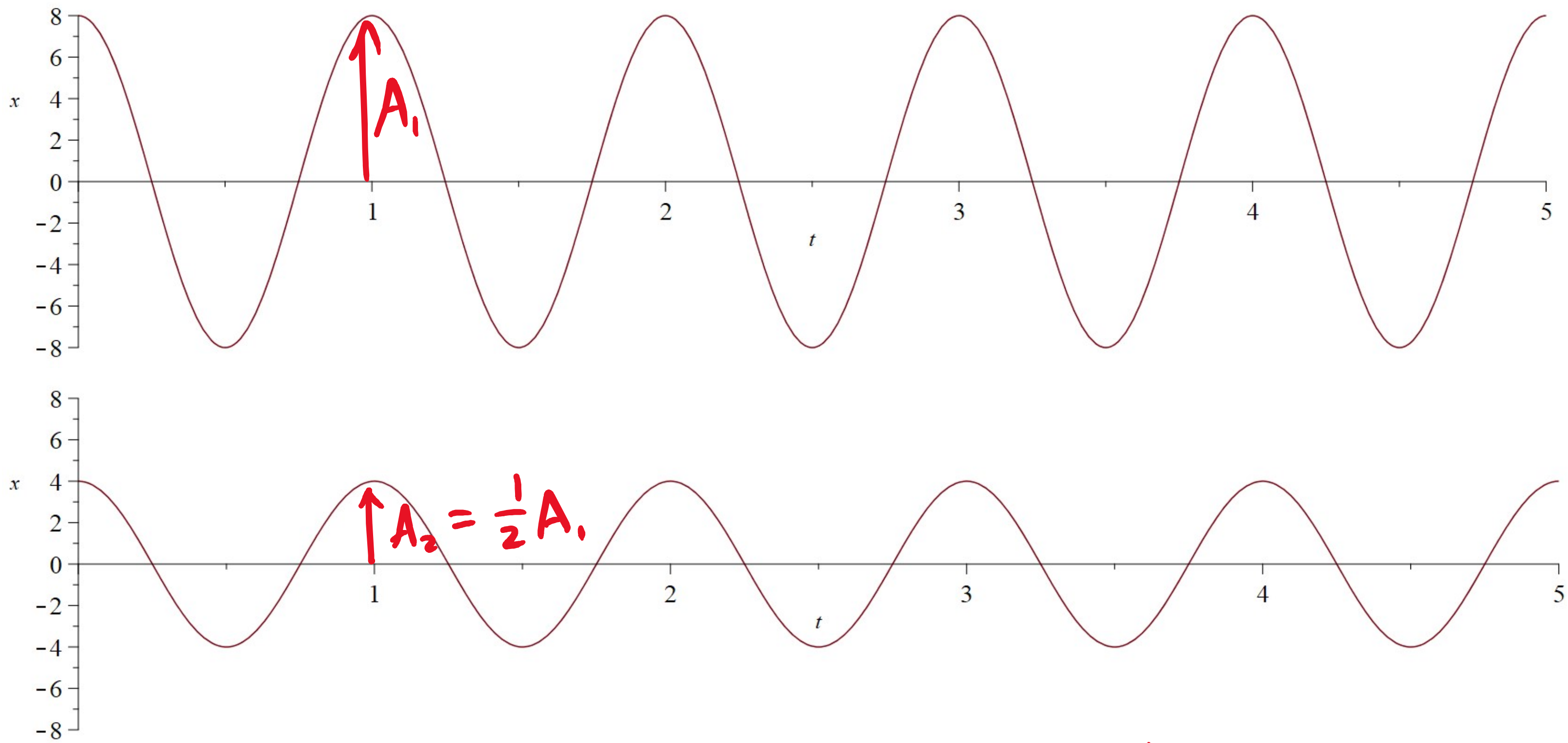
$E_f = \frac{1}{2} M v^2$

Energy conserved: $\frac{1}{2} M v^2 = \frac{1}{2} k(\Delta x)^2 \Rightarrow v = \sqrt{\frac{k}{m}} \cdot \Delta x = 2 m/s$



The two graphs show different oscillations for the same system. Compared with the first case, the total potential plus kinetic energy in the second case is

A) The same B) Twice as big C) Half as big D) One quarter as big E) One 16th as big



for each: energy same at all times. Look at time when $v=0$.

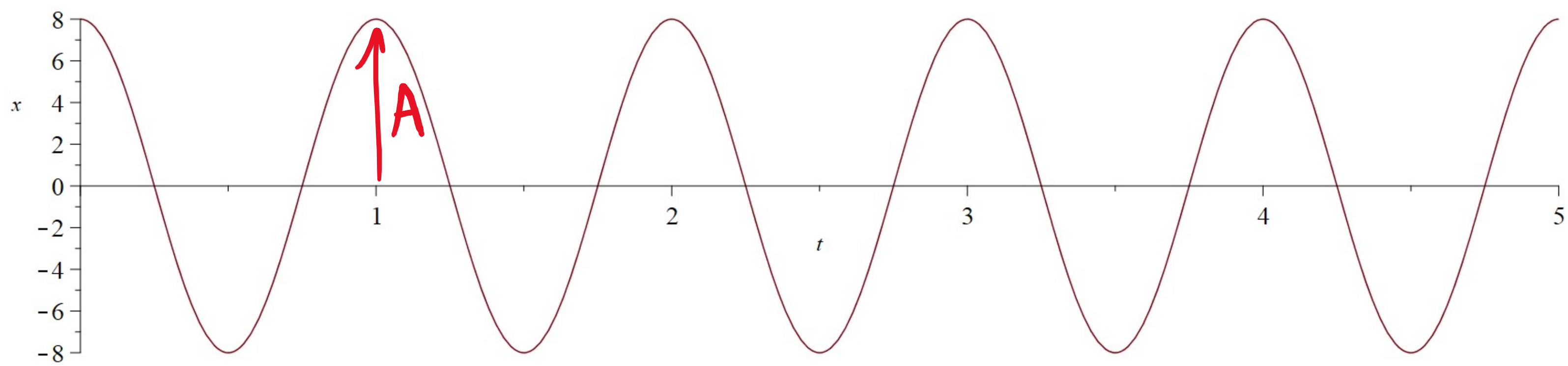
The two graphs show different oscillations for the same system. Compared with the first case, the total potential plus kinetic energy in the second case is

