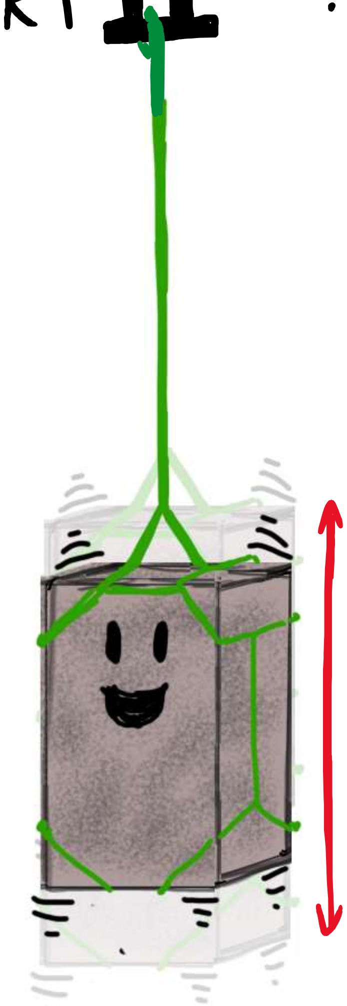
Office hours today: 4-5pm (Zoom), 8-9pm (Zoom)
Midterm Q&A: 5-7pm (Zoom link on main Canvas page)

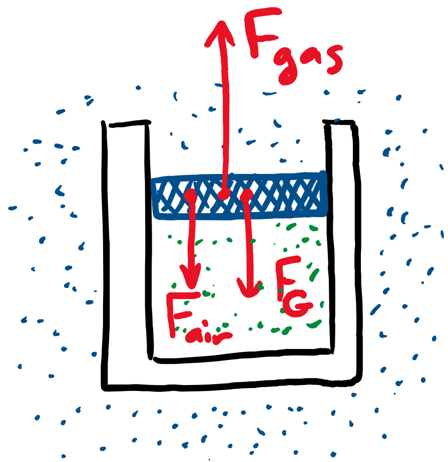
Learning Goals for Today:

- To define what is meant by mechanical equilibrium
- To explain the idea of restoring forces for a system perturbed from a stable mechanical equilibrium situation and how these lead to periodic oscillations
- To explain why Hooke's law applies in general for mechanical systems that are slightly displaced from a stable equilibrium configuration
- To mathematically describe the displacement vs time for mechanical system oscillating under the influence of restoring forces obeying Hooke's law
- To relate the parameters appearing in the sinusoidal function describing an oscillation to the physical properties of the oscillation, including the period, frequency, amplitude, and phase

Physics 157 Part II : Oscillations & Waves

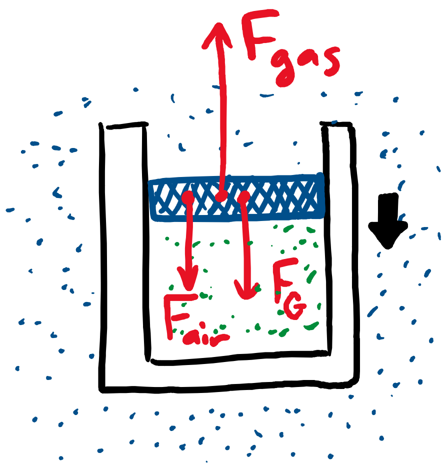


MECHANICAL EQUILIBRIUM: occurs when forces (and torques) on each part of the system add to zero



example: $\vec{F}_{gas} + \vec{F}_{gravity} + \vec{F}_{air} = 0$

- piston is in equilibrium



What happens to the forces if we move the piston downward a little (assume the cylinder is insulated)?

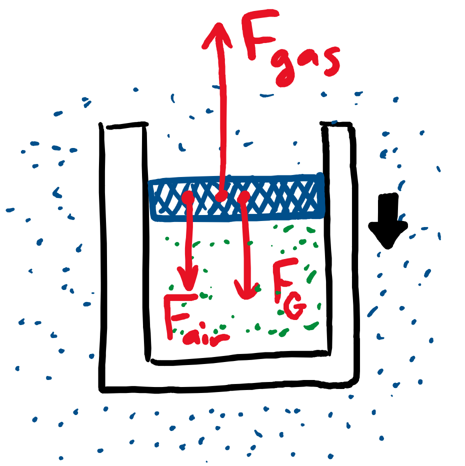
A) $|F_{gas}|$ increases a little while the other forces remain the same.

B) $|F_{gas}|$ increases a little and $|F_{air}|$ increases to compensate.

C) $|F_{gas}|$ decreases a little and the other forces remain the same.

D) $|F_{gas}|$ decreases a little and $|F_{air}|$ decreases to compensate.

E) Nothing: all forces remain the same.



Adiabatic compression:

$PV^\gamma = \text{const}$

$V\downarrow$ so $P\uparrow$

so $F_{air}\uparrow$

What happens to the forces if we move the piston downward a little (assume the cylinder is insulated)?

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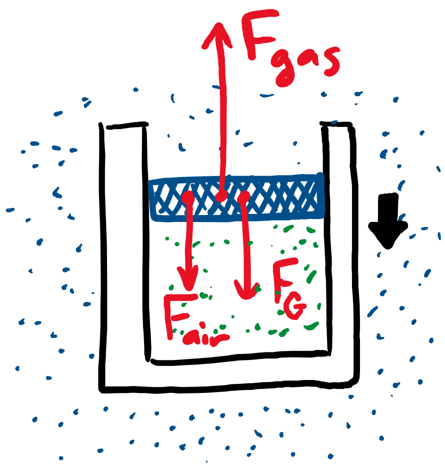
D) $|F_{gas}|$ decreases a little and $|F_{air}|$ decreases to compensate.