Then $E = \frac{1}{2} k A^2$ so $\frac{E_2}{E_1} = \frac{A_2^2}{A_1^2} = \frac{1}{4}$

A) The same B) Twice as big C) Half as big D) One quarter as big E) One 16th as big

Key fact about oscillating systems:

Energy is proportional to amplitude squared



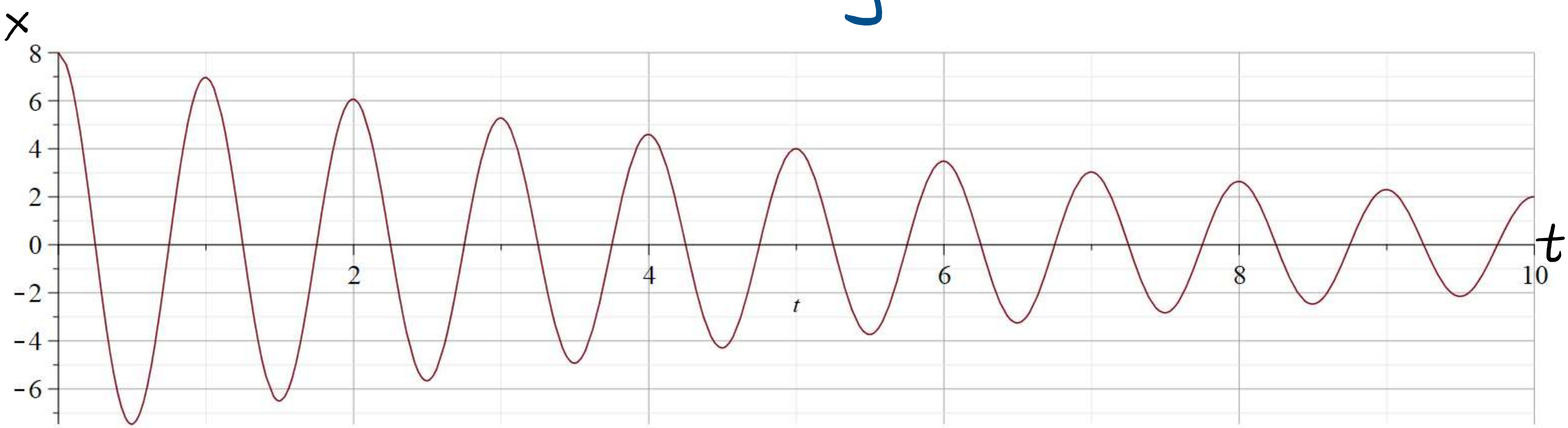
$$E_{TOT} \propto A^2$$

Real oscillators: energy is lost

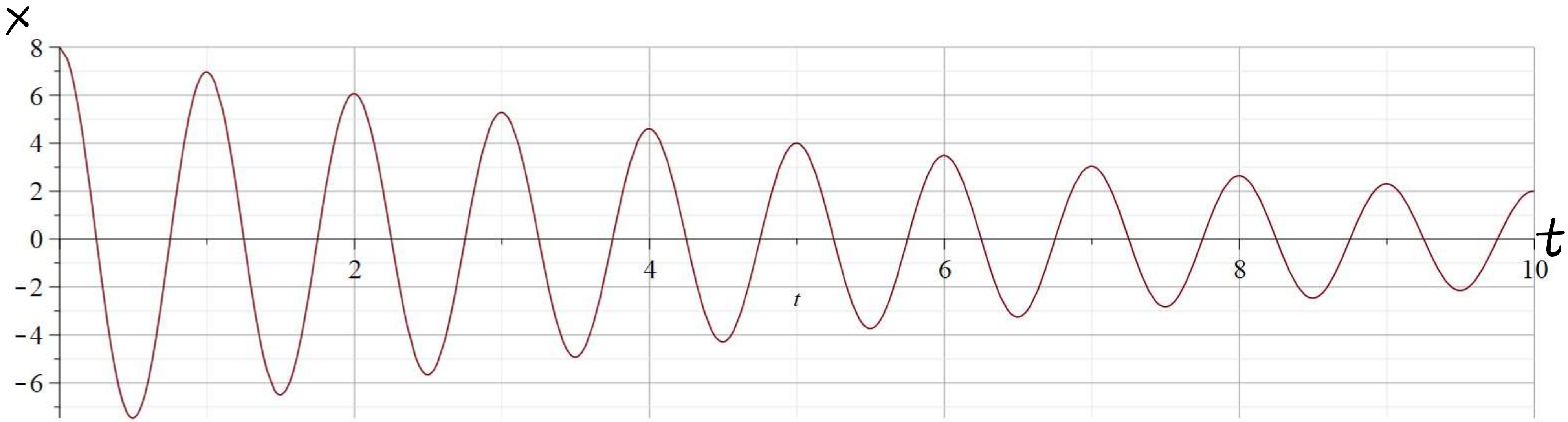
friction

air/fluid drag

heating of system



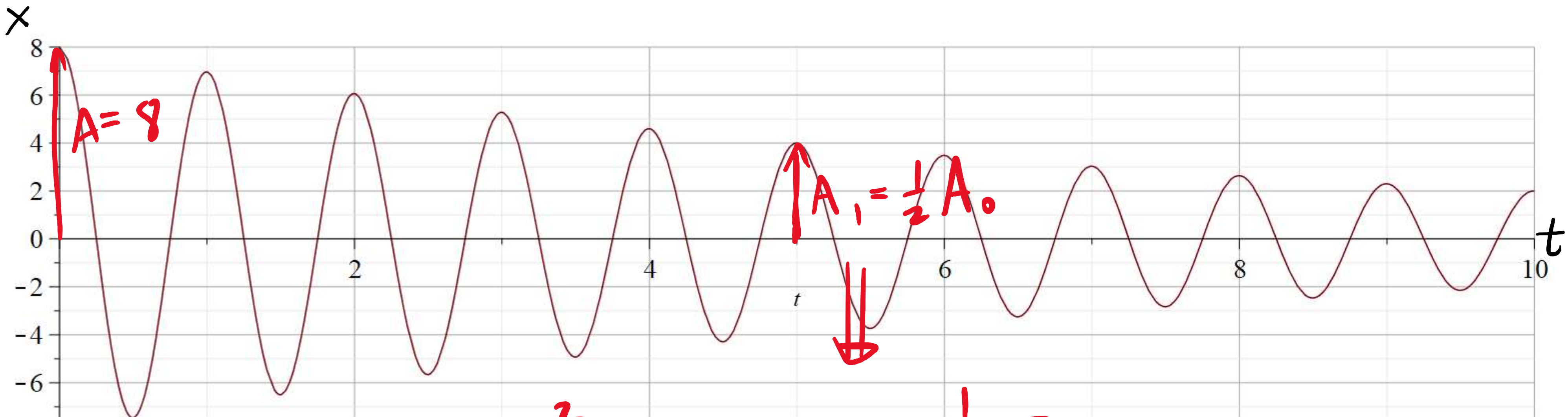
amplitude decreases with time



What fraction of the original kinetic + potential energy remains in the oscillator at t=5s?

A) All of it.
B) Half of it.
C) One quarter of it.
D) $1/\sqrt{2}$ of it.

EXTRA: what fraction of the energy at t=5s remains at t=10s?



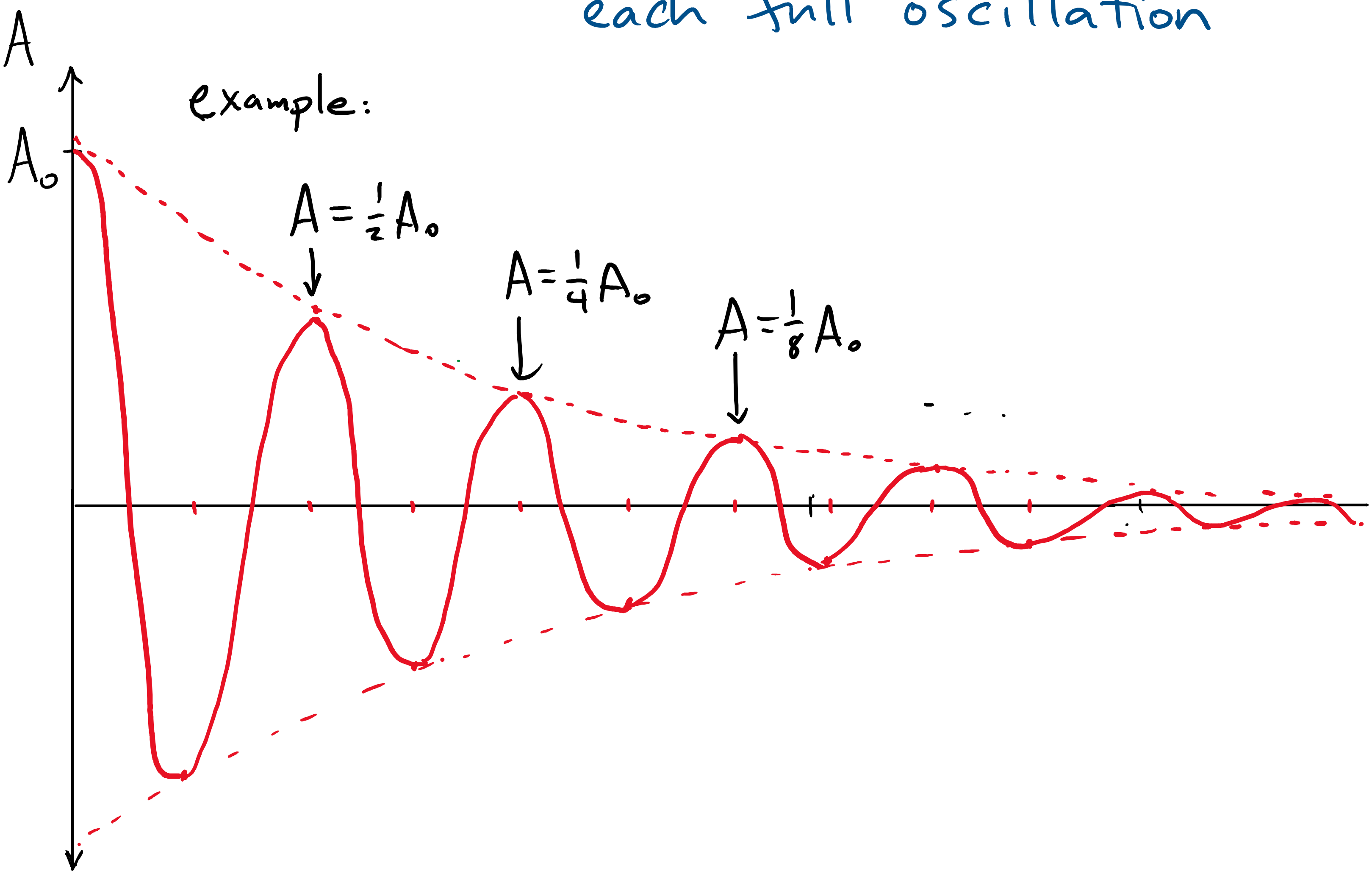
$E \propto \text{amplitude}^2$ $\quad E_1 = \frac{1}{4} E_0$