E) Nothing: all forces remain the same.

gravity & outside air pressure remain constant



What happens to the forces if we move the piston upward a little (assume the cylinder is insulated)?

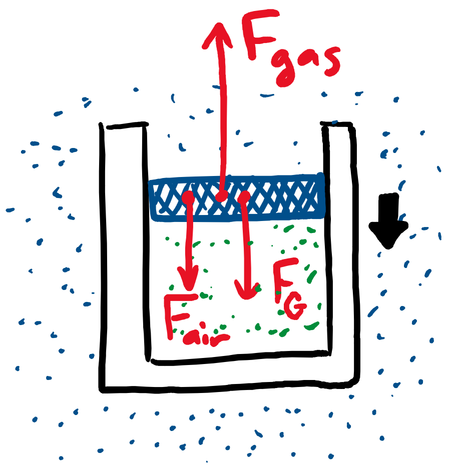
A) $|F_{gas}|$ increases a little while the other forces remain the same.

B) $|F_{gas}|$ increases a little and $|F_{air}|$ increases to compensate.

C) $|F_{gas}|$ decreases a little and the other forces remain the same.

D) $|F_{gas}|$ decreases a little and $|F_{air}|$ decreases to compensate.

E) Nothing: all forces remain the same.



Adiabatic expansion

$P\downarrow$ so $|F_{gas}|\downarrow$

What happens to the forces if we move the piston upward a little (assume the cylinder is insulated)?

A) $|F_{gas}|$ increases a little while the other forces remain the same.

B) $|F_{gas}|$ increases a little and $|F_{air}|$ increases to compensate.

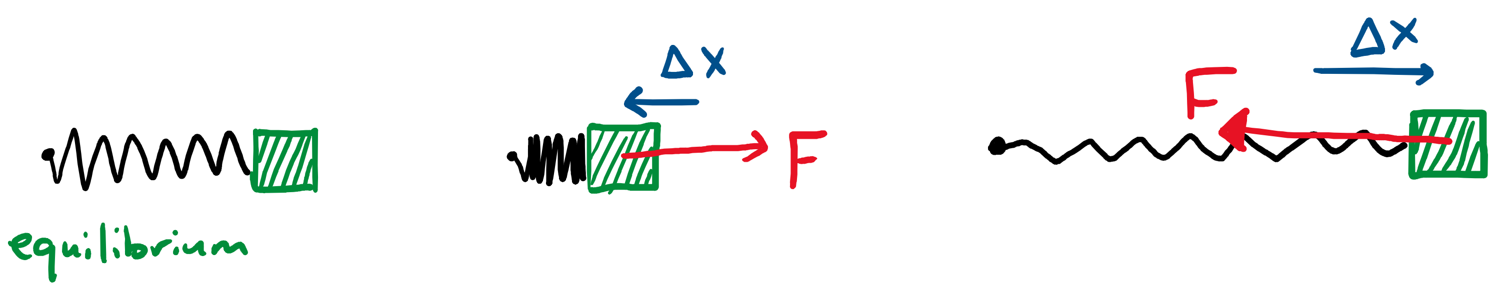
C) $|F_{gas}|$ decreases a little and the other forces remain the same.

D) $|F_{gas}|$ decreases a little and $|F_{air}|$ decreases to compensate.

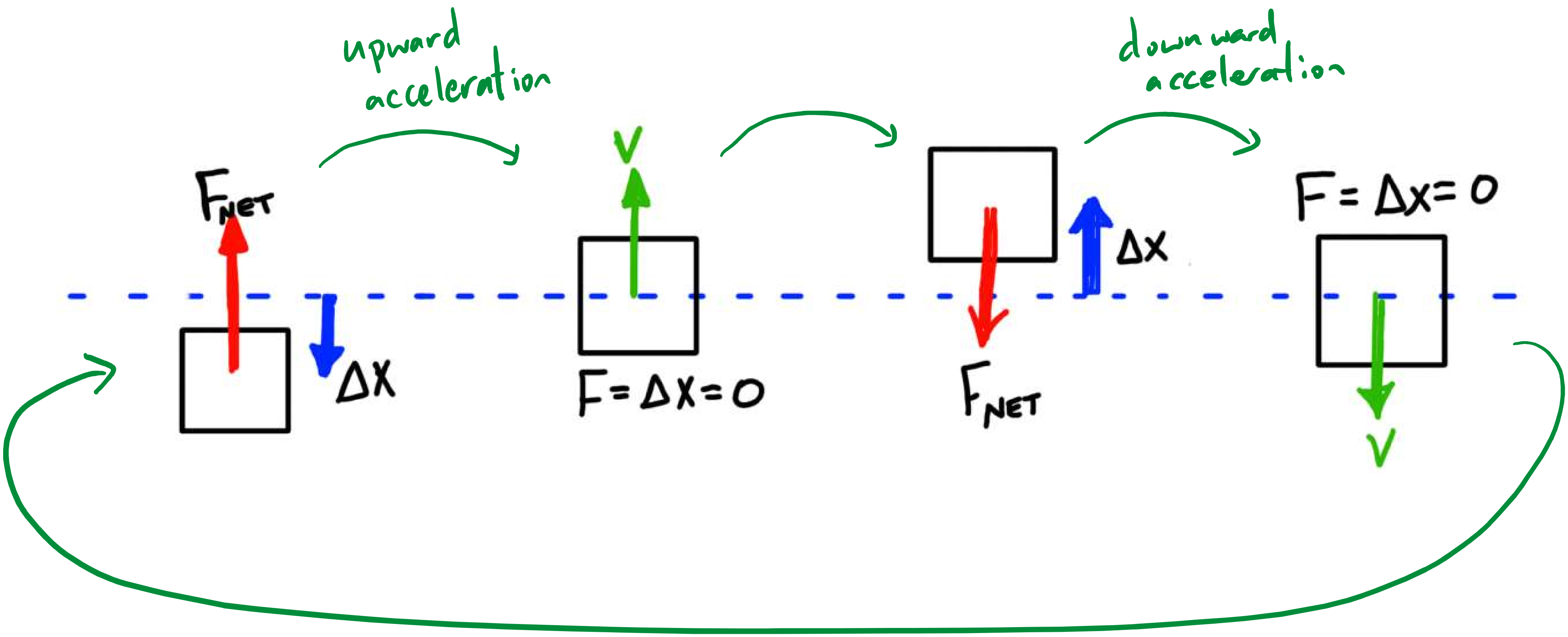
E) Nothing: all forces remain the same.