What fraction of the original kinetic + potential energy remains in the oscillator at t=5s?

A) All of it.
B) Half of it.
C) One quarter of it.
D) $1/\sqrt{2}$ of it.

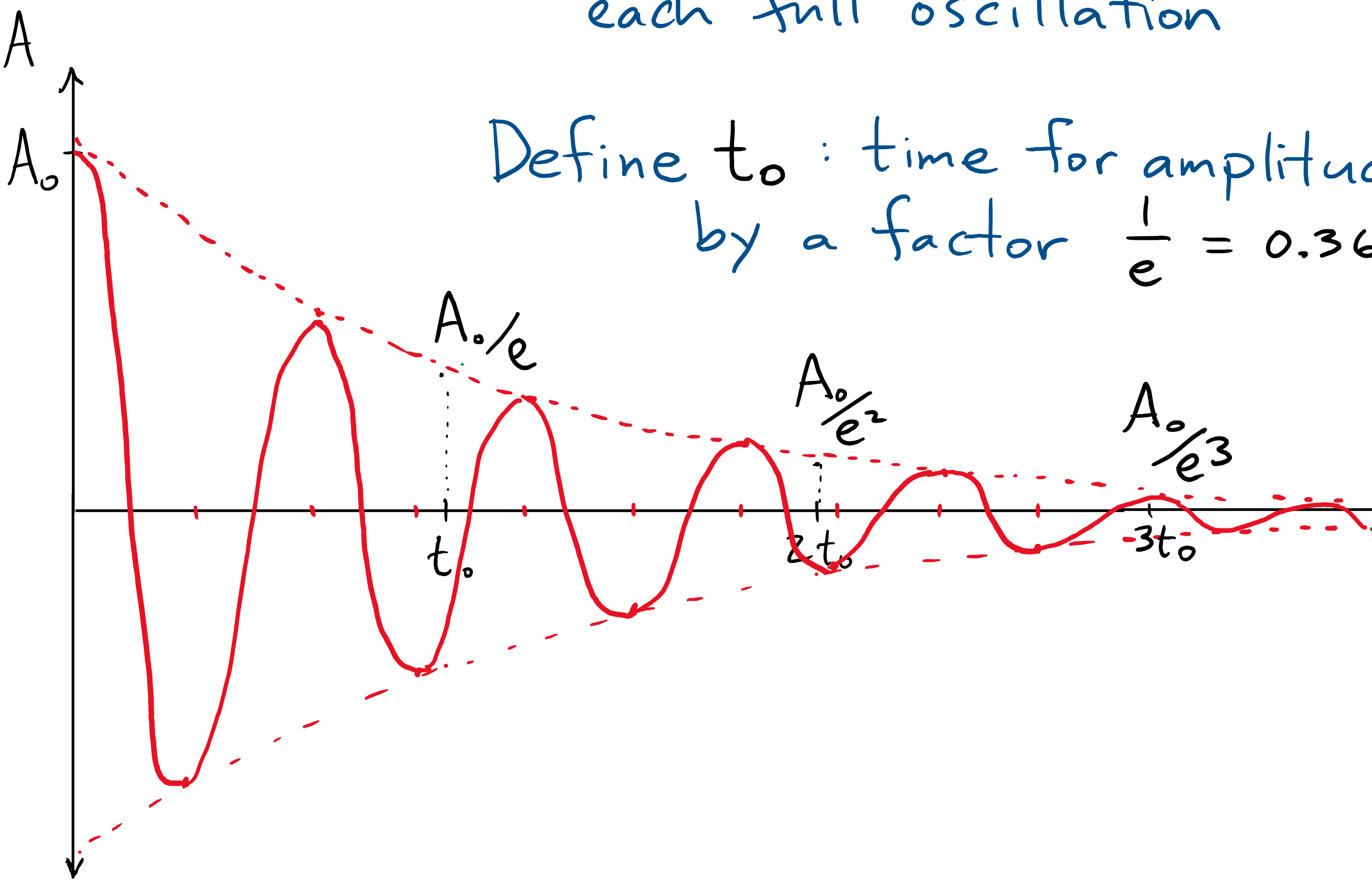
EXTRA: what fraction of the energy at t=5s remains at t=10s? $\frac{1}{4}$

Common situation: amplitude decreases by same fraction each full oscillation

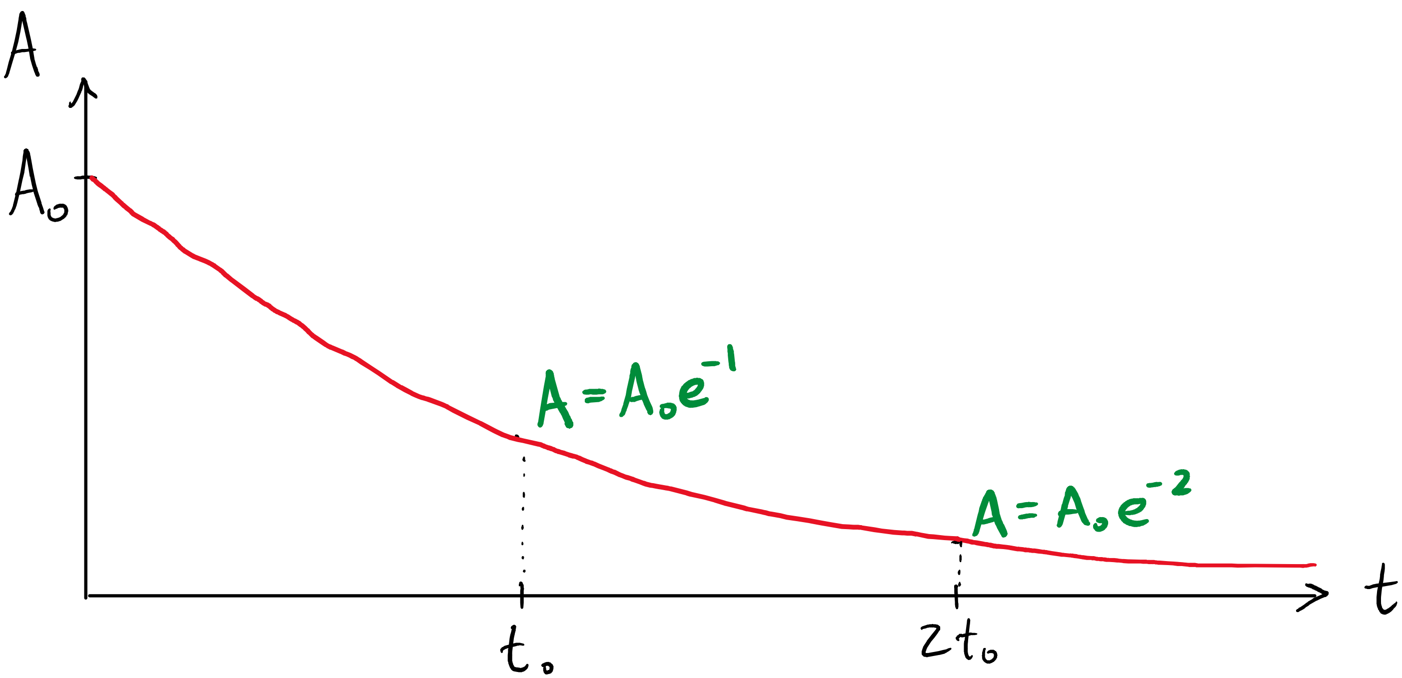


Common situation: amplitude decreases by same fraction each full oscillation

Define t_0 : time for amplitude to decrease by a factor $\frac{1}{e} = 0.3679$



Exponential decay: Amplitude is $A_0 \times \frac{1}{e^n}$ where n is # multiples of t_0.