RESTORING FORCES: For a STABLE equilibrium configuration, a displacement in one direction leads to a net force in the other direction.

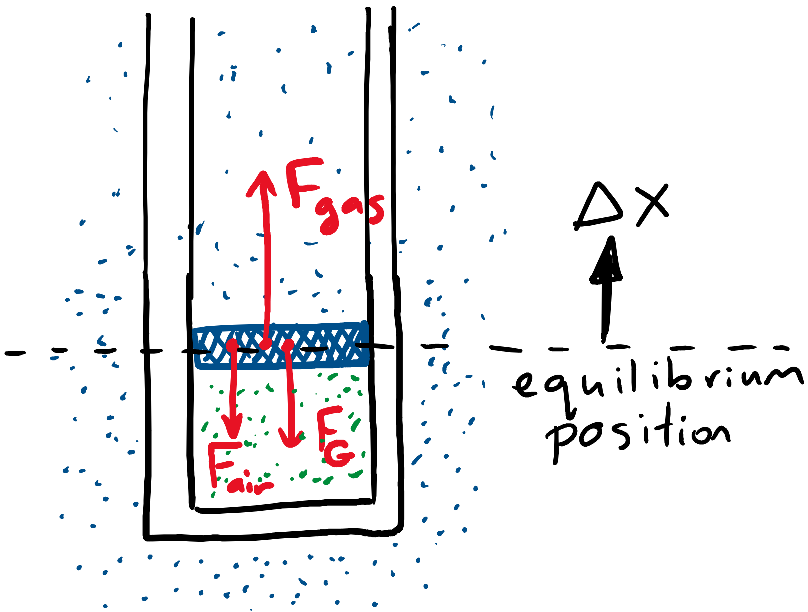
e.g.