Exponential decay: Amplitude is $A_0 \times \frac{1}{e^n}$ where

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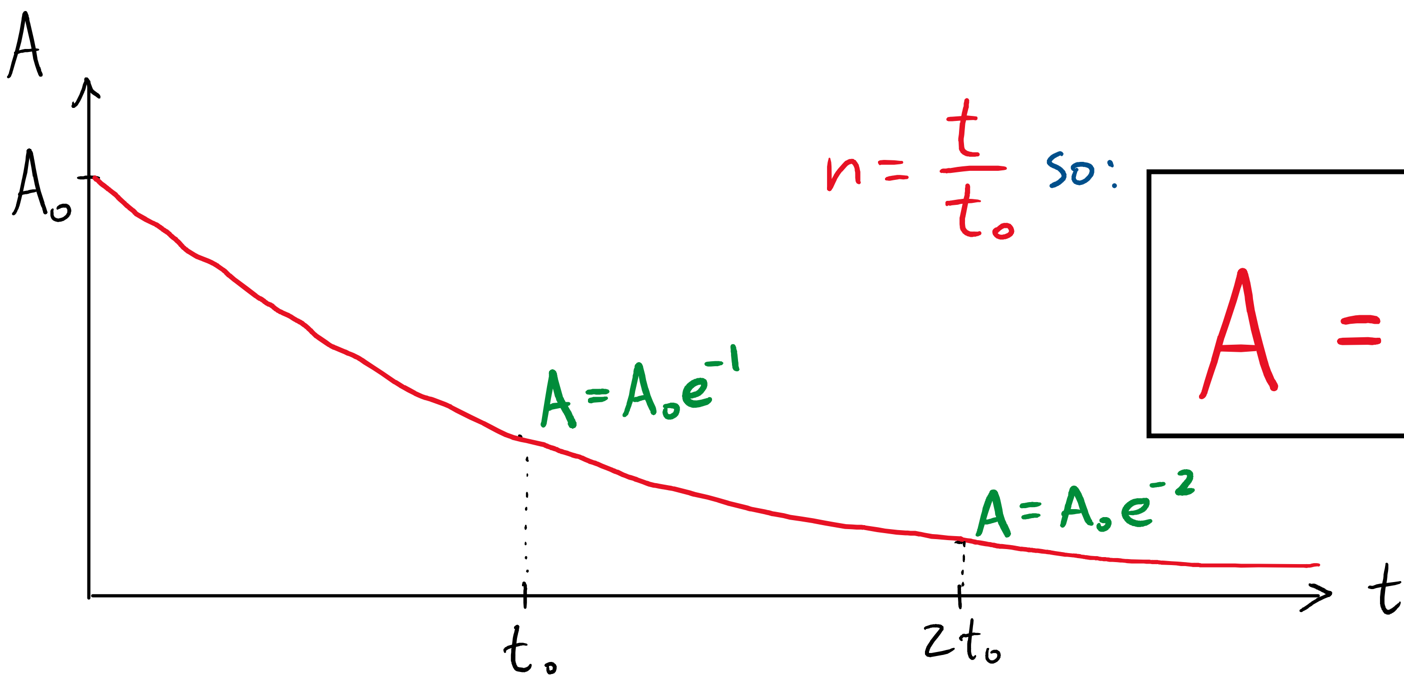
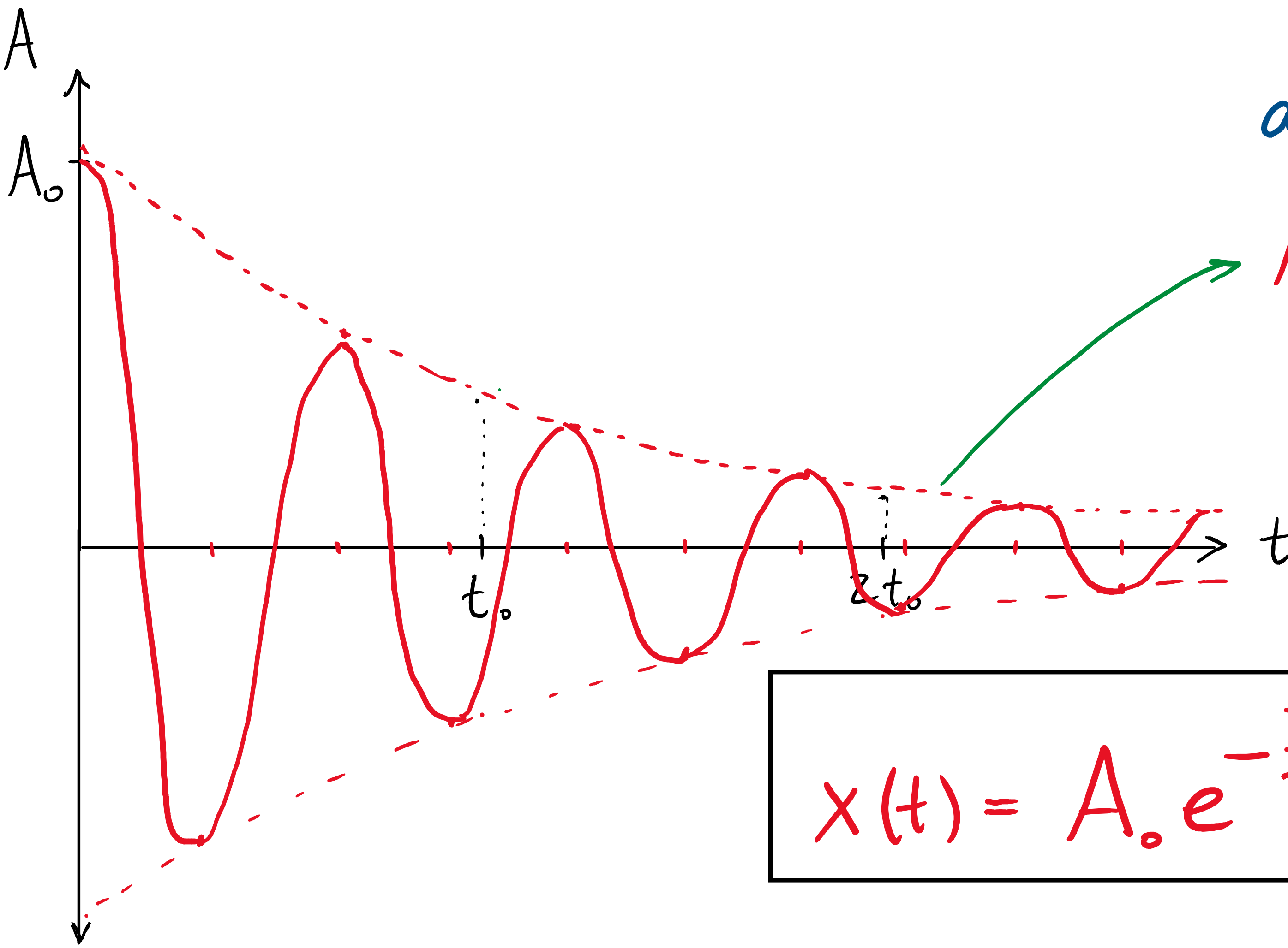