This leads to OSCILLATIONS = periodic motion



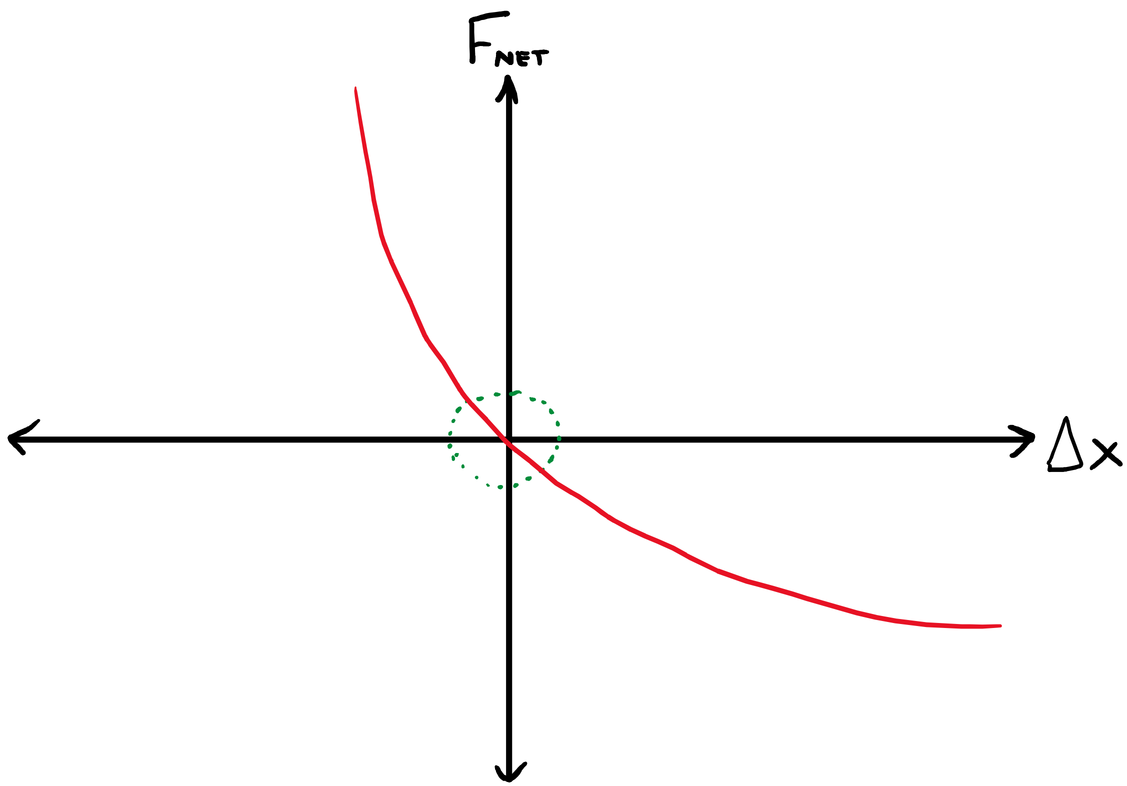
Demo



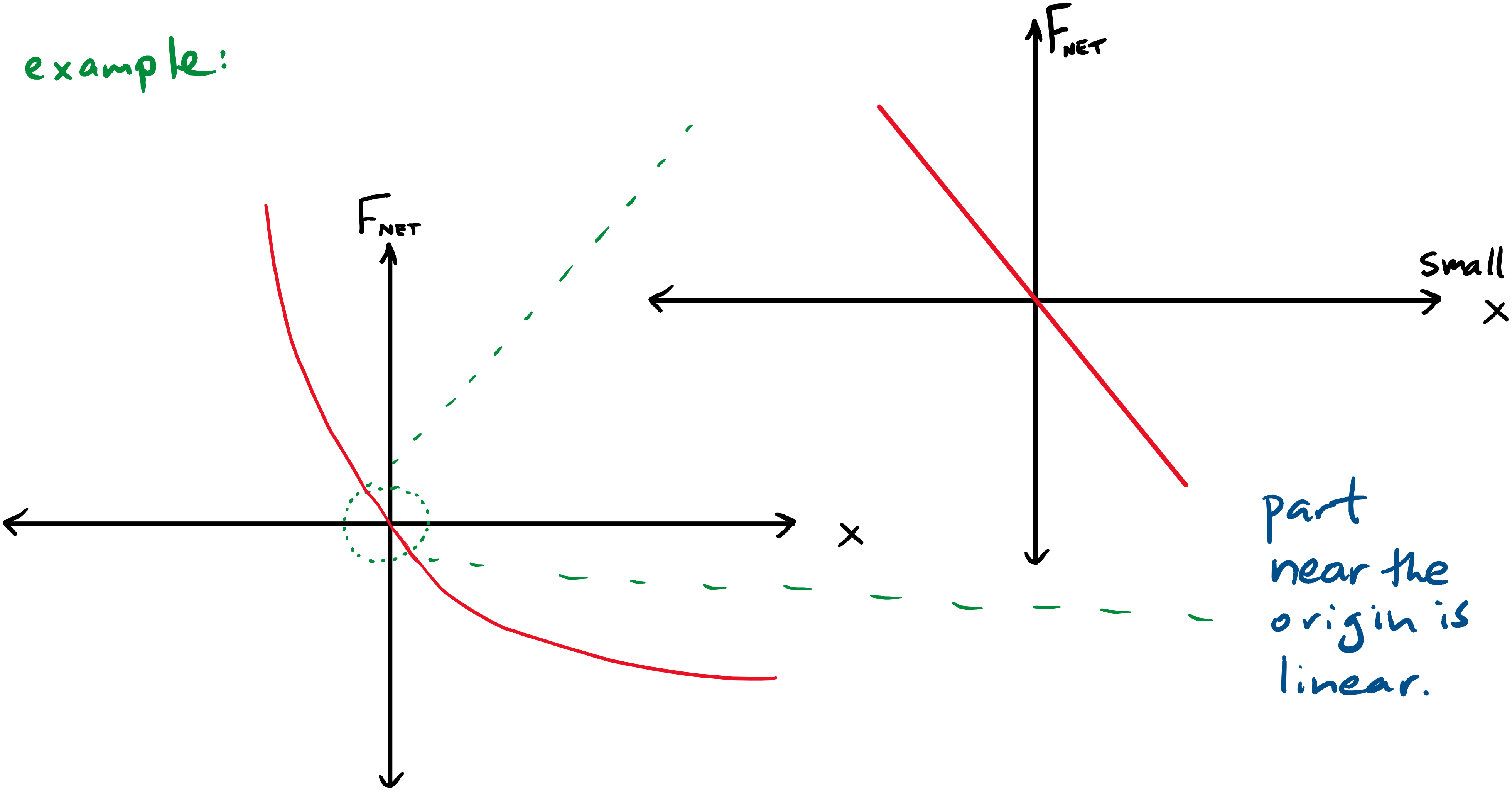
Exercise: For the system shown, sketch a graph of the net upward force on the piston as a function of the displacement Δx from the equilibrium position.

EXTRA: what does your graph look like if you zoom in to the region of small Δx. Can you write down an equation that describes F vs Δx in this region?

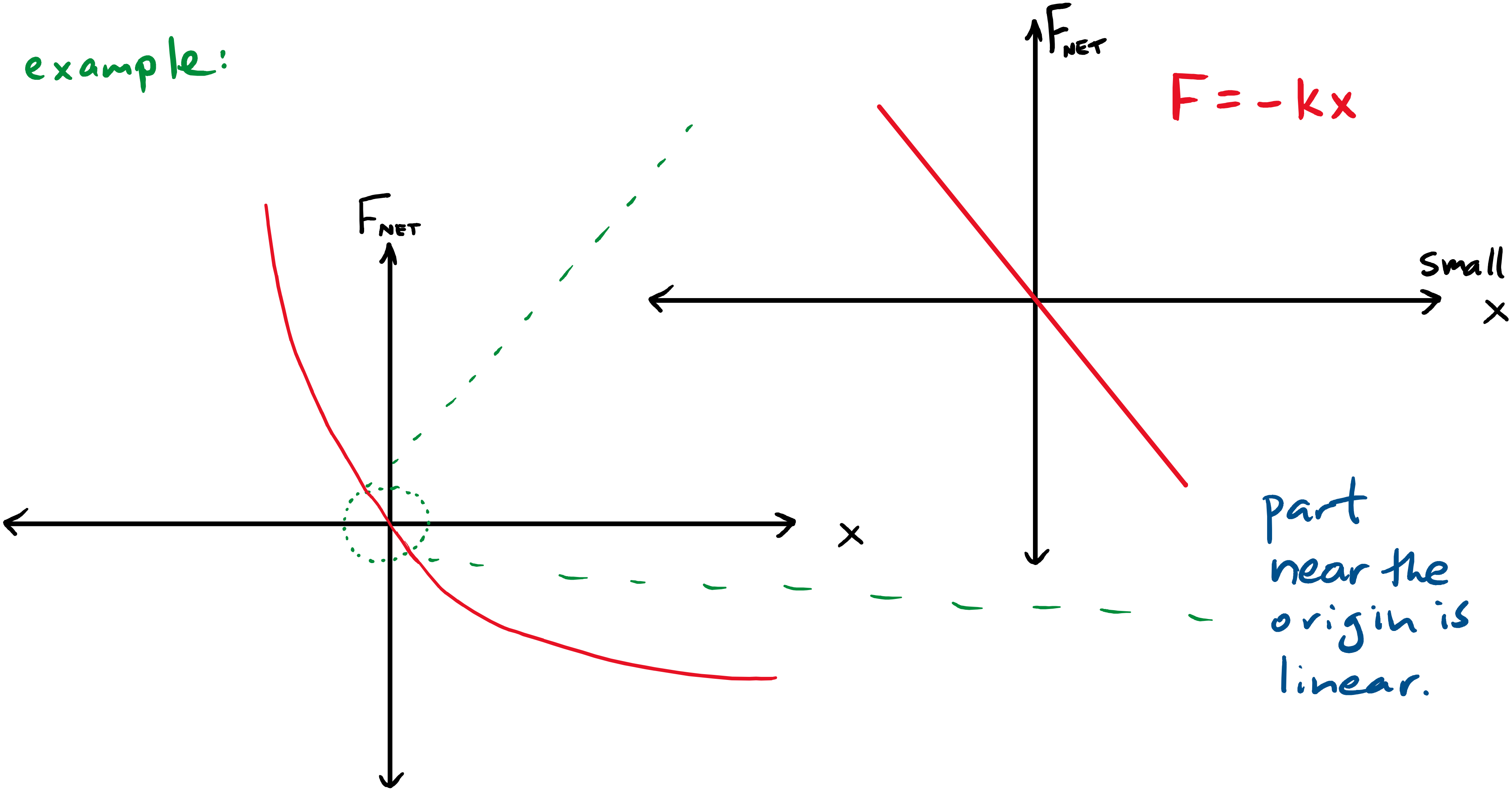
example:



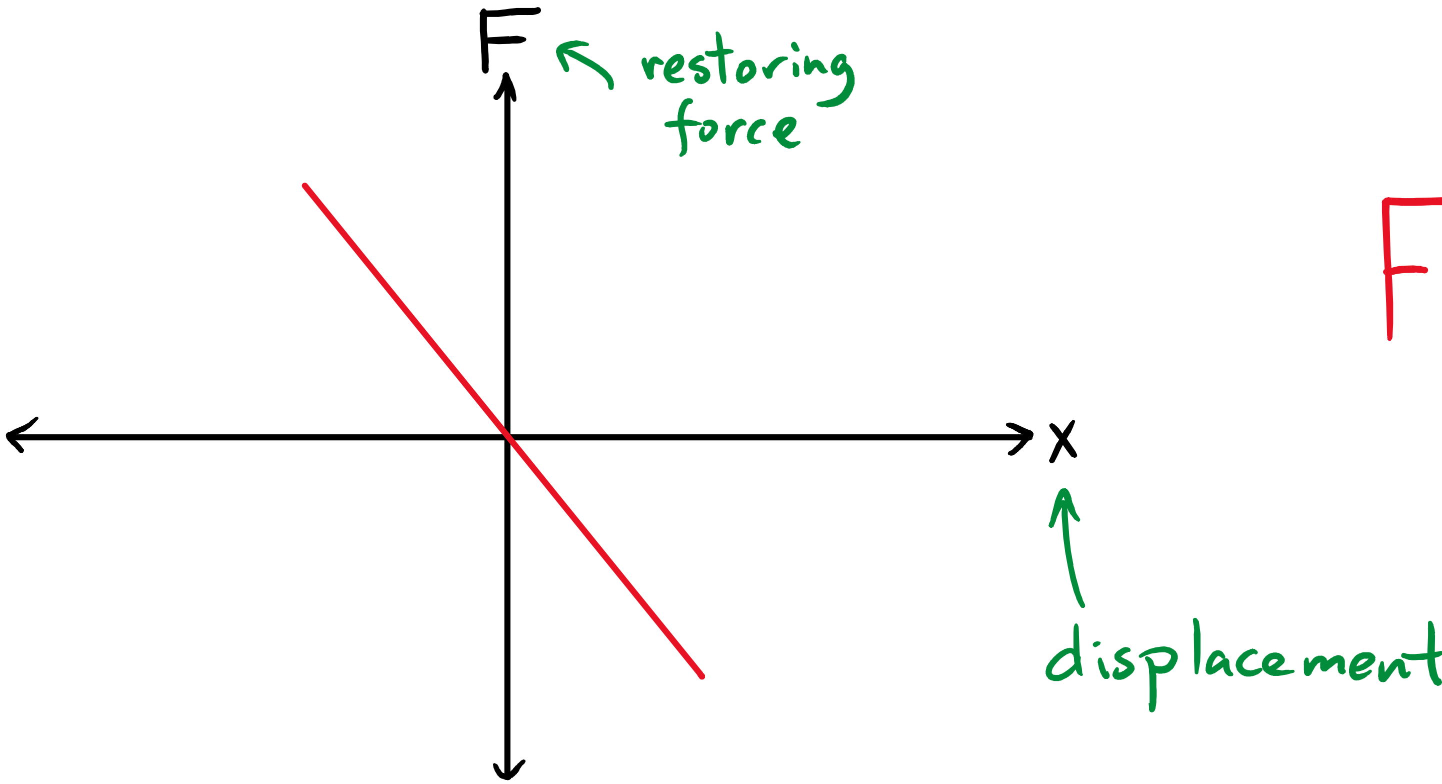
example:



example:



HOOKE'S LAW: Applies to almost any system perturbed a small amount from stable equilibrium

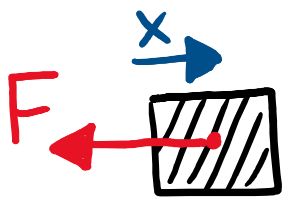


$$F = -kx$$

exact for "ideal spring"

Oscillations with Hooke's Law:

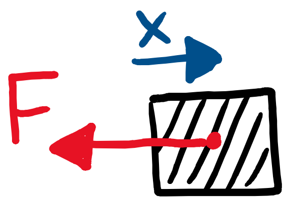
Newton: $a = \frac{F}{m}$



$F = -kx$

Oscillations with Hooke's Law:

Newton: $a = \frac{F}{m} = -\frac{k}{m} \cdot x$



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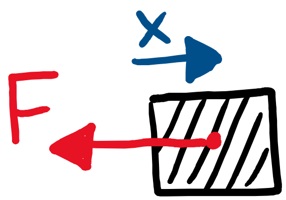
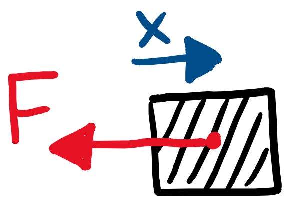


$F = -kx$

$$\frac{dv}{dt} = -\frac{k}{m} x$$

Oscillations with Hooke's Law:



$$F = -kx$$

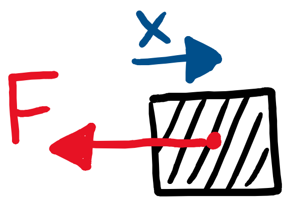
Newton: $$a = \frac{F}{m} = -\frac{k}{m}\cdot x$$

$$\frac{dv}{dt} = -\frac{k}{m}x$$

$$\frac{dx}{dt} = v$$

We can predict how velocity and position change with time.

Oscillations with Hooke's Law:



$F = -kx$

Newton: $a = \dfrac{F}{m} = -\dfrac{k}{m} \cdot x$

$$\frac{dv}{dt} = -\frac{k}{m} x$$

$$\frac{dx}{dt} = v$$

We can predict how velocity and position change with time.