$n = \frac{t}{t_0}$ so:

$$\boxed{A = A_0 e^{-t/t_0}}$$

Damped oscillations

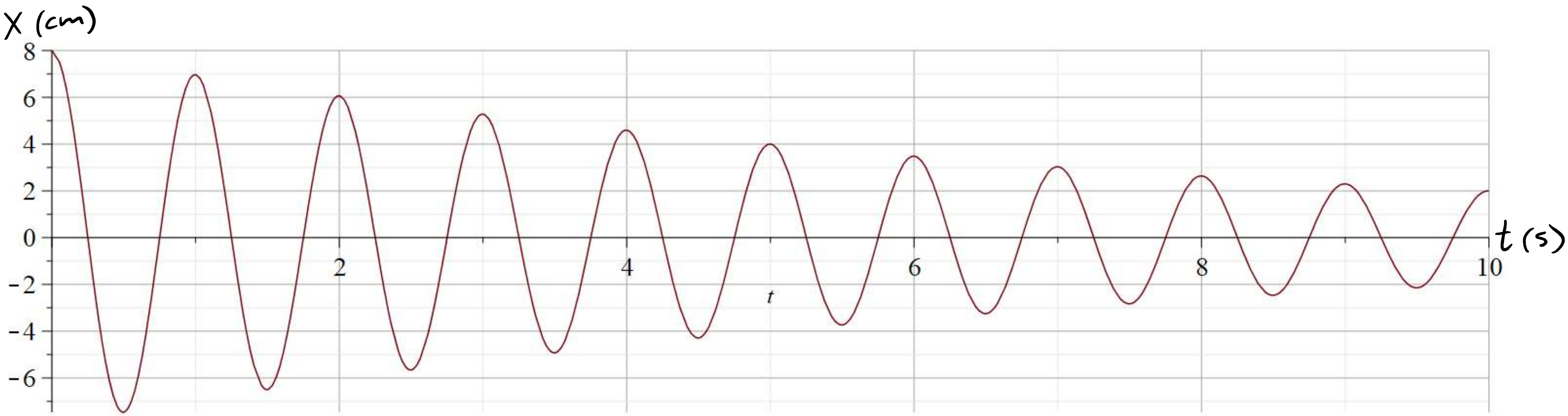


amplitude:

$$A = A_0 e^{-\frac{t}{t_0}}$$

multiplies cosine

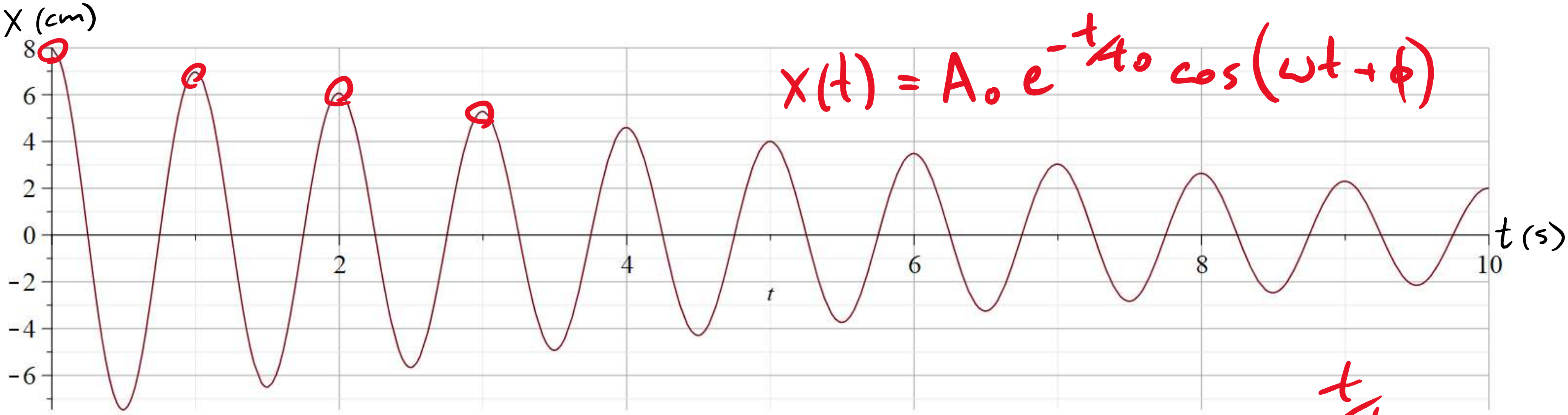
$$\boxed{x(t) = A_0 e^{-\frac{t}{t_0}} \cos(\omega t + \phi)}$$



The graph shows displacement vs time for a damped oscillation. The time constant t_0 in this case is nearest to

A) 1s B) 3s C) 5s D) 7s E)9s

EXTRA: Can you find t_0 exactly?



$x(t) = A_0 e^{-t/t_0} \cos(\omega t + \phi)$

At circled points, $\cos = 1$ so $x(t) = A_0 e^{-t/t_0}$

The graph shows displacement vs time for a damped oscillation. The time constant t_0 in this case is nearest to

At $t = 0$, $x = 8\text{cm}$. At $t = 2\text{s}$, $x = 6\text{cm}$.