Solution is $x(t) = A\cos(\omega t + \phi)$ with $\omega = \sqrt{\dfrac{k}{m}}$

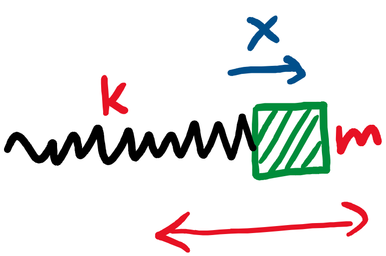
Simple Harmonic Motion

$$x(t) = A\cos(\omega t + \phi)$$

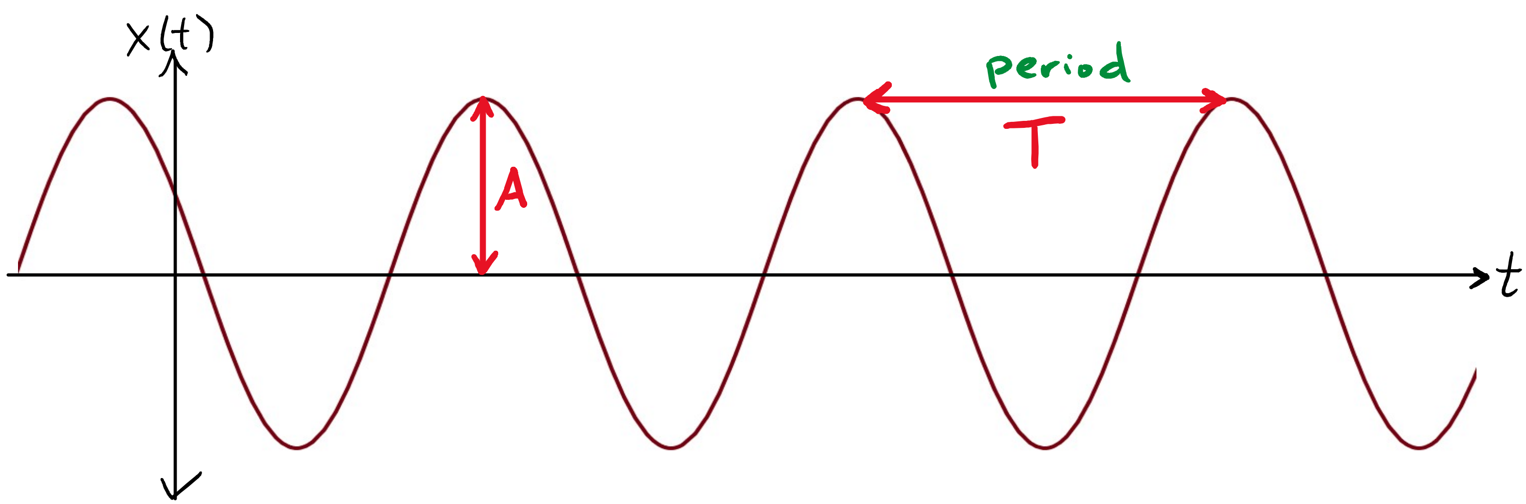
Amplitude

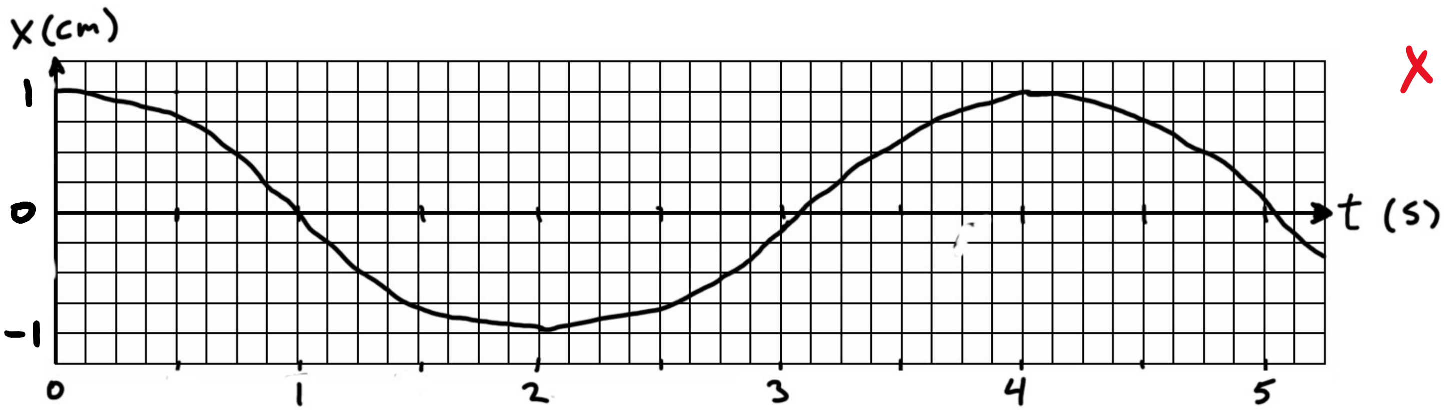
angular frequency

phase



$$\omega = \sqrt{\frac{k}{m}}$$



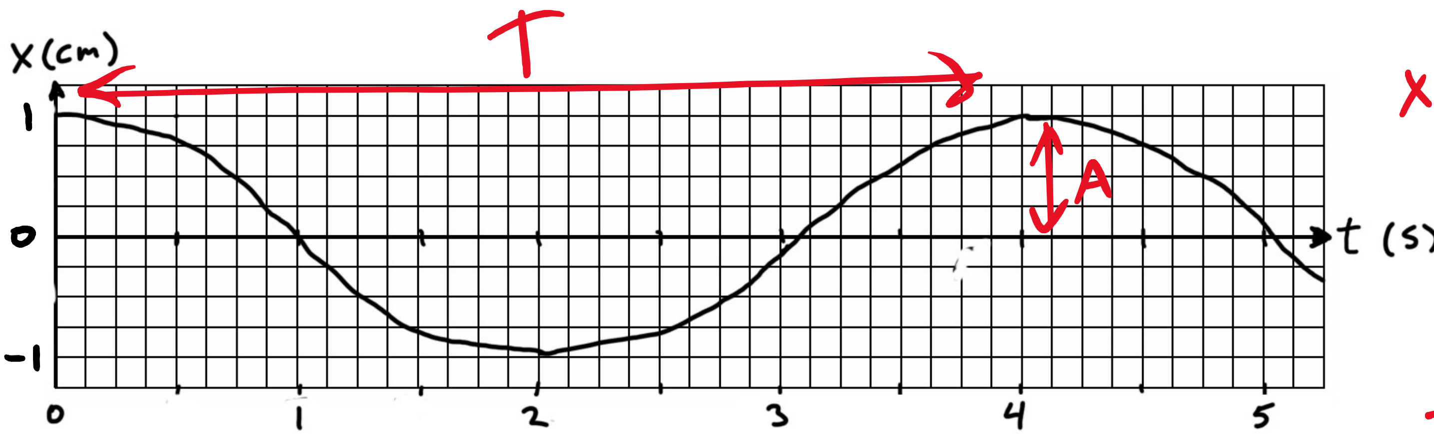


$x(t) = A\cos(\omega t + \phi)$

A plot of ***displacement*** (in cm) as a function of time (in s) is shown above. What are the ***period*** and ***amplitude*** of this simple harmonic motion?

A) T = 1s, A = 2cm
B) T = 2s, A = 2cm
C) T = 4s, A = 2cm
D) T = 2s, A = 1cm
E) T = 4s, A = 1cm

EXTRA: what is ω?



$x(t) = A\cos(\omega t + \phi)$

$A = 1\,\text{cm}$

$T = 4\,\text{s}$

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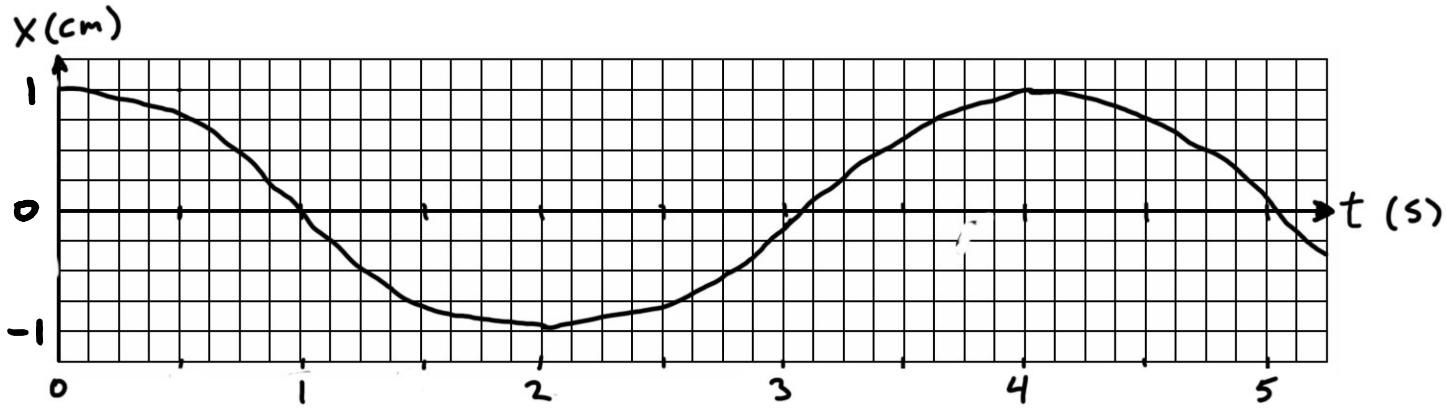
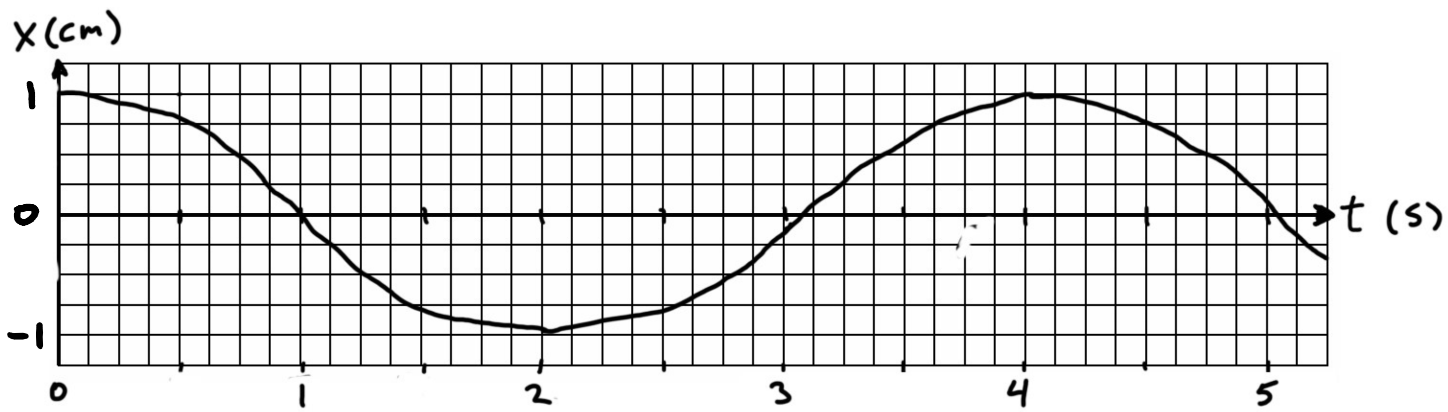


A plot of ***displacement*** (in cm) as a function of time (in s) is shown above. Which function below describes this motion?

A) $x(t) = \cos(t)$
B) $x(t) = \cos(4t)$
C) $x(t) = \cos(2\pi t)$
D) $x(t) = \cos(\pi t)$
E) $x(t) = \cos(\pi/2\ t)$



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A) x(t) = cos(t)
B) x(t) = cos(4t)
C) x(t) = cos(2 π t)
D) x(t) = cos(π t)
E) x(t) = cos(π/2 t)

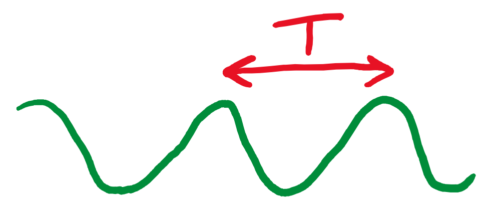
period of cos is 2π
graph is cos(ωt): when t=4s,
graph goes back to 1, so must
have ωt = 2π here.

$$\omega = \frac{2\pi}{4s} = \frac{\pi}{2}\ s^{-1}$$

Frequency & Period

angular frequency

$$x(t) = A\cos(\omega t + \phi)$$



Period T: time from max $\rightarrow$ max

$$T = \frac{2\pi}{\omega}$$ since cos repeats every 2π.

Frequency f: oscillations per time $f = \frac{1}{T}$

gives: $\omega = 2\pi f$