A) 1s B) 3s C) 5s D) 7s E) 9s

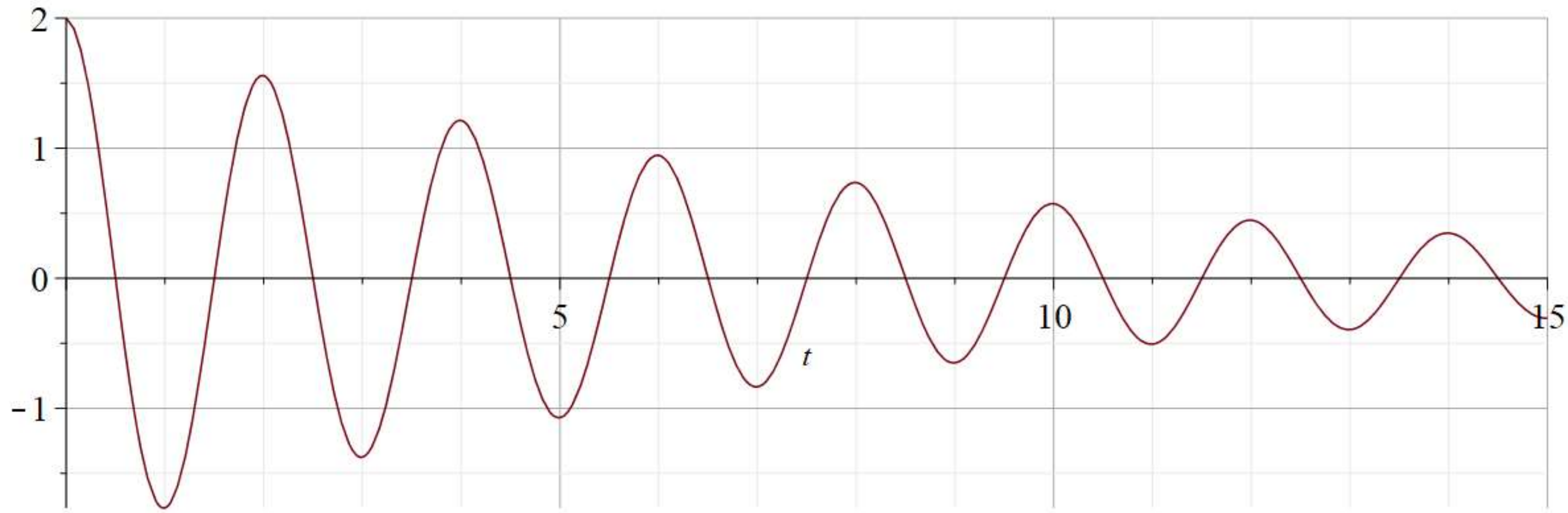
$6\text{cm} = 8\text{cm} \times e^{-\frac{(2s)}{t_0}}$

$\Rightarrow e^{-2s/t_0} = 0.75$

$\Rightarrow -\frac{2s}{t_0} = \ln(0.75)$

$\Rightarrow t_0 \approx 7s$

EXTRA: Can you find t_0 exactly?

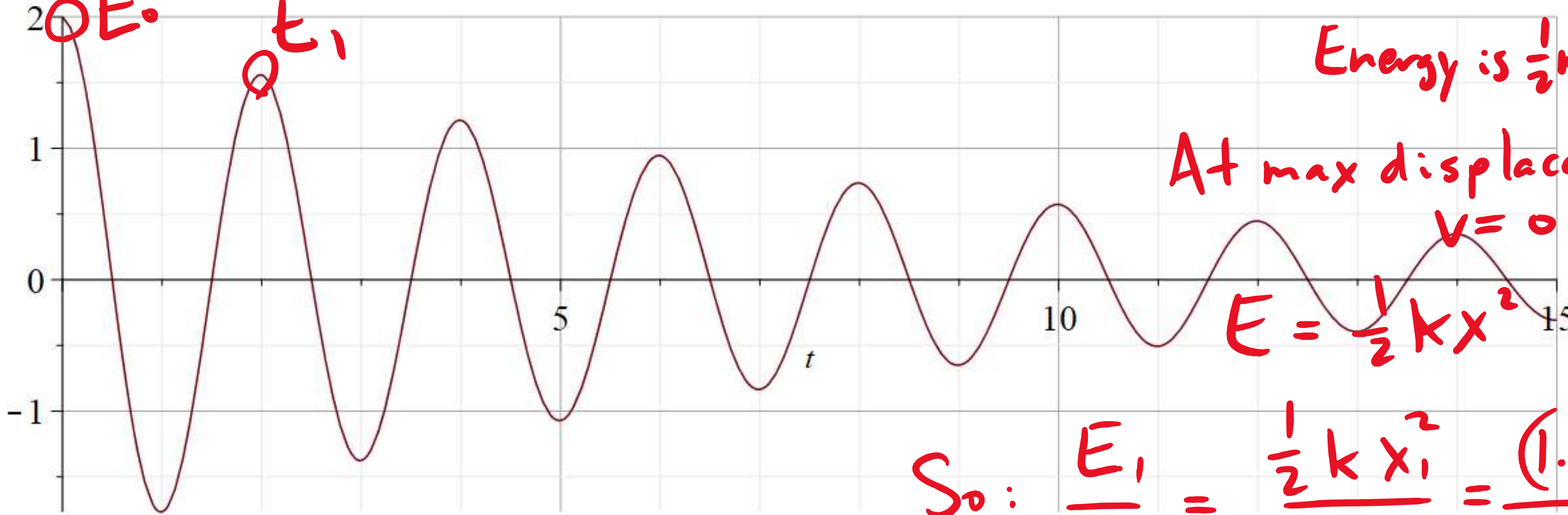


EXTRA: An object with mass 2kg oscillates on a spring with a small amount of damping.

Roughly what fraction of the energy is lost in one complete oscillation?

A) 6%　　B) 12%　　C) 23%　　D) 40%　　E) 72%

E_0 E_1



Energy is $\frac{1}{2}mv^2 + \frac{1}{2}kx^2$

At max displacement, $v = 0$ so:

$$E = \frac{1}{2}kx^2$$

So: $\frac{E_1}{E_0} = \frac{\frac{1}{2}kx_1^2}{\frac{1}{2}kx_0^2} = \frac{(1.55)^2}{2^2}$

An object with mass 2kg oscillates on a spring with a small amount of damping. ≈ 0.6

Roughly what fraction of the energy is lost in one complete oscillation?

so 40% has been lost

A) 6%   B) 12%   C) 23%   D) 40%   E) 72%